Intersection Management for Connected Autonomous Vehicles: A Game Theoretic Framework

Haoran Wei, Lena Mashayekhy, and Jake Papineau

Abstract— **This paper addresses the problem of optimally coordinating connected vehicles crossing an intersection without any explicit traffic signals. We propose a game-in-game framework that utilizes Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) technologies in order to maximize intersection throughput and to minimize traffic accidents and congestion. A Platoon Structure Formation Algorithm (PSFA) is proposed to form coalitions for CAVs at the intersection boundary, and a strategic game for CAVs in the interior is proposed to avoid predicted accidents inside the intersection. We perform extensive experiments to evaluate our proposed game-in-game framework under different traffic conditions. The results show that our proposed game-in-game framework reduces the accidents by 99%, while increasing the intersection throughput significantly.**

I. INTRODUCTION

The number of vehicles on the road is predicted to double from 1.1 billion to 2 billion in the next 15 years, making traffic accidents and congestion to be two major concerns. In 2015, 2.44 million nonfatal injuries and 35,092 deaths were reported on U.S. roadways. In addition, traffic congestion made U.S. drivers spend 6.9 billion hours more on the road and purchase an extra 3.1 billion gallons of fuel in 2014 [1]. By utilizing rich vehicle sensor data shared among vehicles (V2V) and between vehicles and roadside infrastructures (V2I), we can significantly reduce traffic accidents and traffic congestion.

Intersections are the primary sources of traffic bottlenecks. Traffic lights are considered as one of the most efficient ways to control traffic at intersections. However, with the emergence of connected autonomous vehicles (CAVs), a new way of coordination and cooperation among vehicles is possible leading to design of smart virtual traffic lights. CAVs are equipped with Internet access, usually with a wireless local area network, that facilitates the exchange of real-time data between transportation system users, operators, and infrastructure. CAVs will use the data shared by other vehicles and infrastructure to adjust their speeds and avoid accidents. We believe in the near future, every vehicle on the road will be connected, and thus it will be possible to remove signals from intersections, while significantly improve traffic operations and safety.

In this paper, we regulate signal-free intersections for CAVs using game theory. We propose a hierarchical game-in-game framework to improve the traffic safety by reducing collisions, while increasing the intersection throughput. This is an interconnected and layered approach considering both coalitional and non-cooperative games. In the first layer, we propose a platoon formation game at the boundary of the intersection that groups vehicles in forms of platoons and schedules them to pass the intersection in order to maximize the throughput and traffic transient smoothness. In the second layer, a strategic game at the interior is proposed that runs in realtime to avoid collisions inside the intersection. Our proposed game-in-game framework finds equilibrium solutions for intersection traffic management.

Related Work. Transportation systems are experiencing a paradigm shift with the emergence of connected and autonomous vehicles and more intelligent infrastructures. This paradigm is gaining traction after extensive research on intelligent transportation systems and electric vehicles (e.g., [2], [3], [4]). Several research efforts have focused on intersection control management using machine learning [5] and optimization methods [6]. For a survey, the reader is referred to [1]. These methods are the common mathematical tools to study centralized and decentralized systems, but they do not consider the possibility of interactions among system's elements. The common mathematical tool to study complex interactions among elements of systems is game theory.

Game theory provides an analytical framework to study conflicts and/or cooperations of systems' elements as intelligent decision makers. In the past few years, researchers mainly have used game theory models to adjust the traffic light durations in order to improve the intersection throughput [7], [8], [9]. Khanjary [7] modeled a single intersection with Cournot's oligopoly game to adjust the duration of red and green lights, where the payoff is calculated by the number of passed vehicles. Elhenawy et al. [8] proposed a 2-player game to model two conflicting drivers. However, their approach is a small-scale game that cannot be applicable to a busy traffic intersection. To the best of our knowledge, this is the first study that designs a game-in-game framework to regulate an intersection with connected vehicles.

Organization. The rest of the paper is organized as follows. In Section II, we describe the system model. In Section III, we describe our hierarchical game-in-game approach for regulating intersections. In Section IV, we evaluate the game-in-game approach by extensive experiments. In Section V, we summarize our results and present possible directions for future research.

