Secure k-Nearest Neighbor Query over Encrypted Data in Outsourced Environments

Yousef Elmehdwi, Bharath K. Samanthula, and Wei Jiang

Department of Computer Science, Missouri S&T
500 West 15th Street, Rolla, MO 65409, USA
Email: {ymez76, bspq8, wjiang}@mst.edu

***Abstract*— For the past decade, query processing on relational data has been studied extensively, and many theoretical and practical solutions to query processing have been proposed under various scenarios. With the recent popularity of cloud computing, users now have the opportunity to outsource their data as well as the data management tasks to the cloud. However, due to the rise of various privacy issues, sensitive data (e.g., medical records) need to be encrypted before outsourcing to the cloud. In addition, query processing tasks should be handled by the cloud; otherwise, there would be no point to outsource the data at the first place. To process queries over encrypted data without the cloud ever decrypting the data is a very challenging task. In this paper, we focus on solving the k-nearest neighbor (kNN) query problem over encrypted database outsourced to a cloud: a user issues an encrypted query record to the cloud, and the cloud returns the k closest records to the user. We first present a basic scheme and demonstrate that such a naive solution is not secure. To provide better security, we propose a secure kNN protocol that protects the confidentiality of the data, user's input query, and data access patterns. Also, we empirically analyze the efficiency of our protocols through various experiments. These results indicate that our secure protocol is very efficient on the user end, and this lightweight scheme allows a user to use any mobile device to perform the kNN query.**

I. Introduction

As an emerging computing paradigm, cloud computing attracts many organizations to consider utilizing the benefits of a cloud in terms of cost-efficiency, flexibility, and offload of administrative overhead. In cloud computing model [1], [2], a data owner outsources his/her database T and the DBMS functionalities to the cloud that has the infrastructure to host outsourced databases and provides access mechanisms for querying and managing the hosted database. On one hand, by outsourcing, the data owner gets the benefit of reducing the data management costs and improves the quality of service. On the other hand, hosting and query processing of data out of the data owner control raises security challenges such as preserving data confidentiality and query privacy.

One straightforward way to protect the confidentiality of the outsourced data from the cloud as well as from the unauthorized users is to encrypt data by the data owner before outsourcing [3]. By this way, the data owner can protect the privacy of his/her own data. In addition, to preserve query privacy, authorized users require encrypting their queries before sending them to the cloud for evaluation. Furthermore, during query processing, the cloud can also derive useful and sensitive information about the actual data items by observing the data access patterns even if the data and query are encrypted [4], [5]. Therefore, following from the above discussions, secure query processing needs to guarantee (1) confidentiality of the encrypted data (2) confidentiality of a user's query record and (3) hiding data access patterns.

Using encryption as a way to achieve data confidentiality may cause another issue during the query processing step in the cloud. In general, it is very difficult to process encrypted data without ever having to decrypt it. The question here is how the cloud can execute the queries over encrypted data while the data stored at the cloud are encrypted at all times. In the literature, various techniques related to query processing over encrypted data have been proposed, including range queries [6]–[8] and other aggregate queries [9], [10]. However, these techniques are either not applicable or inefficient to solve advanced queries such as the k-nearest neighbor (kNN) query.

In this paper, we address the problem of secure processing of k-nearest neighbor query over encrypted data (SkNN) in the cloud. Given a user's input query Q, the objective of the SkNN problem is to securely identify the k-nearest data tuples to Q using the encrypted database of T in the cloud, without allowing the cloud to learn anything regarding the actual contents of the database T and the query record Q. More specifically, when encrypted data are outsourced to the cloud, we observe that an effective SkNN protocol needs to satisfy the following properties:

- Preserve the confidentiality of T and Q at all times
- Hiding data access patterns from the cloud
- Accurately compute the k-nearest neighbors of query Q
- Incur low computation overhead on the end-user

In the past few years, researchers have proposed various methods [1], [11]–[13] to address the SkNN problem. However, we emphasize that the existing SkNN methods violate at least one of the above mentioned desirable properties of a SkNN protocol. On one hand, the methods in [1], [11] are insecure because they are vulnerable to chosen and known plaintext attacks. On the other hand, recent method in [13] returns non-accurate kNN result to the end-user. More precisely, in [13], the cloud retrieves the relevant encrypted partition instead of finding the encrypted exact k-nearest neighbors. Furthermore, in [1], [12], [13], the end-user involves in heavy computations during the query processing step. By doing so, these methods utilize cloud as just a storage medium, i.e., no significant work

TABLE I
SAMPLE HEART DISEASE DATASET T

record-id	age	sex	cp	trestbps	chol	fbs	slope	ca	thal	num
t_1	63	1	1	145	233	1	3	0	6	0
t_2	56	1	3	130	256	1	2	1	6	2
t_3	57	0	3	140	241	0	2	0	7	1
t_4	59	1	4	144	200	1	2	2	6	3
t_5	55	0	4	128	205	0	2	1	7	3

is done on the cloud side. Additionally, the existing SkNN methods do not protect data access patterns from the cloud. More details about the existing SkNN methods are provided in Section II.

Along this direction, with the goal of providing better security, this paper proposes a novel SkNN protocol that satisfies the above properties altogether. The protocols developed in this paper are secure under the semi-honest model [14]. However, they can easily be extended to secure protocols under other adversary models, such as malicious and covert, using threshold based cryptosystem and zero-knowledge proofs.

A. Problem Definition

Suppose the data owner Alice owns a database T of n records, denoted by $t_1, \ldots, t_n$, and m attributes. Let $t_{i,j}$ denote the j^{th} attribute value of record t_i. In our problem setting, we assume that Alice initially encrypts her database attribute-wise, that is, she computes $E_{pk}(t_{i,j})$, for $1 \leq i \leq n$ and $1 \leq j \leq m$, where E_{pk} denotes the encryption function of a public-key cryptosystem that is semantically secure [15]. Let the encrypted database be denoted by $E_{pk}(T)$. We assume that Alice outsources $E_{pk}(T)$ as well as the future querying processing services to the cloud.

Consider an authorized user Bob who wants to ask the cloud for k-neighbor records that are closest to his input query $Q = \langle q_1, \ldots, q_m \rangle$ based on $E_{pk}(T)$. During this process, Bob's query Q and contents of database T should not be revealed to the cloud. In addition, the access patterns to the data should be protected from the cloud. We refer to such a process as Secure kNN (SkNN) query over encrypted data in the cloud. Without loss of generality, let $\langle t'_1, \ldots, t'_k \rangle$ denote the k-nearest records to Q. Then, we formally define the SkNN protocol as follows:

$$\text{S}k\text{NN}(E_{pk}(T), Q) \rightarrow \langle t'_1, \ldots, t'_k \rangle$$

We emphasize that, at the end of the SkNN protocol, the output $\langle t'_1, \ldots, t'_k \rangle$ should be revealed only to Bob. We now present a real-life application of the SkNN protocol.

Example 1: Consider a physician who wants to know the risk factor of heart disease in a specific patient. Let T denote the sample heart disease dataset with attributes *record-id*, *age*, *sex*, *cp*, *trestbps*, *chol*, *fbs*, *slope*, *ca*, *thal*, and *num* as shown in Table I. The heart disease dataset given in Table I is obtained from the UCI machine learning repository [16].

Initially, the data owner (hospital) encrypts T attribute-wise, outsources the encrypted database $E_{pk}(T)$ to the cloud for easy management. In addition, the data owner delegates the future query processing services to the cloud. Now, we consider a doctor working at the hospital, say Bob, who would like to know the risk factor of heart disease in a specific patient based on T. Let the patient medical information be $Q = \langle 58, 1, 4, 133, 196, 1, 2, 1, \ 6 \rangle$. In the S$k$NN protocol, Bob first need to encrypt Q (to preserve the privacy of his query) and send it to the cloud. Then the cloud searches on the encrypted database $E_{pk}(T)$ to figure out the k-nearest neighbors to the user's request. For simplicity, let us assume $k = 2$. Under this case, the 2 nearest neighbors to Q are t_4 and t_5 (by using Euclidean distance as the similarity metric). After this, the cloud sends t_4 and t_5 (in encrypted form) to Bob. Here, the cloud should identify the nearest neighbors of Q in an oblivious manner without knowing any sensitive information, i.e., all the computations have to be carried over encrypted records. Finally, Bob receives t_4 and t_5 that will help him to make medical decisions. □

B. Our Contribution

In this paper, we propose a novel SkNN protocol to facilitate the k-nearest neighbor search over encrypted data in the cloud that preserves both the data privacy and query privacy. In our protocol, once the encrypted data are outsourced to the cloud, Alice does not participate in any computations. Therefore, no information is revealed to Alice. In particular, the proposed protocol meets the following requirements:

- **Data confidentiality -** Contents of T or any intermediate results should not be revealed to the cloud.
- **Query privacy -** Bob's input query Q should not be revealed to the cloud.
- **Correctness -** The output $\langle t'_1, \ldots, t'_k \rangle$ should be revealed only to Bob. In addition, no information other than $t'_1, \ldots, t'_k$ should be revealed to Bob.
- **Low computation overhead on Bob -** After sending his encrypted query record to the cloud, our protocols incur low computation overhead on Bob compared with the existing works [1], [11]–[13].
- **Hidden data access patterns -** Access patterns to the data, such as the records corresponding to the k-nearest neighbors of Q, should not be revealed to Alice and the cloud (to prevent any inference attacks).

