

Effects of Traffic Network Dynamics on Hierarchical Community-based Representations of Large Road Networks

Mahyar Movahed Nejad, Lena Mashayekhy, and Ratna Babu Chinnam

Abstract—Efficient representation of traffic networks, including congestion states, plays an important role in the effectiveness of routing algorithms incorporating Intelligent Transportation Systems (ITS) data. We employ an emerging concept in analyzing complex networks called “community structure detection” to capture traffic network dynamics in the form of hierarchical community-based representations of road networks. A key strength of these community (structure) detection methods is their computational efficiency. We investigate the impact of traffic dynamics on the hierarchical community-based representations of large road networks. The resulting hierarchical community representations and their evolution over varying traffic conditions with time can aid the computational performance of real-time routing algorithms. We analyze the performance of hierarchical community detection methods on the metropolitan road networks of New York City, Detroit, and San Francisco Bay area.

I. INTRODUCTION

EFFICIENT representation of traffic networks, including congestion states, plays an important role in the effectiveness of routing algorithms incorporating real-time Intelligent Transportation Systems (ITS) data. While road transportation network capacity is not growing fast enough to cope with increasing demand [1], the quickly expanding ITS coverage in the US can be a key enabler for reducing or controlling traffic congestion. The ITS coverage in the US and around the world provides valuable data about traffic network dynamics, and this requires routing algorithms to not only extract information from traffic data in real-time but also incorporate this information for a possible change in the optimal route and response in real-time.

Current state-of-the-art dynamic routing algorithms are incapable of computing these updated directions in an acceptable time as the network size increases. This is particularly true for algorithms that attempt to account for the non-stationary aspects of traffic network dynamics (fluctuations in traffic speeds/densities over time and/or explicit treatment of congestion states).

Dynamic programming methods (both deterministic and stochastic) are prevalent in the literature and suffer from curse of dimensionality in dealing with the scale and complexity of transportation networks in urban areas and require unacceptable run times for computing routing policies and offering rerouting options once the vehicle is en route [2, 3]. Naïve policies that arbitrarily limit the degree of “look ahead” to few links ahead of the vehicle can on the other hand lead to inferior performance (in reducing travel times and/or cost).

Vast majority of the current literature still revolves around relatively small road networks or unrealistic assumptions and are yet to demonstrate any resemblance of a practical real-world algorithm that can support the realities of current day transportation network dynamics [4, 5]. Overcoming these challenges not only depends on effectiveness of routing algorithms but also on the efficient representation of traffic network in a timely fashion which leads to reducing the run time and improving the performance of routing algorithms.

In this paper, to efficiently represent the road network and its traffic conditions, we employ an emerging concept in analyzing complex networks called community structure detection. We use this method to capture traffic network dynamics (congestion states) in the form of hierarchical community-based representations of road networks. While it has been shown that the community detection method is effective in path finding in static networks [6], there is no study on dynamic networks. In this study, we present the results for both static and dynamic road networks.

We first explain structure of road networks in section II based on road’s functional class. Section III proposes the application of community detection methods to road networks. Section IV presents experimental results from applying community detection algorithm to both static and dynamic road networks. Finally, section V offers some concluding remarks and directions for future research.

II. NETWORK STRUCTURE BASED ON HIGHWAY FUNCTIONAL CLASSIFICATION SYSTEM

Road networks commonly inherit a functional classification (e.g., arterial highways, collector roads, local roads...) that can be employed for hierarchical representation of the network.

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M.M. Nejad (corresponding author) and R.B. Chinnam are with the Industrial and Systems Engineering Department, Wayne State University, Detroit, MI 48202 USA (email: mahyar@wayne.edu; r_chinnam@wayne.edu).

L. Mashayekhy is with the Computer Science Department, Wayne State University, Detroit, MI 48202 USA (e-mail: mlена@wayne.edu).