II. PROBLEM STATEMENT

An intersection consists of two parts: a boundary zone and an interior zone (Figure 1). The region at the center of

H. Wei, L. Mashayekhy, and J. Papineau are with the Department of Computer and Information Sciences, University of Delaware, Newark, DE, 19716. {nancywhr,mlena,jakep}@udel.edu

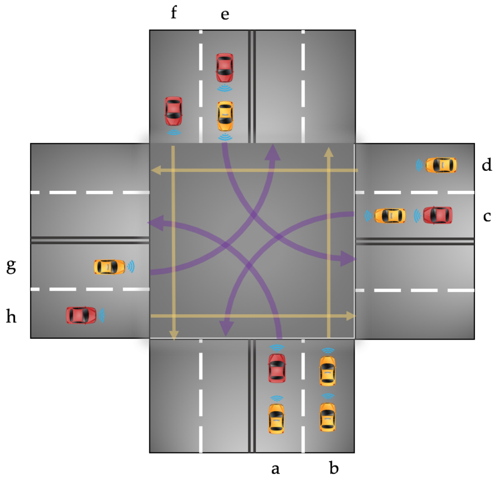


Fig. 1: A general 4-way intersection and possible conflicts

the intersection is called an interior zone and has a length of L. This is the area of potential collisions of vehicles. The intersection has a control zone, called boundary of an intersection, where CAVs can communicate with each other and the infrastructure. The distance between the entry of the boundary zone and the entry of the interior zone is B. We consider a set of CAVs, $C = \{c_1, c_2, ..., c_i, ..., c_N\}$, in an intersection. Our objective is to maximize the intersection throughput by coordinating the CAVs to cross the intersection without any collisions.

Signalization is one of the main intersection traffic management approaches for safe and efficient movement of traffic, where it assigns the right-of-way to one set of non-conflicting traffic movements at a time, forcing all movements in conflict with the active traffic movements to stop. Figure 1 shows a general four-way intersection and possible traffic conflicts. Note that right turning movements are always compatible and are eliminated in this representation. We abstract the intersection into a compatible graph representation $G(\mathcal{L}, E)$ shown in Figure 2, where vertices are lanes $\mathcal{L}$ and edges E represent their compatibility. We define the following terms:

*Definition 1 (**Compatible Lanes**):* A group of lanes not conflicting one another. For example, lane a and lane b are a pair of compatible lanes.

*Definition 2 (**Traffic Stream Rate** (r_i)):* The average number of CAVs passing lane $i \in \mathcal{L}$ at each time unit. For example, $r_a = 5$ means 5 CAVs pass lane a in each time unit (e.g., 1 min). We assume $r_i > 0$ for all lanes.

*Definition 3 (**Platoon Stream Rate** ($r_{\mathbb{P}}$)):* The summation of the traffic stream rates of a set of lanes in $\mathbb{P}$. For example, given a set of lanes $\mathbb{P} = \{a, b, d\}$ and stream rates of $r_a = 5$, $r_b = 8$, $r_d = 5$, the platoon stream rate is $r_{\mathbb{P}} = 18$.

*Definition 4 (**Throughput Threshold** (τ)):* The maximum number of vehicles that can go through the intersection during a time unit considering the lane conflicts. This number has a limit due to the capacity restriction of the intersection infrastructure.

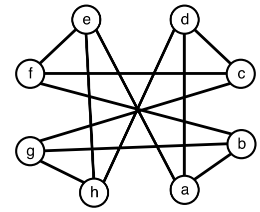


Fig. 2: Graph of traffic-compatible lanes

There are two measurements for the system's performance: safety and throughput, that are defined in Section IV.

III. Hierarchical Game-in-Game Framework

In this section, we model and formulate the intersection management for CAVs as a novel hierarchical game-in-game framework, and we describe our proposed games. We find equilibrium solutions of our proposed games for intersection traffic management.

Our proposed game-in-game framework aims to regulate a normal 4-way intersection with CAVs, specifically by sending virtual light signals to each CAV. The virtual lights signal includes stop, move forward, and speed change. All CAVs communicate with a central controller inside the intersection, and they follow the virtual signals sent by the central controller. We propose a coalitional game for CAVs in the boundary (platoon formation game) in order to maximize traffic transient smoothness and throughput. CAVs in a platoon work cooperatively for speed harmonization while keeping safe distances among themselves. We propose a strategic game for CAVs in the interior to avoid accidents. As an independent decision maker, each CAV will observe other vehicles around itself and will obtain data in real-time for the immediate future trajectories (e.g., adjusting speed).