We emphasize that the intermediate results seen by the cloud in our protocol are either newly generated randomized encryptions or random numbers. Thus, which data records correspond to the k-nearest neighbors of Q are not known to the cloud. Also, after sending his encrypted query record to the cloud, Bob does not involve in any computations (low cost on Bob). Hence, data access patterns are further protected from Bob.

The rest of the paper is organized as follows. We discuss the existing related work and some background concepts in Section II. A set of security primitives that are utilized in the proposed protocols and their possible implementations are provided in Section III. The proposed protocols are explained in detail in Section IV. Section V discusses the performance of the proposed protocols based on various experiments. We conclude the paper along with future work in Section VI.

II. Related Work and Background

In this section, we present an overview of the existing secure k-nearest neighbor techniques. Then, we discuss the security definition adopted in this paper along with the homomorphic properties of the Paillier cryptosystem as a background.

A. Existing SkNN Techniques

Retrieving the k-nearest neighbors to a given query Q is one of the most fundamental problem in many application domains such as similarity search, pattern recognition, and data mining. In the literature, many techniques have been proposed to address the SkNN problem, which can be classified into two categories based on whether the data are encrypted or not: *centralized* and *distributed*.

1) Centralized Methods: In the centralized methods, we assume that the data owner outsources his/her database and DBMS functionalities (e.g., kNN query) to an untrusted external service provider which manages the data on behalf of the data owner where only trusted users are allowed to query the hosted data. By outsourcing data to an untrusted server, many security issues arise, such as data privacy (protecting the confidentiality of the data from the server and query issuer). To achieve data privacy, data owner is required to use data anonymization models (e.g., k-anonymity) or cryptographic (e.g., encryption and data perturbation) techniques over his/her data before outsourcing them to the server.

Encryption is a traditional technique used to protect the confidentiality of sensitive data such as medical records. Due to data encryption, the process of query evaluation over encrypted data becomes challenging. Along this direction, various techniques have been proposed for processing range [6]–[8] and aggregation queries [9], [10] over encrypted data. However, in this paper, we restrict our discussion to secure evaluation of kNN query.

In the past few years, researchers have proposed different methods [1], [11]–[13] to address the SkNN problem. Wong et al. [11] proposed a new encryption scheme called asymmetric scalar-product-preserving encryption (ASPE) that preserves scalar product between the query vector Q and any tuple vector t_i from database T for distance comparison which is sufficient to find kNN. In [11], data and query are encrypted using slightly different encryption schemes before outsourcing to the server and all the query users know the decryption key. As an improvement, Zhu et al. [12] proposed a novel SkNN method in which the key of the data owner is not disclosed to the user. However, their architecture requires the participation of data owner during query encryption. As an alternative, Hu et al. [1] proposed a method based on provably secure privacy homomorphism encryption scheme from [17] that supports modular addition, subtraction and multiplication over encrypted data. They addressed the SkNN problem under the following setting: the client has the ciphertexts of all data points in database T and the encryption function of T whereas the server has the decryption function of T and some auxiliary information regarding each data point. However, both methods in [1], [11] are not secure because they are vulnerable to chosen-plaintext attacks. Also, all the above methods leak data access patterns to the server.

Recently, Yao et al. [13] proposed a new SkNN method based on partition-based secure Voronoi diagram (SVD). Instead of asking the cloud to retrieve the exact kNN, they required, from the cloud, to retrieve a relevant encrypted partition $E_{pk}(G)$ for $E_{pk}(T)$ such that G is guaranteed to contain the k-nearest neighbors of Q. However, in our work, we are able to solve the SkNN problem accurately by letting the cloud to retrieve the exact k-nearest neighbors of Q (in encrypted form). In addition, most of the computations during the query processing step in [1], [12], [13] are performed locally by the end-user which conflicts the very purpose of outsourcing the DBMS functionalities to the cloud. Furthermore, the protocol in [13] leaks data access patterns, such as the partition ID corresponding to a user query, to the cloud.

2) Data Distribution Methods: In the data distributed methods, data are assumed to be partitioned either vertically or horizontally and distributed among a set of independent, non-colluding parties. In the literature, the data distributed methods rely on secure multiparty computation (SMC) techniques that enable multiple parties to securely evaluate a function using their respective private inputs without disclosing the input of one party to the others. Many efforts have been made to address the problem of kNN query in a distributed environment. Shaneck et al. [18] proposed privacy-preserving algorithm to perform k-nearest neighbor search. The protocol in [18] is based on secure multiparty computation for privately computing kNN points in a horizontally partitioned dataset. Qi et al. [19] proposed a single-step kNN search protocol that is provably secure with linear computation and communication complexities. Vaidya et al. [20] studied privacy-preserving top-k queries in which the data are vertically partitioned. Ghinita et al. [21] proposed a private information retrieval (PIR) based framework for answering kNN queries in location-based services. We emphasize that, in [21], the data residing at the server are in plaintext format. However, if the data are encrypted to ensure data confidentiality, it is not clear how a user can obliviously retrieve the output records because he/she does not know the indexes that match his/her input query. Nevertheless, even if a user can retrieve the records using PIR, the user still needs to perform local computations to identify the k-nearest neighbors. However, in our framework, the users computation is completely outsourced to a cloud.

In summary, we emphasize that the above data distribution methods are not applicable to perform kNN queries over encrypted data for two reasons: (1). In our work, we deal with encrypted form of database and query which is not the case in the above methods (2). The database in our case is encrypted and stored on the cloud whereas in the above methods it is partitioned (in plaintext format) among different parties.

B. Security Definition

In this paper, privacy/security is closely related to the amount of information disclosed during the execution of a protocol. There are many ways to define information disclo-

TABLE II
COMMON NOTATIONS

$E_{pk}(T)$	Attribute-wise encryption of T
$\langle n, m\rangle$	Number of data records and attributes in T
$\langle t_i, Q\rangle$	i^{th} record in T and Bob's query record
t'_i	i^{th} nearest record to Q based on T
l	Domain size (in bits) of the squared Euclidean distance
$\langle z_1, z_l\rangle$	The most and least significant bits of integer z
$[z]$	Vector of encryptions of the individual bits of z

sure. To maximize privacy or minimize information disclosure, we adopt the security definitions in the literature of secure multiparty computation (SMC) first introduced by Yao's Millionaires' problem for which a provably secure solution was developed [14]. In this paper, we assume that parties are semi-honest (or honest-but-curious); that is, a semi-honest party follows the rules of the protocol using its correct input, but is free to later use what it sees during execution of the protocol to compromise security. In general, secure protocols under the semi-honest model are more efficient than those under the malicious adversary model, and almost all practical SMC protocols proposed in the literature are secure under the semi-honest model. Due to space limitations, we refer the reader to [14] for detailed security definitions and models.

C. Paillier Cryptosystem

The Paillier cryptosystem is an additive homomorphic and probabilistic asymmetric encryption scheme [15]. Let E_{pk} be the encryption function with public key pk given by (N, g), where N is a product of two large primes and g is in $\mathbb{Z}^*_{N^2}$. Also, let D_{sk} be the decryption function with secret key sk. Given $a, b \in \mathbb{Z}_N$, the Paillier encryption scheme exhibits the following properties:

a. **Homomorphic Addition**

$$E_{pk}(a + b) \leftarrow E_{pk}(a) * E_{pk}(b) \bmod N^2;$$

b. **Homomorphic Multiplication**

$$E_{pk}(a * b) \leftarrow E_{pk}(a)^b \bmod N^2;$$

c. **Semantic Security -** The encryption scheme is semantically secure [22], i.e., given a set of ciphertexts, an adversary cannot deduce any information about the plaintext.

In this paper, we assume that a data owner encrypted his or her data using Paillier cryptosystem before outsourcing them to a cloud. Some common notations that are used extensively in this paper are shown in Table II.

III. BASIC SECURITY PRIMITIVES

In this section, we present a set of generic protocols that will be used as sub-routines while constructing our proposed SkNN protocol in Section IV-B. All of the below protocols are considered under two-party semi-honest setting. In particular, we assume the existence of two semi-honest parties P_1 and P_2 such that the Paillier's secret key sk is known only to P_2 whereas pk is treated as public.