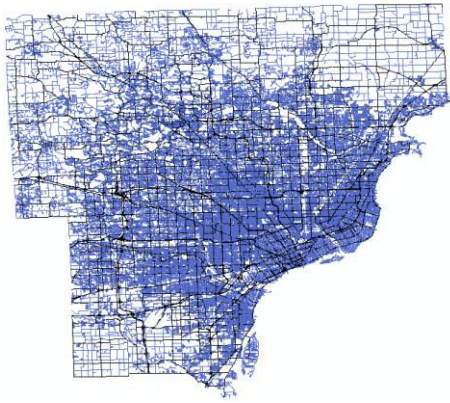


Fig. 1. Metro Detroit road network with 232,969 road arcs and 168,806 cross-section nodes

Functional classification (FC) groups streets and highways into classes based on the character of service they are intended to provide. The classification is rooted in the road network design and helps determine the speed category and travel time (cost) of passing through the road when there is no congestion. There are commonly five different FCs in road networks. For instance, FC1 roads are very long distance routes between major cities, which form a sparse higher level network. This higher level network essentially partitions the whole network into several sub-networks. Each FC extends the coverage of its preceding FC network (e.g., FC2 extends FC1). For the Metro Detroit road network shown in Fig. 1, we present the different FCs in Fig. 2.

Using the functional classification of roads through a hierarchical search algorithm accelerates the routing process for the search algorithm exploits (whenever possible) the sparser and faster higher level roads between the closest entry and exit points given the origin and destination. This

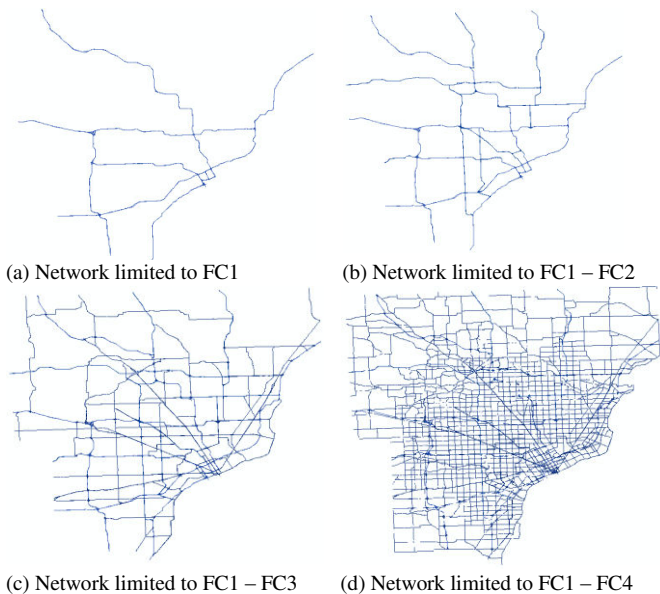


Fig. 2. Functional class hierarchy of Metro Detroit road network

approach is commonly used in speeding up the routing algorithms for large road networks, and has proven to be effective [6-16].

One of the weak points of hierarchical methods based on fixed FCs is that they prefer travel over higher level arcs (e.g., highways) without considering the congestion state at that level. Although the speed limit is higher at higher levels, and the optimal route might pass through higher levels when there is no congestion at those levels, this route may not necessarily be optimal under recurrent (rush hours) or non-recurrent (accident) congestion. Therefore, incorporating just the fixed topology of road networks and its functional classes might not always be adequate for efficient hierarchical routing.

To address the above-mentioned challenges, we use a community detection algorithm, in particular, the Louvain’s method [17] for the hierarchical representation of road networks with consideration for network congestion status. Community detection methods for directed networks consider the weight of each arc (cost of traversing the arc) for extracting the hierarchical structure. In the next section, we first explain the basic principles behind community detection methods.

III. COMMUNITY-BASED REPRESENTATION OF ROAD NETWORKS

Complex networks have attracted a great deal of attention across many fields of science [18-20]. A recently proposed concept in analyzing complex networks is their “community structure” [21, 22]. Many networks can be decomposed into communities such that the densely connected subsets of nodes form communities with only sparser connections between them. A wide variety of methods have been lately developed for detecting communities in networks (see [23] for a recent review).