A. Platoon Formation Game at Boundary

We now introduce the platoon formation game at the boundary as a coalitional graph game in partition form. Coalitional game theory studies the interactions between groups of players, where they can cooperate and form alliances, while ultimately trying to maximize utility. We define the *platoon formation game* as follows:

*Definition 5 (**Platoon Formation Game at Boundary**):* It is a 3-tuple $\mathcal{BG}(\mathcal{L}, G, f)$, where $\mathcal{L}$ is the set of lanes, $G(\mathcal{L}, E)$ is their compatible graph, and f is the *characteristic function* defined on any coalition $\mathbb{P} \subseteq \mathcal{L}$, such that $f : \mathbb{P} \rightarrow \mathbb{R}^+$ and $f(\emptyset) = 0$.

We define a coalition or a platoon as a subset of vehicles in the lanes. If all lanes form a coalition, we call it the *grand coalition*. We define the characteristic function of a coalition $\mathbb{P}$ as follows:

$$f(\mathbb{P}) = \begin{cases} 0 & \text{if } |\mathbb{P}| = 0 \text{ or any lanes in } \mathbb{P} \text{ conflict,} \\ r_{\mathbb{P}} & \text{if } |\mathbb{P}| > 0 \text{ and all non-conflicting} \end{cases} \quad (1)$$

The platoon formation game studies how to form platoons of CAVs considering lane conflicts. The platoon formation game should satisfy two main properties, fairness and stability. The core (the most popular solution concept of a

Algorithm 1 Platoon Structure Formation Algorithm (PSFA)

Input: $\mathcal{BG}(\mathcal{L}, G, f)$, τ
$\mathcal{PS} = \emptyset$ */*initial set of the platoon structure*/*
$\bar{G} = G$ */*the compatibility graph*/*
repeat
    $Q \leftarrow \emptyset$ */*create an empty queue*/*
    for all $i \in \mathcal{L}$ belong to $\bar{G}$ **do**
        $\mathbb{K}_i = \{i\}$ (i.e., $i \in$ group $\mathbb{K}_i$)
        $f(\mathbb{K}_i) = f(\{i\}) = r_i$ */*initialization*/*
        add i to Q */*unvisited vertices*/*
    $\mathbb{P} = \emptyset$ */*initial platoon*/*
    while Q is not empty **do**
        Find i in Q with maximum $f(\mathbb{K}_i)$
        remove i from Q
        if $f(\mathbb{K}_i) \leq \tau$ **then**
            $\mathbb{P} \leftarrow \mathbb{P} \cup i$
            $f(\mathbb{P}) += f(\{i\})$
            for all neighbor $j \in Q$ of i ($ij \in E$) **do**
                if $f(\mathbb{K}_j) < f(\{j\}) + f(\mathbb{P})$ **then**
                    $\mathbb{K}_j = \mathbb{K}_j \cup \mathbb{P}$
                    $f(\mathbb{K}_j) = f(\{j\}) + f(\mathbb{P})$
    $\mathcal{PS} = \mathcal{PS} \cup \mathbb{P}$
    update $\bar{G}$ by removing $\mathbb{P}$ from $\bar{G}$
until all vertices are assigned to platoons in $\mathcal{PS}$
output: $\mathcal{PS}$

coalitional game) of the platoon formation game is empty due to the lane conflicts and the capacity restriction of the intersection. As a result, the grand coalition is not stable and does not form, leading to the formation of independent and disjoint coalitions.

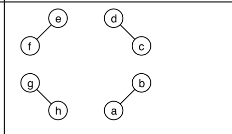
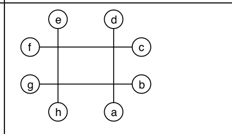
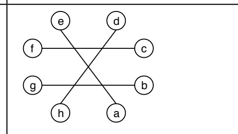
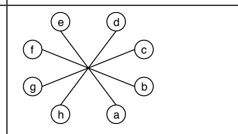
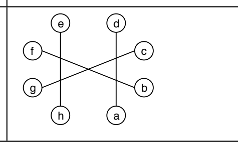
*Definition 6 (**Platoon structure**):* A platoon structure $\mathcal{PS} = \{\mathbb{P}_1, \mathbb{P}_2, \ldots, \mathbb{P}_h\}$ is a partitioning of $\mathcal{L}$ such that each lane is a member of exactly one coalition, *i.e.*, $\mathbb{P}_i \cap \mathbb{P}_j = \emptyset$ for all i and j, where $i \neq j$ and $\bigcup_{\mathbb{P}_i \in \mathcal{PS}} \mathbb{P}_i = \mathcal{L}$.