- Secure Multiplication (SM) Protocol:
  This protocol considers P_1 with input $(E_{pk}(a), E_{pk}(b))$ and outputs $E_{pk}(a * b)$ to P_1, where a and b are not known to P_1 and P_2. During this process, no information regarding a and b is revealed to P_1 and P_2. The output $E_{pk}(a * b)$ is known only to P_1.
- Secure Squared Euclidean Distance (SSED) Protocol:
  P_1 with input $(E_{pk}(X), E_{pk}(Y))$ and P_2 securely compute the encryption of squared Euclidean distance between vectors X and Y. Here X and Y are m dimensional vectors where $E_{pk}(X) = \langle E_{pk}(x_1), \ldots, E_{pk}(x_m)\rangle$ and $E_{pk}(Y) = \langle E_{pk}(y_1), \ldots, E_{pk}(y_m)\rangle$. At the end, the output $E_{pk}(|X - Y|^2)$ is known only to P_1.
- Secure Bit-Decomposition (SBD) Protocol:
  P_1 with input $E_{pk}(z)$ and P_2 securely compute the encryptions of the individual bits of z, where $0 \leq z < 2^l$. The output $[z] = \langle E_{pk}(z_1), \ldots, E_{pk}(z_l)\rangle$ is known only to P_1. Here z_1 and z_l denote the most and least significant bits of integer z respectively.
- Secure Minimum (SMIN) Protocol:
  P_1 with input $([u], [v])$ and P_2 with sk securely compute the encryptions of the individual bits of minimum number between u and v. That is, the output is $[\min(u, v)]$ which will be known only to P_1. During this protocol, no information regarding u and v is revealed to P_1 and P_2.
- Secure Minimum out of n Numbers (SMIN$_n$) Protocol:
  P_1 has n encrypted vectors $([d_1], \ldots, [d_n])$ and P_2 has sk. Here $[d_i] = \langle E_{pk}(d_{i,1}), \ldots, E_{pk}(d_{i,l})\rangle$ such that $d_{i,1}$ and $d_{i,l}$ are the most and least significant bits of integer d_i respectively, for $1 \leq i \leq n$. P_1 and P_2 jointly compute the output $[\min(d_1, \ldots, d_n)]$. At the end, $[\min(d_1, \ldots, d_n)]$ is known only to P_1. During SMIN$_n$, no information about d_i's is revealed to P_1 and P_2.
- Secure Bit-OR (SBOR) Protocol:
  P_1 with input $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 securely compute $E_{pk}(o_1 \vee o_2)$, where o_1 and o_2 are two bits. The output $E_{pk}(o_1 \vee o_2)$ is known only to P_1.

We now discuss each of these protocols in detail. Also, we either propose new solution or refer to the most efficient known implementation to each one of them.

Secure Multiplication (SM). Consider a party P_1 with private input $(E_{pk}(a), E_{pk}(b))$ and a party P_2 with the secret key sk. The goal of the secure multiplication (SM) protocol is to return the encryption of $a * b$, i.e., $E_{pk}(a * b)$ as output to P_1. During this protocol, no information regarding a and b is revealed to P_1 and P_2. The basic idea of SM is based on the following property which holds for any given $a, b \in \mathbb{Z}_N$:

$$a * b = (a + r_a) * (b + r_b) - a * r_b - b * r_a - r_a * r_b \quad (1)$$

where all the arithmetic operations are performed under $\mathbb{Z}_N$. The overall steps in SM are shown in Algorithm 1. Briefly, P_1 initially randomizes a and b by computing $a' = E_{pk}(a) * E_{pk}(r_a)$ and $b' = E_{pk}(b) * E_{pk}(r_b)$, and sends them to P_2. Here r_a and r_b are random numbers in

Algorithm 1 SM$(E_{pk}(a), E_{pk}(b)) \rightarrow E_{pk}(a * b)$

Require: P_1 has $E_{pk}(a)$ and $E_{pk}(b)$; P_2 has sk

1: P_1:
   (a). Pick two random numbers $r_a, r_b \in \mathbb{Z}_N$
   (b). $a' \leftarrow E_{pk}(a) * E_{pk}(r_a)$
   (c). $b' \leftarrow E_{pk}(b) * E_{pk}(r_b)$; send a', b' to P_2
2: P_2:
   (a). Receive a' and b' from P_1
   (b). $h_a \leftarrow D_{sk}(a')$; $h_b \leftarrow D_{sk}(b')$
   (c). $h \leftarrow h_a * h_b \bmod N$
   (d). $h' \leftarrow E_{pk}(h)$; send h' to P_1
3: P_1:
   (a). Receive h' from P_2
   (b). $s \leftarrow h' * E_{pk}(a)^{N-r_b}$
   (c). $s' \leftarrow s * E_{pk}(b)^{N-r_a}$
   (d). $E_{pk}(a * b) \leftarrow s' * E_{pk}(r_a * r_b)^{N-1}$

$\mathbb{Z}_N$ known only to P_1. Upon receiving, P_2 decrypts and multiplies them to get $h = (a + r_a) * (b + r_b) \bmod N$. Then, P_2 encrypts h and sends it to P_1. After this, P_1 removes extra random factors from $h' = E_{pk}((a + r_a) * (b + r_b))$ based on Equation 1 to get $E_{pk}(a*b)$. Note that, for any given $x \in \mathbb{Z}_{\mathbb{N}}$, "$N - x$" is equivalent to "$-x$" under $\mathbb{Z}_N$. Hereafter, we use the notation $r \in_R \mathbb{Z}_N$ to denote r as a random number in $\mathbb{Z}_N$.

Secure Squared Euclidean Distance (SSED). Suppose P_1 holds two encrypted vectors $(E_{pk}(X), E_{pk}(Y))$ and P_2 holds sk. Here X and Y are two m-dimensional vectors such that $E_{pk}(X) = \langle E_{pk}(x_1), \ldots, E_{pk}(x_m)\rangle$ and $E_{pk}(Y) = \langle E_{pk}(y_1), \ldots, E_{pk}(y_m)\rangle$. The goal of SSED is to securely compute $E_{pk}(|X - Y|^2)$, where $|X - Y|$ denotes the Euclidean distance between X and Y. During this protocol, no information regarding X and Y is revealed to P_1 and P_2. The basic idea of SSED follows from the following equation:

$$|X - Y|^2 = \sum_{i=1}^{m} (x_i - y_i)^2 \tag{2}$$

The main steps involved in SSED are shown in Algorithm 2. Briefly, for $1 \leq i \leq m$, P_1 initially computes $E_{pk}(x_i - y_i)$ by using the homomorphic properties. Then P_1 and P_2 jointly compute $E_{pk}((x_i - y_i)^2)$ using the SM protocol, for $1 \leq i \leq m$. Note that the outputs of SM are known only to P_1. By applying homomorphic properties on $E_{pk}((x_i - y_i)^2)$, P_1 computes $E_{pk}(|X - Y|^2)$ locally based on Equation 2.

Secure Bit-Decomposition (SBD). Suppose P_1 has $E_{pk}(z)$ and P_2 has sk, where z is not known to both parties and $0 \leq z < 2^l$. The goal of SBD is to compute the encryptions of the individual bits of binary representation of z [23]. The output is $[z] = \langle E_{pk}(z_1), \ldots, E_{pk}(z_l)\rangle$, where z_1 and z_l denote the most and least significant bits of z respectively. At the end, the output $[z]$ is known only to P_1. Since the goal of this paper is not to investigate the existing SBD protocols, we simply use the most efficient SBD protocol that was recently proposed in [23].

Algorithm 2 SSED$(E_{pk}(X), E_{pk}(Y)) \rightarrow E_{pk}(|X - Y|^2)$

Require: P_1 has $E_{pk}(X)$ and $E_{pk}(Y)$; P_2 has sk

1: P_1, **for** $1 \leq i \leq m$ **do**:
   (a). $E_{pk}(x_i - y_i) \leftarrow E_{pk}(x_i) * E_{pk}(y_i)^{N-1}$
2: P_1 and P_2, **for** $1 \leq i \leq m$ **do**:
   (a). Compute $E_{pk}((x_i - y_i)^2)$ using the SM protocol
3: P_1 computes $E_{pk}(|X - Y|^2) \leftarrow \prod_{i=1}^{m} E_{pk}((x_i - y_i)^2)$

Secure Minimum (SMIN). In this protocol, P_1 with input $([u], [v])$ and P_2 with sk securely compute the encryptions of the individual bits of $\min(u, v)$, i.e., the output is $[\min(u, v)]$. Here $[u] = \langle E_{pk}(u_1), \ldots, E_{pk}(u_l)\rangle$ and $[v] = \langle E_{pk}(v_1), \ldots, E_{pk}(v_l)\rangle$, where u_1 (resp., v_1) and u_l (resp., v_l) are the most and least significant bits of u (resp., v). At the end, the output $[\min(u, v)]$ is known only to P_1.

We assume that $0 \leq u, v < 2^l$ and propose a novel SMIN protocol. The basic idea of the proposed SMIN protocol is for P_1 to randomly choose the functionality F (by flipping a coin), where F is either $u > v$ or $v > u$, and to obliviously execute F with P_2. Since F is randomly chosen and known only to P_1, the output of the functionality F is oblivious to P_2. Based on the output and chosen F, P_1 computes $[\min(u, v)]$ locally using homomorphic properties.