Road networks are commonly represented by directed graphs where streets form the arcs and intersections are considered as nodes. To capture the dynamics of road networks, travel time or congestion state of each arc can be considered as the arc “weight”. Meaning, a road segment with a low weight generally indicates that traffic flows freely through that arc segment, and if selected for travel, leads to a lower overall travel time. Then, community detection methods can be employed to decompose the road network (with arc weights) to effectively represent the network structure and connectivity. To date, there aren’t any comprehensive studies to show the effectiveness of these methods for traffic road networks. In particular, we investigate the ability of community detection methods to represent the traffic dynamics of road networks. This is being done with the hope that the resulting community structure will aid the development and execution performance of routing algorithms. The rationale being that by applying community detection on a road network, it is decomposed into communities, such that, adjacent communities are connected by the inter-community arcs.

Therefore, for routing sort of applications, the shortest path between any pair of nodes in different communities should generally involve traversing one of these inter-community arcs [21]. As a result, detecting such arcs can be used to accelerate the route computation process, which justifies the detection of communities in road networks.

To model a road network in the form of communities, we use a graph representation of the road network. We consider a graph $G(V, A, W)$ as a directed graph where V is a set of nodes representing road intersections, A is a set of arcs representing roads, and W is a set of arc weights representing travel time of passing through that arc. Community detection methods partition the graph into disjoint communities (subgraphs). If G is partitioned into n communities $C^p(V_p, A_p, W_p)$, $p = 1 \dots n$, these communities have the following properties:

$$\begin{cases} \bigcup_{p=1}^n V_p = V \\ \bigcup_{p=1}^n A_p \subseteq A \end{cases} \quad (1)$$

$\forall p, q, V_p \cap V_q = \emptyset, A_p \cap A_q = \emptyset, 1 \leq p, q \leq n$, and $p \neq q$ such that the union of all nodes in all communities is V .

In each community C^p , a subset of A , A_p , connects its nodes, V_p . In addition to these arcs, $A / \bigcup_{p=1}^n A_p$ is a subset of arcs representing the inter-community arcs, which connect pairs of communities.

To partition the graph into communities, a modularity measure was first introduced in [21]. This measure gives a value, Q , between -1 and 1 for a partition based on the density of arcs inside communities in comparison with the density of arcs between communities. A higher value of Q indicates a better partitioning of the network. In the case of weighted networks, the modularity measure is defined as in Eq. (2):

$$Q = \frac{1}{2m} \sum_{i,j} [W_{ij} - \frac{k_i k_j}{2m}] \delta(C_i, C_j) \quad (2)$$

$$\delta(C_i, C_j) = \begin{cases} 1, & \text{if } C_i = C_j \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$m = \frac{1}{2} \sum_{i,j} W_{ij} \quad (4)$$

where W_{ij} represents the weight of the arc between i and j , k_i is the sum of the weights of the arcs attached to vertex i , C_i is the community to which vertex $i \in V$ is assigned.

As explained in Section II, road networks have a valuable characteristic, which is their hierarchical structure. However, as stated earlier, the fixed hierarchal representation of road networks based on their fixed topology / functional classes may not always be efficient. This is the case when dynamics of traffic networks change over time (e.g., due to recurrent or non-recurrent congestion). Building a hierarchy from a representation that considers the traffic dynamics would be more efficient. Since, traffic dynamics are considered in the community representation of road networks, a hierarchical structure can be built based on that. Therefore, the detected

communities in a road network can be an input to another iteration of community detection, which extracts a hierarchical representation of the road network.

Recently, fast algorithms are proposed to detect hierarchical community structure in networks [17, 24]. In these approaches, each hierarchy forms a directed graph in itself with fewer arcs and nodes as we go up the levels. These higher levels are abstractions of their lower levels graphs.