We denote by Π the set of all platoon structures. In the next subsection, we present our proposed algorithm for forming a stable platoon structure, and we investigate its properties.

B. Platoon Structure Formation Algorithm

We propose a Platoon Structure Formation Algorithm (PSFA) to form a stable platoon structure for the proposed game. The proposed algorithm maximizes the throughput of the intersection considering the capacity of the intersection and avoiding accidents. In doing so, the algorithm forms independent and disjoint platoons of CAVs maximizing the platoon stream rates of the formed platoons considering the throughput threshold constraint. Our proposed algorithm, PSFA, is given in Algorithm 1. The algorithm receives the compatible graph of an intersection and the throughput threshold as the inputs. The algorithm finds a vertex i with the highest traffic stream rate and marks all the other vertices as unvisited. This vertex i is the first member of a platoon $\mathbb{P}_1$. Then, the algorithm iteratively explores the neighbors of the current vertex and calculates the platoon stream rates considering if the neighbor vertex of any vertex in $\mathbb{P}_1$ joins the platoon. The group of vertices whose platoon stream rate is mostly closed to the threshold form the final platoon $\mathbb{P}_1$, and they are removed from the compatible graph for the next iteration to find $\mathbb{P}_2$. After all iterations are explored, the platoon structure $\mathcal{PS} = \{\mathbb{P}_1, \mathbb{P}_2, \ldots, \mathbb{P}_h\}$ is the output of the algorithm.

TABLE I: Coalition Formation Baseline

	compatible combination	baseline	throughputs
baseline-1		{a, b} {c, d} {e, f} {g, h}	$\{13, 7, 6, 11\}$
baseline-2		{a, d} {h, e} {c, f} {g, b}	$\{10, 10, 6, 11\}$
baseline-3		{a, e} {h, d} {c, f} {g, b}	$\{7, 13, 6, 11\}$
baseline-4		{a, e} {h, d} {c, g} {f, b}	$\{7, 13, 5, 12\}$
baseline-5		{h, e} {a, d} {b, f} {g, c}	$\{10, 10, 12, 5\}$

For example, consider an intersection shown in Figure 1 with the traffic stream rate for each lane listed in an alphabetic order as: $\{5, 8, 2, 5, 2, 4, 3, 8\}$. The formed platoon structure consists of $\mathbb{P}_1 = \{c, d, h\}$, $\mathbb{P}_2 = \{a, b, e\}$ and $\mathbb{P}_3 = \{f, g\}$.

Notice that the platoon formulation game aims at maximizing the platoon stream rates in order to increase the throughput of the intersection. However, the formed platoon structure may have platoons with conflicting lanes. For example, in $\mathbb{P}_1$, lane h and lane c conflict. We define a strategic game in Section IV.D that coordinates the CAVs in a platoon by solving the conflicts inside each platoon.

A baseline for the throughput maximization problem is to form all compatible groups without any conflicts and find a group with the highest throughput. For the intersection in Figure 1, there are 5 possible compatible groups formation to consider (shown in the first column of Table I). To show the efficiency of platoon formation using PSFA, we will compare its performance with these baselines.

C. Intersection Management

For all the formed platoons in $\mathcal{PS}$, we need to find their time intervals to manage the intersection. The time interval of a platoon is a duration for a virtual green light that CAVs of that platoon are allowed to pass the intersection. For a platoon $\mathbb{P} \in \mathcal{PS}$, its time interval is denoted by $x_{\mathbb{P}}$. We define W as an upper bound on the summation of the interval durations, representing the traffic signal cycle. Each interval's duration is also bounded to be within a range $[b_l, b_h]$.

To maximize the throughput of the intersection across all intervals, while satisfying the above mentioned constrains, we formulate the problem as a Linear Optimization Program as follows:

$$\begin{aligned} \text{Maximize} \quad & \sum_{\mathbb{P} \in \mathcal{PS}} r_{\mathbb{P}} x_{\mathbb{P}} \\ \text{s.t.} \quad & \sum_{\mathbb{P} \in \mathcal{PS}} x_{\mathbb{P}} \leq W \\ & b_l \leq x_{\mathbb{P}} \leq b_h, \quad \forall \mathbb{P} \in \mathcal{PS} \end{aligned} \tag{2}$$

where $x_{\mathbb{P}}$ is a decision variable. The number of CAVs passing the intersection during an interval $x_{\mathbb{P}}$ is $N_{\mathbb{P}} = r_{\mathbb{P}} x_{\mathbb{P}}$, and the number of CAVs passing the intersection from lane $i \in \mathbb{P}$ is $N_i = r_i x_{\mathbb{P}}$.