The overall steps involved in the SMIN protocol are shown in Algorithm 3. To start with, P_1 initially chooses the functionality F as either $u > v$ or $v > u$ randomly. Then, using SM, P_1 computes $E_{pk}(u_i * v_i)$ with the help of P_2. Now, depending on F, P_1 proceeds as follows, for $1 \leq i \leq l$:

- If $F : u > v$, compute

$$W_i = E_{pk}(u_i) * E_{pk}(u_i * v_i)^{N-1} = E_{pk}(u_i * (1 - v_i))$$
$$\Gamma_i = E_{pk}(v_i - u_i) * E_{pk}(\hat{r}_i) = E_{pk}(v_i - u_i + \hat{r}_i)$$

- If $F : v > u$, compute:

$$W_i = E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-1} = E_{pk}(v_i * (1 - u_i))$$
$$\Gamma_i = E_{pk}(u_i - v_i) * E_{pk}(\hat{r}_i) = E_{pk}(u_i - v_i + \hat{r}_i)$$

  where $\hat{r}_i$ is a random number in $\mathbb{Z}_N$
- Observe that if $F : u > v$, then $W_i = E_{pk}(1)$ only if $u_i > v_i$, and $W_i = E_{pk}(0)$ otherwise. Similarly, when $F : v > u$, we have $W_i = E_{pk}(1)$ only if $v_i > u_i$, and $W_i = E_{pk}(0)$ otherwise. Also, depending of F, Γ_i stores the encryption of randomized difference between u_i and v_i which will be used in later computations.
- Compute the encrypted bit-wise XOR between the bits u_i and v_i as $G_i = E_{pk}(u_i \oplus v_i)$ using the below formulation:

$$G_i = E_{pk}(u_i) * E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-2}$$

  In general, for any two given bits o_1 and o_2, we have $o_1 \oplus o_2 = o_1 + o_2 - 2(o_1 * o_2)$
- Compute an encrypted vector H by preserving the first occurrence of $E_{pk}(1)$ (if there exists one) in G by initializing $H_0 = E_{pk}(0)$. The rest of the entries of H are computed as $H_i = H_{i-1}^{r_i} * G_i$. We emphasize that at most one of the entry in H is $E_{pk}(1)$ and the remaining

Algorithm 3 SMIN$([u],[v]) \rightarrow [\min(u,v)]$

Require: P_1 has $[u]$ and $[v]$, where $0 \leq u, v < 2^l$; P_2 has sk

1: P_1:

(a). Randomly choose the functionality F

(b). **for** $i = 1$ to l **do**:

- $E_{pk}(u_i * v_i) \leftarrow \text{SM}(E_{pk}(u_i), E_{pk}(v_i))$
- **if** $F : u > v$ **then**:
  - $W_i \leftarrow E_{pk}(u_i) * E_{pk}(u_i * v_i)^{N-1}$
  - $\Gamma_i \leftarrow E_{pk}(v_i - u_i) * E_{pk}(\hat{r}_i)$; $\hat{r}_i \in_R \mathbb{Z}_N$

  else
  - $W_i \leftarrow E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-1}$
  - $\Gamma_i \leftarrow E_{pk}(u_i - v_i) * E_{pk}(\hat{r}_i)$; $\hat{r}_i \in_R \mathbb{Z}_N$
- $G_i \leftarrow E_{pk}(u_i \oplus v_i)$
- $H_i \leftarrow H_{i-1}^{r_i} * G_i$; $r_i \in_R \mathbb{Z}_N$ and $H_0 = E_{pk}(0)$
- $\Phi_i \leftarrow E_{pk}(-1) * H_i$
- $L_i \leftarrow W_i * \Phi_i^{r_i'}$; $r_i' \in_R \mathbb{Z}_N$

(c). $\Gamma' \leftarrow \pi_1(\Gamma)$

(d). $L' \leftarrow \pi_2(L)$; send Γ' and L' to P_2

2: P_2:

(a). Receive Γ' and L' from P_1

(b). $M_i \leftarrow D_{sk}(L_i')$, for $1 \leq i \leq l$

(c). **if** $\exists\ j$ such that $M_j = 1$ **then** $\alpha \leftarrow 1$
else $\alpha \leftarrow 0$

(d). $M_i' \leftarrow \Gamma_i'^{\alpha}$, for $1 \leq i \leq l$

(e). Send M' and $E_{pk}(\alpha)$ to P_1

3: P_1:

(a). Receive M' and $E_{pk}(\alpha)$ from P_2

(b). $\widetilde{M} \leftarrow \pi_1^{-1}(M')$

(c). **for** $i = 1$ to l **do**:

- $\lambda_i \leftarrow \widetilde{M}_i * E_{pk}(\alpha)^{N-\hat{r}_i}$
- **if** $F : u > v$ **then** $E_{pk}(\min(u,v)_i) \leftarrow E_{pk}(u_i) * \lambda_i$
  else $E_{pk}(\min(u,v)_i) \leftarrow E_{pk}(v_i) * \lambda_i$

entries are encryptions of either 0 or a random number. Also, if there exists an index j such that $H_j = E_{pk}(1)$, then index j is the first position (from the most significant bit) at which the corresponding bits of u and v differ.

- Then, P_1 computes $\Phi_i = E_{pk}(-1) * H_i$. Note that "$-1$" is equivalent to "$N-1$" under $\mathbb{Z}_N$. From the above discussions, it is clear that $\Phi_i = E_{pk}(0)$ at most once since H_i is equal to $E_{pk}(1)$ at most once. Also, if $\Phi_j = E_{pk}(0)$, then index j is the position at which the bits of u and v differ first.
- Compute an encrypted vector L by combining W and Φ. Note that W_i stores the result of $u_i > v_i$ or $v_i > u_i$ which depends on F known only to P_1. Precisely, P_1 computes $L_i = W_i * \Phi_i^{r_i'}$, where r_i' is a random number in $\mathbb{Z}_N$. The observation here is if $\exists$ an index j such that $\Phi_j = E_{pk}(0)$, denoting the first flip in the bits of u and v, then W_j stores the corresponding desired information, i.e., whether $u_j > v_j$ or $v_j > u_j$ in encrypted form.

After this, P_1 permutes the encrypted vectors Γ and L using two random permutation functions π_1 and π_2. Specifically, P_1 computes $\Gamma' = \pi_1(\Gamma)$ and $L' = \pi_2(L)$, and sends them to P_2. Upon receiving, P_2 decrypts L' component-wise to get $M_i = D_{sk}(L_i')$, for $1 \leq i \leq l$, and checks for index j (decide the output of F). That is, if $M_j = 1$, then the output of F is 1, and 0 otherwise. Let the output be α. Note that since F is not known to P_2, the output α is oblivious to P_2. In addition, P_2 computes a new encrypted vector M' where $M_i' = \Gamma_i'^{\alpha}$, for $1 \leq i \leq l$, sends M' and $E_{pk}(\alpha)$ to P_1. After receiving M' and $E_{pk}(\alpha)$, P_1 computes the inverse permutation of M' as $\widetilde{M} = \pi_1^{-1}(M')$. Then, P_1 performs the following homomorphic operations to compute the encryption of i^{th} bit of $\min(u,v)$, i.e., $E_{pk}(\min(u,v)_i)$, for $1 \leq i \leq l$:

- Remove the randomness from $\widetilde{M}_i$ by computing
$$\lambda_i = \widetilde{M}_i * E_{pk}(\alpha)^{N-\hat{r}_i}$$
- If $F : u > v$, compute the i^{th} encrypted bit of $\min(u,v)$ as $E_{pk}(\min(u,v)_i) = E_{pk}(u_i) * \lambda_i = E_{pk}(u_i + \alpha * (v_i - u_i))$. Otherwise, compute $E_{pk}(\min(u,v)_i) = E_{pk}(v_i) * \lambda_i = E_{pk}(v_i + \alpha * (u_i - v_i))$.

In the SMIN protocol, one main observation (upon which we can also justify the correctness of the final output) is that if $F : u > v$, then $\min(u,v)_i = (1-\alpha) * u_i + \alpha * v_i$ always holds, for $1 \leq i \leq l$. Similarly, if $F : v > u$, then $\min(u,v)_i = \alpha * u_i + (1-\alpha) * v_i$ always holds.

Example 2: Consider that $u = 55$, $v = 58$, and $l = 6$. We assume that P_1's random permutation functions are given as below. Suppose P_1 holds

i	=	1	2	3	4	5	6
		↓	↓	↓	↓	↓	↓
$\pi_1(i)$	=	6	5	4	3	2	1
$\pi_2(i)$	=	2	1	5	6	3	4

$[55] = \langle E_{pk}(1), E_{pk}(1), E_{pk}(0), E_{pk}(1), E_{pk}(1), E_{pk}(1)\rangle$ and $[58] = \langle E_{pk}(1), E_{pk}(1), E_{pk}(1), E_{pk}(0), E_{pk}(1), E_{pk}(0)\rangle$. Without loss of generality, suppose P_1 chooses the functionality $F : v > u$. Then, various intermediate results based on the SMIN protocol are as shown in Table III. Following from Table III, we observe that:

- At most one of the entry in H is $E_{pk}(1)$ $(= H_3)$ and the remaining entries are encryptions of either 0 or a random number in $\mathbb{Z}_N$. Index $j = 3$ is the first position at which the corresponding bits of u and v differ.
- $\Phi_3 = E_{pk}(0)$ since H_3 is equal to $E_{pk}(1)$. Also, since $M_5 = 1$, P_2 sets α to 1.