We adapt a hierarchical community detection method, Louvain's method [17], for constructing hierarchical representation of the road network based on the congestion status. This method not only extracts a hierarchical community structure, but exhibits excellent computational performance even for large-scale directed networks. The Louvain's method is a heuristic method based on the gain in modularity, ΔQ , by adding (removing) a vertex i into (from) a community C in each iteration of their proposed method. ΔQ is defined as follows:

$$\Delta Q = \left[\frac{\sum_{j,k \in C} W_{jk} + 2 \sum_{j \in C} W_{ij}}{2m} - \left(\frac{\sum_{j \in C, k \notin C} W_{jk} + k_i}{2m} \right)^2 \right] - \left[\frac{\sum_{j,k \in C} W_{jk}}{2m} - \left(\frac{\sum_{j \in C, k \notin C} W_{jk}}{2m} \right)^2 - \left(\frac{k_i}{2m} \right)^2 \right] \quad (5)$$

where $\sum_{j,k \in C} W_{jk}$ is the sum of the weights of the arcs inside C , $\sum_{j \in C, k \notin C} W_{jk}$ is the sum of the weights of the arcs incident to vertices in C , and $\sum_{j \in C} W_{ij}$ is the sum of the weights of the arcs from i to vertices in C .

In the next section, we analyze the performance of this method for both static and dynamic road networks.

IV. EXPERIMENTAL RESULTS

In this Section, we first evaluate the performance of Louvain's method, a hierarchical community detection method, in static road networks. Then, we analyze the effects of traffic network dynamics on the detected hierarchical communities. We conduct all of our experiments in C++. Experiments are conducted on an Intel 2.53 GHz Dual Core processor with 3G RAM Linux Platform (Fedora).

A. Static Road Networks

We show the performance of the adapted Louvain's method for community structure detection in static road networks with a numerical study carried out on the metropolitan road networks of Detroit, New York City and San Francisco Bay area. We use two different datasets; the first dataset is a real road network of Metro Detroit provided by NAVTEQ [25]. This data consists of recently collected road network features such as coordinates of all intersections, speed limits, etc. The second dataset is from the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) of Rutgers, New Jersey [26]. It includes coordinates of all

Table 1 Properties of sample city road networks

	# of nodes	# of arcs
New York	264,346	733,846
San Francisco	321,270	800,172
Metro Detroit	168,806	232,969

Table 2 Number of communities in each level revealed by the community detection algorithm

	New York	San Francisco	Metro Detroit
Level 0	264,347	321,271	168,806
Level 1	79,261	93,100	67,136
Level 2	18,968	23,104	21,508
Level 3	4,007	5,085	5,833
Level 4	952	1244	1,453
Level 5	438	672	457
Level 6	-	-	368

intersections, the distance graph, and the travel time graph of the U.S. road network.

Table 1 shows the number of nodes and arcs of all three road networks. Table 2 reports the number of communities identified in each level of the hierarchy using the hierarchical community detection algorithm. The algorithm extracted the same number of hierarchy levels in New York and San Francisco while it extracted one more level for Metro Detroit. This is due to the fact that Metro Detroit has a somewhat sparser network with respect to the other two networks. In the first level (level 0), each community contains just one node. As the level increases in the hierarchy, more nodes are merged to construct each community. Therefore, there are fewer communities at the higher levels. For instance, the Metro Detroit network is represented by only 368 communities in the top level (Level 6) of its hierarchy.

This representation can be exploited by hierarchical search strategies in routing algorithms such that they do not need to equally explore the whole network to find the optimal path. That is, the routing algorithms just need to consider only the relevant communities and the inter-community arcs (and ignore the rest of the network), hence, leading to reduced routing computational burden.

In our experiments, the hierarchical community detection algorithm finds the hierarchical communities in less than one second for the studied road networks. This fast execution time enables us to capture changes in the structure of the communities with change in the road network dynamics, for example, due to varying traffic congestion patterns over time. In the next subsection, we analyze the effects of traffic network dynamics on the detected hierarchical communities.