D. CAVs Strategic Game at Interior

We depict all lanes' trajectories inside the intersection (shown in Figure 1), and the crossing points of trajectories are hot spots for potential accidents. Notice that each trajectory only crosscuts one another at each time, thus we only consider the crash between two vehicles. To avoid possible accidents, we formulate a 2-player strategic game, where each pair of CAVs adjusts their speeds once a potential accident predicted according to the sensory observation and prediction. A detector runs on each CAV in real-time and is responsible for observing nearby connected vehicles, predicting the possible accident, and triggering the strategic games.

We propose a strategic game at interior $\mathcal{IG} = \langle \mathbb{C}, (\mathcal{U}_i)_{i\in\mathbb{C}}, (s_i)_{i\in\mathbb{C}} \rangle$, where $\mathbb{C}$ is the player set such that $\mathbb{C} = [C]^2 = \{\{i,j\} | c_i, c_j \in C, c_i \neq c_j\}$ for a 2-player game, $\mathcal{U}_i$ is the utility function (payoff) of CAV $i \in \mathbb{C}$, and s_i is the strategy (speed assignment) of CAV i. Each CAV is able to change its speed with a constant value, denoted by δ, at any time. For CAV i with speed v at time t, its set of strategies is defined as $S_i^t = \{v_{it}^-, v_{it}, v_{it}^+\}$, where $v_{it}^- = v_{it} - \delta$ and $v_{it}^+ = v_{it} + \delta$. We define S_i as the set of feasible strategies (i.e., the strategy space) of CAV i. In addition, $s = (s_i, s_j) \in S$ is a strategy profile (list of strategies for each CAV), where $S = S_i \times S_j$.

When CAV i chooses a strategy, then the CAV obtains a payoff $U_i(s)$. The payoff depends on the strategy profile chosen (i.e., on the strategy chosen by CAV i and the strategies chosen by all the other players). More specifically, the utility (payoff) of CAVs is impacted by the possibility of accident, change in travel time, and cost of speed change. After an accident is predicted, if CAVs choose any strategies and an accident still can happen, those CAVs will be charged ∞ ($U_i = \infty$). When a CAV chooses to decelerate, it also forces others CAVs behind that to slow down and can lead to increase in the total traffic time, and thus it may negatively impact the utility of that CAV. We will analyzing the payoff functions of the CAVs participating in our proposed Strategic Game at Interior in the next subsection.

We use the Nash Equilibrium as the solution concept of our proposed game. A strategy profile $s^* \in S$ is a Nash equilibrium if no unilateral deviation in strategy by any CAV

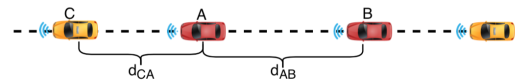


Fig. 3: Scenario 1: Predicted accident on the same lane

is profitable for that CAV:

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*) \quad \forall i, s_i \in S_i \tag{3}$$

where s_{-i} is a strategy profile of all players except for CAV i. Our proposed game $\mathcal{IG}$ has a Nash Equilibrium following Theorem 1.

*Theorem 1 (**Nash Equilibrium Existence**):* If $\mathbb{C}$ is finite and S_i is finite for every CAV i, then our proposed strategic game $\mathcal{IG} = \langle \mathbb{C}, (\mathcal{U}_i)_{i\in\mathbb{C}}, (s_i)_{i\in\mathbb{C}} \rangle$ has at least one Nash Equilibrium.

If there are multiple Nash Equilibria, we select the one with the maximum sum of the payoffs as the solution of the game. The final solution of the game is the Nash Equilibrium $(\widetilde{s}_1^*, \cdots, \widetilde{s}_{\mathbb{C}}^*) = \arg\max \sum_{i\in\mathbb{C}} U_i(s_i^*, s_{-i}^*)$, where $\widetilde{s}_i^*$ is the final strategy for CAV i.

The new speeds of the CAVs are based on the final strategies corresponding to a Nash Equilibrium.

E. Analyzing the Strategic Game at Interior

Now, we analyze the game considering an accident has been predicted. An accident can be caused by i) CAVs on the same lane, and ii) CAVs on two conflicting lanes. In each scenario, we will define the payoff function of the CAVs and will present the Nash Equilibrium of the game.