At the end, only P_1 knows $[\min(u,v)] = [u] = [55]$. □

TABLE III

P_1 CHOOSES F AS $v > u$ WHERE $u = 55$ AND $v = 58$

$[u]$	$[v]$	W_i	Γ_i	G_i	H_i	Φ_i	L_i	Γ_i'	L_i'	M_i	λ_i	$\min_i$
1	1	0	r	0	0	-1	r	$1+r$	r	r	0	1
1	1	0	r	0	0	-1	r	r	r	r	0	1
0	1	1	$-1+r$	1	1	0	1	$1+r$	r	r	-1	0
1	0	0	$1+r$	1	r	r	r	$-1+r$	r	r	1	1
1	1	0	r	0	r	r	r	r	1	1	0	1
1	0	0	$1+r$	1	r	r	r	r	r	r	1	1

All column values are in encrypted form ($E_{pk}(.)$) except M_i column. Also, r is a random in $\mathbb{Z}_N$ which is different for each row and column.

Algorithm 4 $\text{SMIN}_n([d_1], \ldots, [d_n]) \rightarrow [d_{\min}]$

Require: P_1 has $([d_1], \ldots, [d_n])$; P_2 has sk

1: P_1:
   (a). $[d'_i] \leftarrow [d_i]$, for $1 \leq i \leq n$, and $num \leftarrow n$
2: P_1 and P_2, **for** $i = 1$ to $\lceil \log_2 n \rceil$:
   (a). **for** $1 \leq j \leq \lfloor \frac{num}{2} \rfloor$:
   - **if** $i = 1$ **then**:
     - $[d'_{2j-1}] \leftarrow \text{SMIN}([d'_{2j-1}], [d'_{2j}])$
     - $[d'_{2j}] \leftarrow 0$

     else
     - $[d'_{2i(j-1)+1}] \leftarrow \text{SMIN}([d'_{2i(j-1)+1}], [d'_{2ij-1}])$
     - $[d'_{2ij-1}] \leftarrow 0$

   (b). $num \leftarrow \lceil \frac{num}{2} \rceil$
3: P_1 sets $[d_{\min}]$ to $[d'_1]$

Secure Minimum out of n Numbers (SMIN_n). Consider P_1 with private input $([d_1], \ldots, [d_n])$ and P_2 with sk, where $0 \leq d_i < 2^l$ and $[d_i] = \langle E_{pk}(d_{i,1}), \ldots, E_{pk}(d_{i,l}) \rangle$, for $1 \leq i \leq n$. The goal of the SMIN_n protocol is to compute $[\min(d_1, \ldots, d_n)] = [d_{\min}]$ without revealing any information about d_i's to P_1 and P_2. Here we construct a new SMIN_n protocol by utilizing SMIN as the building block. The proposed SMIN_n protocol is an iterative approach and it computes the desired output in an hierarchical fashion. In each iteration, minimum between a pair of values is computed and are fed as input to the next iteration. Therefore, generating a binary execution tree in a bottom-up fashion. At the end, only P_1 knows the final result $[d_{\min}]$.

The overall steps involved in the proposed SMIN_n protocol are highlighted in Algorithm 4. Initially, P_1 assigns $[d_i]$ to a temporary vector $[d'_i]$, for $1 \leq i \leq n$. Also, he/she creates a global variable num and initialize it to n, where num represents the number of (non-zero) vectors involved in each iteration. Since the SMIN_n protocol executes in a binary tree hierarchy (bottom-up fashion), we have $\lceil \log_2 n \rceil$ iterations, and in each iteration, the number of vectors involved varies. In the first iteration (i.e., $i = 1$), P_1 with private input $([d'_{2j-1}], [d'_{2j}])$ and P_2 with sk involve in the SMIN protocol, for $1 \leq j \leq \lfloor \frac{num}{2} \rfloor$. At the end of the first iteration, only P_1 knows $[\min(d'_{2j-1}, d'_{2j})]$ and nothing is revealed to P_2, for $1 \leq j \leq \lfloor \frac{num}{2} \rfloor$. Also, P_1 stores the result $[\min(d'_{2j-1}, d'_{2j})]$ in $[d'_{2j-1}]$, updates $[d'_{2j}]$ to zero and num to $\lceil \frac{num}{2} \rceil$.

During the i^{th} iteration, only the non-zero vectors are involved, for $2 \leq i \leq \lceil \log_2 n \rceil$. For example, during second iteration (i.e., $i = 2$), only $[d'_1], [d'_3]$, and so on are involved. Note that in each iteration, the output is revealed only to P_1 and num is updated to $\lceil \frac{num}{2} \rceil$. At the end of SMIN_n, P_1 assigns the final encrypted binary vector of global minimum value, i.e., $[\min(d_1, \ldots, d_n)]$ which is stored in $[d'_1]$ to $[d_{\min}]$. For example, assume that P_1 holds $\langle [d_1], \ldots, [d_6] \rangle$ (i.e., $n = 6$). Then, based on the SMIN_n protocol, the binary execution tree (in a bottom-up fashion) to compute $[\min(d_1, \ldots, d_6)]$ is as shown in Figure 1. Note that, $[d'_i]$ is initially set to $[d_i]$, for $1 \leq i \leq 6$.

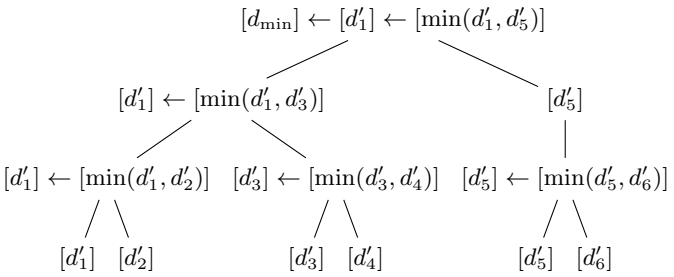


Fig. 1. Binary execution tree for $n = 6$ based on SMIN_n

Secure Bit-OR (SBOR). P_1 holds $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 holds sk, where o_1 and o_2 are two bits not known to both parties. The goal of the SBOR protocol is to securely compute $E_{pk}(o_1 \vee o_2)$. At the end of this protocol, only P_1 knows $E_{pk}(o_1 \vee o_2)$. During this process, no information related to o_1 and o_2 is revealed to P_1 and P_2. Using SM, P_1 and P_2 compute $E_{pk}(o_1 \vee o_2)$ as follows:

- P_1 with input $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 with sk involve in the SM protocol. At the end of this step, the output $E_{pk}(o_1 * o_2)$ is known only to P_1. Note that, since o_1 and o_2 are bits, $E_{pk}(o_1 * o_2) = E_{pk}(o_1 \wedge o_2)$.
- $E_{pk}(o_1 \vee o_2) = E_{pk}(o_1 + o_2) * E_{pk}(o_1 \wedge o_2)^{N-1}$.

We emphasize that, for any given two bits o_1 and o_2, the property $o_1 \vee o_2 = o_1 + o_2 - o_1 \wedge o_2$ always holds.

It is worth pointing out that SMIN, SMIN_n and SBOR are completely new and are not based on any existing protocols. On the other hand, SSED is not new, but our implementation is more efficient. Also, SM and SBD are directly adopted from the literature.

IV. THE PROPOSED PROTOCOLS

In this section, we first present a basic SkNN protocol and demonstrate why such a simple solution is not secure. Then, we discuss our second approach, a fully secure kNN protocol. Both protocols are constructed using the security primitives discussed in Section III as building blocks.

As mentioned earlier, we assume that Alice's database consists of n records, denoted by $T = \langle t_1, \ldots, t_n \rangle$, and m attributes, where $t_{i,j}$ denotes the j^{th} attribute value of record t_i. Initially, Alice encrypts her database attribute-wise, that is, she computes $E_{pk}(t_{i,j})$, for $1 \leq i \leq n$ and $1 \leq j \leq m$. Let the encrypted database be denoted by $E_{pk}(T)$. We assume that Alice outsources $E_{pk}(T)$ as well as the future query processing service to the cloud. Also, we assume that all attribute values and their Euclidean distances lie in $[0, 2^l)$.

In our proposed protocols, we assume the existence of two non-colluding semi-honest cloud service providers, denoted by C_1 and C_2, which together form a federated cloud. We emphasize that such an assumption is not new and has been commonly used in the related problem domains (e.g., [24]). The intuition behind such an assumption is as follows. Most of the cloud service providers in the market are well-established IT companies, such as Amazon and Google. Therefore, a collusion between them is highly unlikely as it will damage their reputation which in turn effects their revenues.

Under this setting, Alice outsources her encrypted database $E_{pk}(T)$ to C_1 and the secret key sk to C_2. The goal of the proposed protocols is to retrieve the top k records that are closest to the user query in an efficient and secure manner. Briefly, consider an authorized user Bob who wants to find k records that are closest to his query record $Q = \langle q_1, \ldots, q_m \rangle$ based on $E_{pk}(T)$ in C_1. Bob initially sends his query Q (in encrypted form) to C_1. After this, C_1 and C_2 involve in a set of sub-protocols to securely retrieve (in encrypted form) the set of k records corresponding to the k-nearest neighbors of the input query Q. At the end of our protocols, only Bob will receive the k-nearest neighbors to Q as the output.

A. Basic Protocol

In the basic secure k-nearest neighbor query protocol, denoted by SkNN$_\text{b}$, we relax the desirable properties to produce an efficient protocol (more details are given in the later part of this section).