B. Dynamic Road Networks

We perform here a set of experiments to investigate the effects of traffic network dynamics (congestion states) on community-based representations of large road networks. We use real road network data from Metro Detroit provided

by NAVTEQ [25] to drive our experiments. This data consists of recently collected road network features including all arcs (232,969), and all nodes (168,806), speed limits of road segments, etc. Using this data, we obtain the travel time of passing through each arc when there is no congestion by dividing the distance with the speed limit. The captured travel time is static. That means, it does not include the impact of varying traffic dynamics by time of day on travel time. Unfortunately, we did not have access to real-time ITS arc travel times for whole of Metro Detroit. Many transportation studies in the literature [8],[27], employed artificially generated time-dependent costs for analysis. In this work, we too generated time-dependent costs based on the same model for Metro Detroit road network, as described in our previous work [28].

We generate six distinct congestion state graphs to capture different levels of congestion severity in the network.

In each case, we divide the speed of traffic going through a fraction of the arcs by a randomly generated value between [1.5, 3]. For example, in congestion state graph #1, 5% of all arcs (randomly selected) are forced to experience congestion; travel time of passing through these arcs is now more than the “base” scenario where there is no congestion. State 2, 3, 4, 5, and 6 correspond to cases where 10%, 15%, 20%, 25%, and 30% of the arcs experience congestion, respectively. Congestion states represent situations such as rush hours, accidents, bad weather conditions, and special events (games), which influence traffic patterns.

Note that the number of communities and number of inter-community arcs play an important role in the efficiency of routing algorithms. This is due to the fact that the performance of the routing algorithms depends on the number of nodes searched by the algorithms. One can reduce the state space by not covering undesirable nodes through a

Table 3 Effects of changes in congestion states on number of communities

	Base	State 1	State 2	State 3	State 4	State 5	State 6
Level 0	168806	168806	168806	168806	168806	168806	168806
Level 1	67137	66248	65444	64441	63331	62414	61211
Level 2	21509	20661	19952	19284	18567	17999	17684
Level 3	5834	5448	5116	4923	4740	4543	4467
Level 4	1454	1301	1250	1235	1129	1138	1120
Level 5	458	437	471	479	493	520	569
Level 6	369	387	430	445	469	508	557

Table 4 Effects of changes in congestion states on number of inter-community arcs

	Base	State 1	State 2	State 3	State 4	State 5	State 6
Level 0	232969	232969	232969	232969	232969	232969	232969
Level 1	122181	121144	120299	119246	118069	117197	116004
Level 2	53749	52465	51677	51042	50354	50214	50600
Level 3	21181	20432	20169	20273	20555	20637	21043
Level 4	8089	7722	7811	8142	8188	8413	8551
Level 5	3758	3763	4038	4237	4595	4726	5061
Level 6	3054	3328	3740	3956	4338	4566	4910

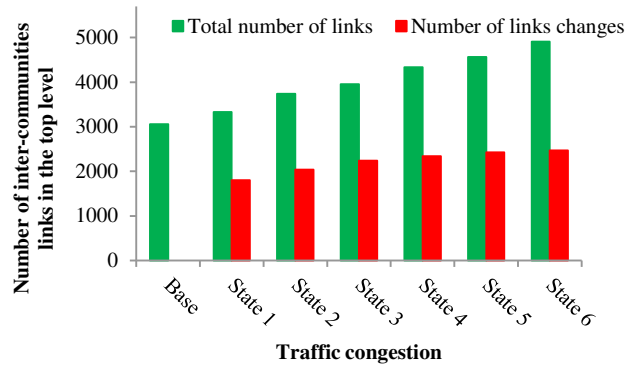


Fig. 3. Effect of changes in the inter-community arcs at the top level at different congestion states

trip [2, 3, 29].

Table 3 shows the number of communities detected in each level of hierarchy under the different congestion states. It is clear from the table that the number of communities decreases in the lower levels of hierarchy with increasing network congestion severity. This is due to the fact that the gaps between speeds of arcs decreases, which leads to merging of some neighbor communities into larger communities. This happens in neighborhoods in which the congestion occurs.