Scenario 1: Predicted accident on the same lane: An accident between two CAVs on the same lane is predicted to happen if the front CAV decelerates or the back CAV accelerates. Figure 3 shows this scenario, where CAVs A and B are the two participants in a predicted accident (either A decelerated or B accelerated or both). We simply represent each CAV's strategies as $\{v^-, v, v^+\}$. Based on the predicted accident, CAVs A and B can choose different strategies as follows:

- CAV A speeds up. CAV B can choose to not change its speed or to slow down. The problem is that if A increases its speed, it possibly causes another accident with CAV C, which is in front of CAV A. As a result, we measure when such an accident can happen (between A and C). The sooner they will crash, the worse the payoff will return to CAV A.
- CAV A slows down or does not change its speed. If B slows down, all other vehicles may need to slow down consequently because of the platoon speed harmonization. If B does not change speed, then A and B will have an accident based on the item above, and this is reflected in the payoff function.

Now we describe these cases in detail.

CAV A accelerates. If A chooses to speed up, it may cause an accident with its front CAV C. As a result, this needs to be reflected in the payoff function. The distance between

TABLE II: Payoff Table of the Strategic Game (Scenario 1)

		CAV B		
		v_B^-	v_B	v_B^+
CAV A	v_A^-	$-\infty$,$-\infty$	$-\infty$,$-\infty$	$-\infty$,$-\infty$
	v_A	0,$-\frac{5}{6}$	$-\infty$,$-\infty$	$-\infty$,$-\infty$
	v_A^+	-5, $-\frac{5}{6}$	-5, 0	$-\infty$,$-\infty$

CAVs A and C is d_{CA}, and their current speeds are v_A and v_C. CAV A increases its speed to v_A^+ at time t and CAV C's speed is v_C:

If $v_A^+ \leq v_C$, then CAV A receives a payoff of ∞ since no accident will be caused while traveling time will also decrease.

If $v_A^+ > v_C$, a crash is predicted to happen in time $t + t^p$, where t^p is calculated as:

$$t_A^p = \frac{d_{CA}}{v_A^+ - v_C} \tag{4}$$

CAV A is penalized for the future crash with CAV C, and its payoff is calculated as:

$$U_A(v_A^+) = -\frac{H}{t_A^p + \epsilon} \tag{5}$$

where H is a constant cost and ϵ is a tuning parameter. If the accident would happen at the moment (i.e., $t_A^p = 0$), the payoff is $-\infty$, while the payoff increases with a later accident time (i.e., higher values of t_A^p).

CAV A does not change its speed. If A chooses to not change its speed, we need to analyze CAV B's choices. When CAV B chooses to slow down, other vehicles behind B whose speeds are higher then v_B^- may need to decrease their speeds in order to avoid future accidents. Therefore, the payoff is calculated based on the average time delay on passing the intersection using the following:

$$U_B(v_B^-) = \frac{L}{v_B} - \frac{L}{v_B^-} \tag{6}$$

where the L is the length of the interior of the intersection. Note that the length of the turning trajectories are very close to the length of the straight trajectories.

Considering a situation as an example, where $v_B = 3$, $v_A = 2$, $v_C = 2$ and the speed of all CAVs following B is 2. The distances between CAVs C, A, and B are $d_{CA} = 2$ and $d_{AB} = 1$. Having $L = 5$, $\delta = 1$, $\epsilon = 0$, and $H = 10$, the payoff table of the game between CAVs A and B is shown in Table II. The game has two Nash Equilibria (NE), (v_A, v_B^-) and (v_A^+, v_B) with the total payoffs of $-\frac{5}{6}$ and -5, respectively. Therefore, the final solution of the game is (v_A, v_B^-), which shows the new speeds of the CAVs to avoid an accident.