The main steps involved in the SkNN$_\text{b}$ protocol are given in Algorithm 5. Bob initially encrypts his query Q attribute-wise, that is, he computes $E_{pk}(Q) = \langle E_{pk}(q_1), \ldots, E_{pk}(q_m) \rangle$ and sends it to C_1. Upon receiving $E_{pk}(Q)$ from Bob, C_1 with private input $(E_{pk}(Q), E_{pk}(t_i))$ and C_2 with the secret key sk jointly involve in the SSED protocol, where $E_{pk}(t_i) = \langle E_{pk}(t_{i,1}), \ldots, E_{pk}(t_{i,m}) \rangle$, for $1 \leq i \leq n$. The output of this step, denoted by $E_{pk}(d_i)$, is the encryption of squared Euclidean distance between Q and t_i, i.e., $d_i = |Q - t_i|^2$. As mentioned earlier, $E_{pk}(d_i)$ is known only to C_1, for $1 \leq i \leq n$. We emphasize that computation of exact Euclidean distance between encrypted vectors is hard to achieve as it involves square root. However, in our problem, it is sufficient to compare the squared Euclidean distances as it preserves relative ordering. After this, C_1 sends $\{\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle\}$ to C_2, where entry $\langle i, E_{pk}(d_i) \rangle$ correspond to data record t_i, for $1 \leq i \leq n$. Upon receiving $\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle$, C_2 decrypts the encrypted distance in each entry to get $d_i = D_{sk}(E_{pk}(d_i))$. Then, C_2 generates an index list $\delta = \langle i_1, \ldots, i_k \rangle$ such that $\langle d_{i_1}, \ldots d_{i_k} \rangle$ are the top k smallest distances among $\langle d_1, \ldots, d_n \rangle$. After this, C_2 sends δ to C_1. Upon receiving δ, C_1 proceeds as follows:

- Select the encrypted records $E_{pk}(t_{i_1}), \ldots, E_{pk}(t_{i_k})$ as the k-nearest records to Q and randomize them attribute-wise. More specifically, C_1 computes $E_{pk}(\gamma_{j,h}) = E_{pk}(t_{i_j,h}) * E_{pk}(r_{j,h})$, for $1 \leq j \leq k$ and $1 \leq h \leq m$. Here $r_{j,h}$ is a random number in $\mathbb{Z}_N$ and $t_{i_j,h}$ denotes the column h attribute value of data record t_{i_j}. Send $\gamma_{j,h}$ to C_2 and $r_{j,h}$ to Bob, for $1 \leq j \leq k$ and $1 \leq h \leq m$.

Upon receiving $\gamma_{j,h}$, for $1 \leq j \leq k$ and $1 \leq h \leq m$, C_2 decrypts it to get $\gamma'_{j,h} = D_{sk}(\gamma_{j,h})$ and sends them to Bob. Note that, due to randomization by C_1, $\gamma'_{j,h}$ is always a random number in $\mathbb{Z}_N$.

Finally, upon receiving $r_{j,h}$ from C_1 and $\gamma'_{j,h}$ from C_2, Bob computes the attribute values of j^{th} nearest neighbor to Q as $t'_{j,h} = \gamma'_{j,h} - r_{j,h} \mod N$, for $1 \leq j \leq k$ and $1 \leq h \leq m$.

Algorithm 5 SkNN$_\text{b}(E_{pk}(T), Q) \rightarrow \langle t'_1, \ldots, t'_k \rangle$

Require: C_1 has $E_{pk}(T)$; C_2 has sk; Bob has Q

1: Bob:
   (a). Compute $E_{pk}(q_j)$, for $1 \leq j \leq m$
   (b). Send $E_{pk}(Q) = \langle E_{pk}(q_1), \ldots, E_{pk}(q_m) \rangle$ to C_1
2: C_1 and C_2:
   (a). C_1 receives $E_{pk}(Q)$ from Bob
   (b). **for** $i = 1$ to n **do**:
   - $E_{pk}(d_i) \leftarrow \text{SSED}(E_{pk}(Q), E_{pk}(t_i))$

   (c). Send $\{\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle\}$ to C_2
3: C_2:
   (a). Receive $\{\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle\}$ from C_1
   (b). $d_i \leftarrow D_{sk}(E_{pk}(d_i))$, for $1 \leq i \leq n$
   (c). Generate $\delta \leftarrow \langle i_1, \ldots, i_k \rangle$, such that $\langle d_{i_1}, \ldots, d_{i_k} \rangle$ are the top k smallest distances among $\langle d_1, \ldots, d_n \rangle$
   (d). Send δ to C_1
4: C_1:
   (a). Receive δ from C_2
   (b). **for** $1 \leq j \leq k$ and $1 \leq h \leq m$ **do**:
   - $\gamma_{j,h} \leftarrow E_{pk}(t_{i_j,h}) * E_{pk}(r_{j,h})$, where $r_{j,h} \in_R \mathbb{Z}_N$
   - Send $\gamma_{j,h}$ to C_2 and $r_{j,h}$ to Bob

5: C_2:
   (a). **for** $1 \leq j \leq k$ and $1 \leq h \leq m$ **do**:
   - Receive $\gamma_{j,h}$ from C_1
   - $\gamma'_{j,h} \leftarrow D_{sk}(\gamma_{j,h})$; send $\gamma'_{j,h}$ to Bob

6: Bob:
   (a). **for** $1 \leq j \leq k$ and $1 \leq h \leq m$ **do**:
   - Receive $r_{j,h}$ from C_1 and $\gamma'_{j,h}$ from C_2
   - $t'_{j,h} \leftarrow \gamma'_{j,h} - r_{j,h} \mod N$

B. Fully Secure kNN Protocol

The above-mentioned SkNN$_\text{b}$ protocol reveals the data access patterns to C_1 and C_2. That is, for any given Q, C_1 and C_2 know which data records correspond to the k-nearest neighbors of Q. Also, it reveals d_i values to C_2. However, leakage of such information may not be acceptable in privacy-sensitive applications such as medical data. Along this direction, we propose a fully secure protocol, denoted by SkNN$_\text{m}$(where m stands for maximally secure), to retrieve the k-nearest neighbors of Q. The proposed SkNN$_\text{m}$ protocol preserves all the desirable properties of a secure kNN protocol as mentioned in Section I.

The main steps involved in the proposed SkNN$_\text{m}$ protocol are as shown in Algorithm 6. Initially, Bob sends his attribute-wise encrypted query Q, that is, $E_{pk}(Q) = \langle E_{pk}(q_1), \ldots, E_{pk}(q_m) \rangle$ to C_1. Upon receiving, C_1 with private input $(E_{pk}(Q), E_{pk}(t_i))$ and C_2 with the secret key sk jointly involve in the SSED protocol. The output of this step is $E_{pk}(d_i) = E_{pk}(|Q - t_i|^2)$ which will be known only to C_1, for $1 \leq i \leq n$. Then, C_1 with input $E_{pk}(d_i)$ and C_2 with sk securely compute the encryptions of the individual bits of d_i using the SBD protocol. Note that the output of this step $[d_i] = \langle E_{pk}(d_{i,1}), \ldots, E_{pk}(d_{i,l}) \rangle$ is known only to C_1,

Algorithm 6 $\text{S}k\text{NN}_\text{m}(E_{pk}(T), Q) \rightarrow \langle t'_1, \ldots, t'_k \rangle$

Require: C_1 has $E_{pk}(T)$ and π; C_2 has sk; Bob has Q

1: Bob sends $E_{pk}(Q) = \langle E_{pk}(q_1), \ldots, E_{pk}(q_m) \rangle$ to C_1

2: C_1 and C_2:

(a). C_1 receives $E_{pk}(Q)$ from Bob

(b). **for** $i = 1$ to n **do:**

- $E_{pk}(d_i) \leftarrow \text{SSED}(E_{pk}(Q), E_{pk}(t_i))$
- $[d_i] \leftarrow \text{SBD}(E_{pk}(d_i))$

3: **for** $s = 1$ to k **do:**

(a). C_1 and C_2:

- $[d_{\min}] \leftarrow \text{SMIN}_n([d_1], \ldots, [d_n])$

(b). C_1:

- $E_{pk}(d_{\min}) \leftarrow \prod_{\gamma=0}^{l-1} E_{pk}(d_{\min,\gamma+1})^{2^{l-\gamma-1}}$
- **if** $s \neq 1$ **then**, for $1 \leq i \leq n$
  - $E_{pk}(d_i) \leftarrow \prod_{\gamma=0}^{l-1} E_{pk}(d_{i,\gamma+1})^{2^{l-\gamma-1}}$
- **for** $i = 1$ to n **do:**
  - $\tau_i \leftarrow E_{pk}(d_{\min}) * E_{pk}(d_i)^{N-1}$
  - $\tau'_i \leftarrow \tau_i^{r_i}$, where $r_i \in_R \mathbb{Z}_N$
- $\beta \leftarrow \pi(\tau')$; send β to C_2

(c). C_2:

- Receive β from C_1
- $\beta'_i \leftarrow D_{sk}(\beta_i)$, for $1 \leq i \leq n$
- Compute U, for $1 \leq i \leq n$:
  - **if** $\beta'_i = 0$ **then** $U_i = E_{pk}(1)$
  - **else** $U_i = E_{pk}(0)$
- Send U to C_1

(d). C_1:

- Receive U from C_2 and compute $V \leftarrow \pi^{-1}(U)$
- $V'_{i,j} \leftarrow \text{SM}(V_i, E_{pk}(t_{i,j}))$, for $1 \leq i \leq n$ and $1 \leq j \leq m$
- $E_{pk}(t'_{s,j}) \leftarrow \prod_{i=1}^{n} V'_{i,j}$, for $1 \leq j \leq m$
- $E_{pk}(t'_s) = \langle E_{pk}(t'_{s,1}), \ldots, E_{pk}(t'_{s,m}) \rangle$

(e). C_1 and C_2, for $1 \leq i \leq n$:

- $E_{pk}(d_{i,\gamma}) \leftarrow \text{SBOR}(V_i, E_{pk}(d_{i,\gamma}))$, for $1 \leq \gamma \leq l$

The rest of the steps are similar to steps 4-6 of $\text{S}k\text{NN}_\text{b}$

where $d_{i,1}$ and $d_{i,l}$ are the most and least significant bits of d_i respectively. Note that $0 \leq d_i < 2^l$, for $1 \leq i \leq n$.