On the other hand, the number of communities increases in the higher levels of hierarchy with congestion severity. Merging communities in lower levels makes strong/dense communities in higher levels. These dense communities merge with fewer neighboring communities in those higher levels. Therefore, it leads to more communities in the top level as the severity of congestion goes up. Interestingly, this is true for all states of congestion in all levels of detected hierarchical communities. This shows that the community detection methods not only represent the road networks efficiently, but also capture traffic network dynamics effectively.

Table 4 shows the number of inter-community arcs in each level of the hierarchy, for the different states of traffic congestion. The data pattern is consistent with the data pattern from Table 3. As we move up the congestion state (from State #1 to State #6), inter-connectivity arcs decrease at lower levels with a decrease in number of communities. We see the opposite trend at higher levels for there are more communities.

Another approach to study the effects of traffic dynamics on the detected communities is to check to see how many inter-community arcs at the top level would change with increasing congestion. To do so, we compare the inter-community arcs of each congestion state with the inter-community arcs of the network without any congestion (base case) in the top level. The results (shown in Fig. 3) show that more than 50% of the inter-community arcs in the top level

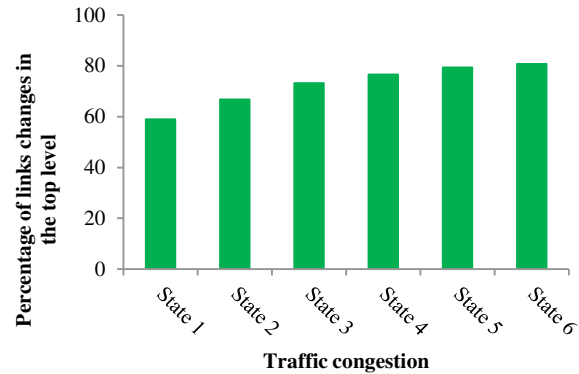


Fig. 4. Percentage of changes in the inter-community arcs at the top level at different congestion states

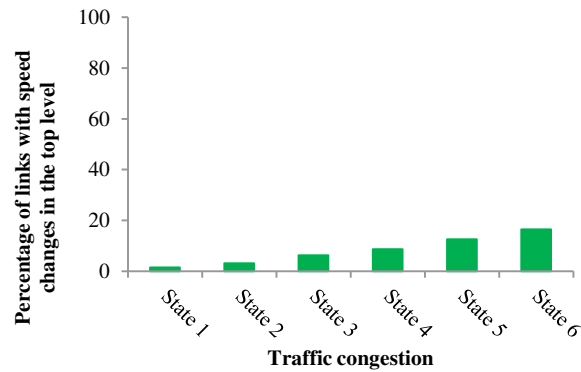


Fig. 5. Number of top level inter-community arcs downgraded to lower levels due to congestion

of the base network are not anymore the inter-community arcs in the top level of the networks with congestion. This is due to the fact that these inter-community arcs no longer play their previous function of connecting communities. They are either impacted by the decrease in the speed of traffic going through them or by empowering other arcs in their traffic mobility regarding changes in community structures. The precise percentage of these changes in the number of inter-community arcs in the top level is presented in Fig. 4.

We show in Fig. 5 that the number of inter-community arcs affected by the decrease in their traffic speed is less than newly formed inter-community arcs under congestion. Although in state 1, only 5% of arcs are experiencing congestion, it leads to changes in the connectivity structure of communities which plays an important role in bringing new arcs to a higher level functionality.

From the above results, we conclude that hierarchical community structure works to provide efficient representation of large-scale road networks with time-varying arc weights. Also, efficiency of such representation is promising for developing hierarchical search strategies in dynamic routing algorithms using real-time ITS data.

V. CONCLUSION

In this paper, we explained how an emerging concept in analyzing complex networks called “community structure detection” can be used to efficiently represent traffic networks with time-varying arc weights. We also demonstrate the computational efficiency of these community detection methods for road networks. We then investigated the effects of traffic network dynamics on the detected hierarchical communities. The resulting hierarchical community representation and their evolution over varying traffic conditions with time can aid the computational performance of real-time routing algorithms. We plan to incorporate the achieved results from this study in developing real-time routing algorithms under ITS in our next study.

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