Scenario 2: Predicted accident on the conflicting lanes: Figure 4 shows this scenario, where CAVs A and D are predicted to crash shortly and considered as the players. The predicted crash point is labeled as I. If both CAVs choose to change their speeds ($v_A^+ = v_D^+$ or $v_A^- = v_D^-$) and the accident is still predicted to happen, their payoffs are set to $-\infty$. Considering any pair of conflicting CAVs i and $j \in \{A, B, \ldots, F\}$, the payoffs are calculated as follows:

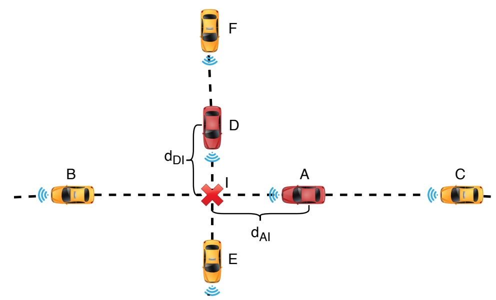


Fig. 4: Scenario 2: Predicted accident on two crossing lanes

$$U_i(v_i^+) = \begin{cases} -\frac{H}{t_p + \epsilon} & v_i \neq v_j \\ -\infty & v_i = v_j \end{cases} \tag{7}$$

$$U_i(v_i^-) = \begin{cases} \frac{L}{v_i} - \frac{L}{v_i^-} & v_i \neq v_j \\ -\infty & v_i = v_j \end{cases} \tag{8}$$

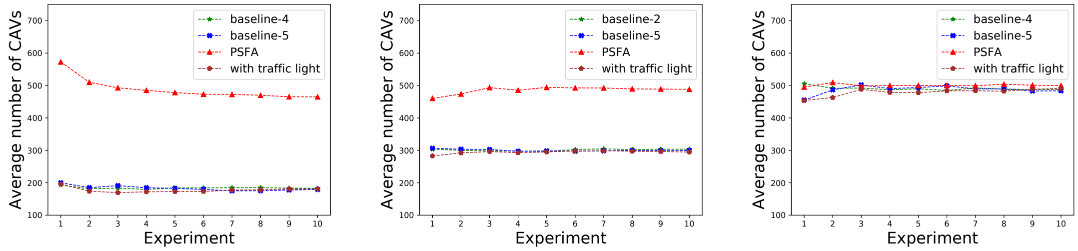
IV. Experimental Results

A. Experimental Setup

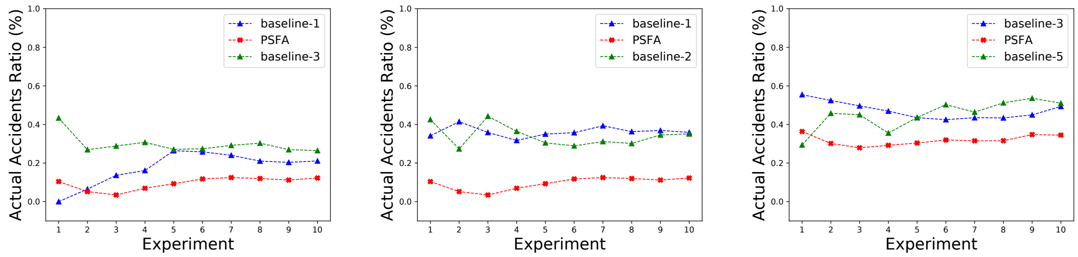
We build a 4-way intersection simulator considering the intersection layout presented in Figure 1. The intersection's size is 25meter $\times$ 25meter. Each vehicle's size is 2meter $\times$ 5meter, and the safety distance between two vehicles is $\geq$ 0.5 meters. Following [10], we classify traffic conditions by traffic stream rates: light traffic, heavy traffic, and jam traffic. If the average jam density is 150 vehicles per lane per kilometer ($\approx$ 240 vehicles/lane/mile) [10], and the average speed is assumed to be 45km/h($\approx$ 30mph), then the stream rate is $\approx$ 20 vehicles per lane per 10 seconds accordingly. With the average speed of 30mph, the stream rates (vehicles/lane/10 sec) for three conditions (light, heavy, and jam) are [1,10], [10,15], and [15,25], respectively, considering their densities [60,120], [120,180], [180,300] (vehicles/lane/mile). Each CAV arrives to the boundary with a speed, uniformly distributed between 20mph-50mph. We set the throughput threshold to 50 vehicles/10 sec for all lanes. Finally, the traffic light's cycle is 120 seconds.