After this, C_1 and C_2 compute the top k (in encrypted form) records that are closest to Q in an iterative manner. More specifically, they compute $E_{pk}(t'_1)$ in the first iteration, $E_{pk}(t'_2)$ in the second iteration, and so on. Here t'_s denotes the s^{th} nearest neighbor to Q, for $1 \leq s \leq k$. At the end of k iterations, only C_1 knows $\langle E_{pk}(t'_1), \ldots, E_{pk}(t'_k) \rangle$. To start with, in the first iteration, C_1 and C_2 jointly compute the encryptions of the individual bits of the minimum value among $d_1, \ldots, d_n$ using SMIN_n. That is, C_1 with input $\langle [d_1], \ldots, [d_n] \rangle$ and C_2 compute $[d_{\min}]$, where $d_{\min}$ is the minimum value among $d_1, \ldots, d_n$. The output $[d_{\min}]$ is known only to C_1. Now, C_1 performs the following operations locally:

- Compute the encryption of $d_{\min}$ from its encrypted individual bits as below

$$\begin{aligned} E_{pk}(d_{\min}) &= \prod_{\gamma=0}^{l-1} E_{pk}(d_{\min,\gamma+1})^{2^{l-\gamma-1}} \\ &= E_{pk}(d_{\min,1} * 2^{l-1} + \cdots + d_{\min,l}) \end{aligned}$$

  where $d_{\min,1}$ and $d_{\min,l}$ are the most and least significant bits of $d_{\min}$ respectively.
- Compute the encryption of difference between $d_{\min}$ and each d_i. That is, C_1 computes $\tau_i = E_{pk}(d_{\min}) * E_{pk}(d_i)^{N-1} = E_{pk}(d_{\min} - d_i)$, for $1 \leq i \leq n$.
- Randomize τ_i to get $\tau'_i = \tau_i^{r_i} = E_{pk}(r_i * (d_{\min} - d_i))$, where r_i is a random number in $\mathbb{Z}_N$. Note that τ'_i is an encryption of either 0 or a random number, for $1 \leq i \leq n$. Also, permute τ' using a random permutation function π (known only to C_1) to get $\beta = \pi(\tau')$ and send it to C_2.

Upon receiving β, C_2 decrypts it component-wise to get $\beta'_i = D_{sk}(\beta_i)$, for $1 \leq i \leq n$. After this, he/she computes an encrypted vector U of length n such that $U_i = E_{pk}(1)$ if $\beta'_i = 0$, and $E_{pk}(0)$ otherwise. Here we assume that exactly one of the entries in β equals to zero and rest of them are random. This further implies that exactly one of the entries in U is an encryption of 1 and rest of them are encryptions of 0's. However, we emphasize that if β' has more than one 0's, then C_2 can randomly pick one of those indexes and assign $E_{pk}(1)$ to the corresponding index of U and $E_{pk}(0)$ to the rest. Then, C_2 sends U to C_1. After receiving U, C_1 performs inverse permutation on it to get $V = \pi^{-1}(U)$. Note that exactly one of the entry in V is $E_{pk}(1)$ and the remaining are encryption of 0's. In addition, if $V_i = E_{pk}(1)$, then t_i is the closest record to Q. However, C_1 and C_2 do not know which entry in V corresponds to $E_{pk}(1)$.

Finally, C_1 computes $E_{pk}(t'_1)$, encryption of the closest record to Q, and updates the distance vectors as follows:

- C_1 and C_2 jointly involve in the secure multiplication (SM) protocol to compute $V'_{i,j} = V_i * E_{pk}(t_{i,j})$, for $1 \leq i \leq n$ and $1 \leq j \leq m$. The output V' from the SM protocol is known only to C_1. After this, by using homomorphic properties, C_1 computes the encrypted record $E_{pk}(t'_1) = \langle E_{pk}(t_{1,1}), \ldots, E_{pk}(t_{1,m}) \rangle$ locally, $E_{pk}(t'_{1,j}) = \prod_{i=1}^{n} V'_{i,j}$, where $1 \leq j \leq m$. Note that $t'_{1,j}$ denotes the j^{th} attribute value of record t'_1.
- It is important to note that the first nearest tuple to Q should be obliviously excluded from further computations. However, since C_1 does not know the record corresponding to $E_{pk}(t'_1)$, we need to obliviously eliminate the possibility of choosing this record again in next iterations. For this, C_1 obliviously updates the distance corresponding to $E_{pk}(t'_1)$ to the maximum value, i.e., $2^l - 1$. More specifically, C_1 updates the distance vectors with the help of C_2 using the SBOR protocol as $E_{pk}(d_{i,\gamma}) = \text{SBOR}(V_i, E_{pk}(d_{i,\gamma}))$, for $1 \leq i \leq n$ and $1 \leq \gamma \leq l$. Note that when $V_i = E_{pk}(1)$, the corresponding distance vector d_i is set to the maximum value. That is, under this case, $[d_i] = \langle E_{pk}(1), \ldots, E_{pk}(1) \rangle$. However, when $V_i = E_{pk}(0)$, the OR operation has no affect on d_i.

The above process is repeated until k iterations, and in each iteration $[d_i]$ corresponding to the current chosen record is set to the maximum value. However, since C_1 does not know which $[d_i]$ is updated, he/she has to re-compute $E_{pk}(d_i)$ in each iteration using the corresponding $[d_i]$, for $1 \leq i \leq n$. In iteration s, $E_{pk}(t'_s)$ is known only to C_1.

At the end of the iterative step (i.e., step 3 of Algorithm 6), C_1 has $\langle E_{pk}(t'_1), \ldots, E_{pk}(t'_k)\rangle$ - the list of encrypted records of k-nearest neighbors to the input query Q. The rest of the process is similar to steps 4 to 6 of Algorithm 5. Briefly, C_1 randomizes $E_{pk}(t'_j)$ attribute-wise to get $\gamma_{j,h} = E_{pk}(t'_{j,h}) * E_{pk}(r_{j,h})$ and sends $\gamma_{j,h}$ to C_2 and $r_{j,h}$ to Bob, for $1 \leq j \leq k$ and $1 \leq h \leq m$. Here $r_{j,h}$ is a random number in $\mathbb{Z}_N$. Upon receiving $\gamma_{j,h}$'s, C_2 decrypts them to get the randomized k-nearest records as $\gamma'_{j,h} = D_{sk}(\gamma_{j,h})$ and sends them to Bob, for $1 \leq j \leq k$ and $1 \leq h \leq m$. Finally, upon receiving $r_{j,h}$ from C_1 and $\gamma'_{j,h}$ from C_2, Bob computes the j^{th} nearest neighboring record to Q, as $t'_{j,h} = \gamma'_{j,h} - r_{j,h} \bmod N$, for $1 \leq j \leq k$ and $1 \leq h \leq m$.

C. Security Analysis

First, due to the encryption of Q and by semantic security of the Paillier cryptosystem, Bob's input query Q is protected from Alice, C_1 and C_2 in both protocols.

In the SkNN$_\text{b}$ protocol, the decryption operations at step 3(b) of Algorithm 5 reveal d_i values to C_2. In addition, since C_2 generates the top k index list (at step 3(c) of Algorithm 5) and sends it to C_1, the data access patterns are revealed to C_1 and C_2. Therefore, our basic SkNN$_\text{b}$ protocol is secure under the assumption that d_i values can be revealed to C_2 and data access patterns can be revealed to C_1 and C_2.