B. Analysis of Results

We conduct 10 independent experiments and 20 traffic rounds in each experiment. The system's performance is measured by 1) the average throughput in each traffic round; and 2) the ratio of the number of actual accidents to the total number of predicted accidents (without using our proposed strategic game). The throughout is the total number of vehicles passing the intersection safely. We compared the PSFA's performance with the baselines 1-5 presented in Table I at the boundary. PSFA and the baselines are implemented along with the CAV Strategic Game at the interior to minimize the number of accidents at the intersection. We selected the top



(a) Light Traffic: Throughput (b) Heavy Traffic: Throughput (c) Jam Traffic: Throughput



(d) Light Traffic: Accident (e) Heavy Traffic: Accident (f) Jam Traffic: Accident

Fig. 5: Comparison between throughput and accident under light, heavy and jam traffic conditions

two baselines in terms of performance to compare with the PSFA. The results are shown in Figure 5. We also presented the results of current practice, which uses only traffic light signals (called: with traffic light).

Using PSFA, the performance is dramatically improved compared with that of the baselines and with the traffic light only. The throughput with traffic light signal is marginally increased by the baselines. However, there is a significant improvement when using our game-in-game framework (PSFA). Under the light traffic condition, the throughput increases over 2.4 times as shown in Figure 5a (from ≈ 180 CAVs/lane/round to $\approx$ 430 CAVs/lane/round). Under the heavy traffic condition, the increase in throughput reaches to ≈ 1.43 times as shown in Figure 5b. Even under the jam condition, there are still over 20 vehicles able to go through the intersection at each traffic round (Figure 5c).

Figures 5d-5f show that the accident ratio of PSFA is always lower than that of the baselines. This ratio is below 0.15% for PSFA under light and heavy conditions and lower than 0.38% under jam condition. The results show that using our proposed game-in-game framework, 99% of the accidents can be avoided without the existence of a traffic light.

V. Conclusion

In this paper, we proposed a hierarchical game-in-game framework to regulate signal-free intersections for CAVs. We measured the intersection performance by the throughput and ratio of accidents under three traffic conditions. The results demonstrate that our proposed framework efficiently improves the traffic safety, while significantly increases the intersection throughput. For our future work, we plan to investigate more complex situations, where a more than 2-player strategic game needs to be defined.

Acknowledgment. This research was supported in part by NSF grant CNS-1755913.

References

[1] J. Rios-Torres and A. A. Malikopoulos, “A survey on the coordination of connected and automated vehicles at intersections and merging at highway on-ramps,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 5, pp. 1066–1077, 2017.

[2] M. Nejad, L. Mashayekhy, R. Chinnam, and A. Phillips, “Hierarchical time-dependent shortest path for routing on dynamic road networks under its,” *IIE Transactions on Scheduling and Logistics*, vol. 48, no. 2, pp. 158–169, 2016.

[3] M. Nejad, L. Mashayekhy, R. Chinnam, and D. Grosu, “Online scheduling and pricing for electric vehicle charging,” *IIE Transactions on Scheduling and Logistics*, vol. 49, no. 2, pp. 178–193, 2017.

[4] M. Nejad, L. Mashayekhy, D. Grosu, and R. Chinnam, “Optimal routing for plug-in hybrid electric vehicles,” *Transportation Science*, vol. 51, no. 4, pp. 1304–1325, 2017.

[5] P. Mannion, J. Duggan, and E. Howley, “Parallel reinforcement learning for traffic signal control,” *Procedia Computer Science*, vol. 52, pp. 956–961, 2015.

[6] N. Shah, S. Kumar, F. Bastani, and I.-L. Yen, “Optimization models for assessing the peak capacity utilization of intelligent transportation systems,” *European Journal of Operational Research*, vol. 216, no. 1, pp. 239–251, 2012.

[7] M. Khanjary, “Using game theory to optimize traffic light of an intersection,” in *Proc. of the 14th IEEE International Symposium on Computational Intelligence and Informatics*, 2013, pp. 249–253.

[8] M. Elhenawy, A. A. Elbery, A. A. Hassan, and H. A. Rakha, “An intersection game-theory-based traffic control algorithm in a connected vehicle environment,” in *Proc. of the 18th International IEEE Conference on Intelligent Transportation Systems*, 2015, pp. 343–347.

[9] L. Banjanovic-Mehmedovic, E. Halilovic, I. Bosankic, M. Kantardzic, and S. Kasapovic, “Autonomous vehicle-to-vehicle (V2V) decision making in roundabout using game theory,” *Int. J. Adv. Comput. Sci. Appl*, vol. 7, pp. 292–298, 2016.

[10] V. L. Knoop and W. Daamen, “Automatic fitting procedure for the fundamental diagram,” *Transportmetrica B: Transport Dynamics*, vol. 5, no. 2, pp. 129–144, 2017.