On the other hand, the security analysis of SkNN$_\text{m}$ is as follows. At step 2 of Algorithm 6, the outputs of SSED and SBD are in encrypted format, and are known only to C_1. In addition, all the intermediate results decrypted by C_2 in SSED are uniformly random in $\mathbb{Z}_N$. Also, as mentioned in [23], the SBD protocol is secure. Thus, no information is revealed during step 2 of Algorithm 6. In each iteration, the output of SMIN$_n$ is known only to C_1 and no information is revealed to C_2. Also, C_1 and C_2 do not know which record belongs to current global minimum. Thus, data access patterns are protected from both C_1 and C_2. At step 3(c) of Algorithm 6, a component-wise decryption of β reveals the tuples that satisfy the current global minimum distance to C_2. However, due to permutation by C_1, C_2 cannot trace back to the corresponding data records. Also, note that decryption of β gives either encryptions of 0's or random numbers in $\mathbb{Z}_N$. Similarly, since U is an encrypted vector, C_1 cannot know which tuple corresponds to current global minimum distance. Thus, data access patterns are further protected at this step from C_1. In addition, the update process at step 3(e) of Algorithm 6 does not leak any information to C_1 and C_2. In summary, C_1 and C_2 do not know which data records correspond to the output set $\langle t'_1, \ldots, t'_k\rangle$.

Based on the above discussions, it is clear that the proposed SkNN$_\text{m}$ protocol protects the confidentiality of the data, privacy of user's input query, and hides the data access patterns.

D. Complexity Analysis

The computation complexity of SkNN$_\text{b}$ is bounded by $O(n * m + k)$ encryptions, decryptions and exponentiations. In practice $k \ll n * m$; therefore, the computation complexity of SkNN$_\text{b}$ is bounded by $O(n * m)$ encryptions and exponentiations (assuming that encryption and decryption operations under Paillier cryptosystem take similar amount of time). On the other hand, the computation complexity of SkNN$_\text{m}$ is bounded by $O(n * (l + m + k * l * \log_2 n))$ encryptions and exponentiations. Due to space limitations, we refer the reader to our technical report [25] for a detailed complexity analyses.

Depending on the encryption key size, the overall computation cost of the proposed SkNN$_\text{m}$ (more expensive than SkNN$_\text{b}$) is between 2 and 3 orders of magnitude higher than the non-crypto cases (the related works acknowledged in the paper). This is the cost we need to pay to maximize data confidentiality. However, on the user or client side, the running time is comparable to the non-crypto case since the user only performs a very small number of encryption operations (bounded by the number of attributes) which was done in less than a second as shown in our experiments. Our goal is to outsource all or most computations to the cloud so that the user can issue queries using any mobile device with limited storage and computing capability. Note that, data confidentiality is fully protected under the proposed SkNN$_\text{m}$ protocol.

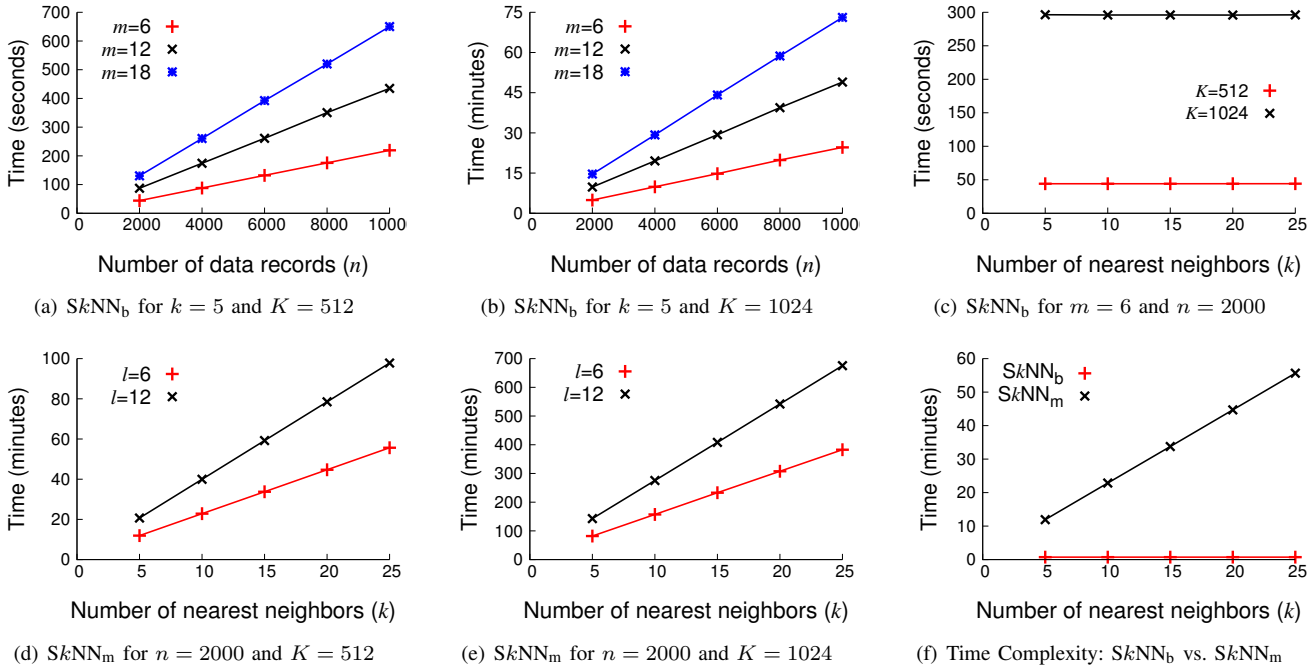
V. Empirical Results

In this section, we discuss the performances of the proposed protocols in detail under different parameter settings. We used Paillier cryptosystem [15] and implemented the proposed protocols in C. Various experiments were conducted on a Linux machine with an Intel® Xeon® Six-Core™ CPU 3.07 GHz processor and 12GB RAM running Ubuntu 10.04 LTS.

Since it is difficult to control the parameters in a real dataset, we randomly generated synthetic datasets depending on the parameter values in consideration. Using these synthetic datasets we can perform a more elaborated analysis on the computation costs of the proposed protocols under different parameter settings. We encrypted these datasets attribute-wise, using the Paillier encryption whose key size is varied in our experiments, and the encrypted data were stored on our machine. Based on the proposed protocols, we then executed a random query over this encrypted data. For the rest of this section, we do not discuss about the performance of Alice since it is a one-time cost. Instead, we evaluate and analyze the performances of SkNN$_\text{b}$ and SkNN$_\text{m}$ separately. In addition, we compare the two protocols. In our experiments, the Paillier encryption key size K is set to either 512 or 1024 bits.

A. Performance of SkNN$_b$

In this sub-section, we analyze the computation costs of SkNN$_\text{b}$ by varying the number of data records (n), number of attributes (m), number of nearest neighbors (k), and encryption



(a) SkNN$_b$ for $k = 5$ and $K = 512$

(b) SkNN$_b$ for $k = 5$ and $K = 1024$

(c) SkNN$_b$ for $m = 6$ and $n = 2000$

(d) SkNN$_m$ for $n = 2000$ and $K = 512$

(e) SkNN$_m$ for $n = 2000$ and $K = 1024$

(f) Time Complexity: SkNN$_b$ vs. SkNN$_m$

Fig. 2. Time complexities of SkNN$_b$ and SkNN$_m$ for varying values of n, m, l, k and encryption key size K

key size (K). Note that SkNN$_b$ is independent of the domain size of attributes (l).

First, by fixing $k = 5$ and $K = 512$, we evaluated the computation costs of SkNN$_b$ for varying n and m. As shown in Figure 2(a), the computation costs of SkNN$_b$ grows linearly with n and m. For example, when $m = 6$, the computation time of SkNN$_b$ increases from 44.08 to 87.91 seconds when n is varied from 2000 to 4000. A similar trend is observed for $K = 1024$ as shown in Figure 2(b). For any fixed parameters, we observed that the computation time of SkNN$_b$ increases almost by a factor of 7 when K is doubled.

Next, by fixing $m = 6$ and $n = 2000$, we evaluated the running times of SkNN$_b$ for varying k and K. The results are shown in Figure 2(c). Irrespective of K, the computation time of SkNN$_b$ does not change much with varying k. This is because most of the cost in SkNN$_b$ comes from the SSED protocol which is independent of k. E.g., when $K = 512$ bits, the computation time of SkNN$_b$ changes from 44.08 to 44.14 seconds when k is changed from 5 to 25. Based on the above discussions, it is clear that the running time of SkNN$_b$ mainly depends on (or grows linearly with) n and m which further justifies our complexity analysis in Section IV-D.

B. Performance of SkNN$_m$

We also evaluated the computation costs of SkNN$_m$ for varying values of k, l and K. Throughout this sub-section, we fix $m = 6$ and $n = 2000$. However, we observed that the running time of SkNN$_m$ grows almost linearly with n and m.

For K = 512 bits, the computation costs of SkNN$_m$ for varying k and l are as shown in Figure 2(d). Following from Figure 2(d), for $l = 6$, the running time of SkNN$_m$ varies from 11.93 to 55.65 minutes when k is changed from 5 to 25 respectively. Also, for $l = 12$, the running time of SkNN$_m$ varies from 20.68 to 97.8 minutes when k is changed from 5 to 25 respectively. In either case, the cost of SkNN$_m$ grows almost linearly with k and l.

A similar trend is observed for $K = 1024$ as shown in Figure 2(e). In particular, for any given fixed parameters, we identified that the computation cost of SkNN$_m$ increases by almost a factor of 7 when K is doubled. For example, when k = 10, SkNN$_m$ took 22.85 and 157.17 minutes to generate the 10 nearest neighbors of Q under K = 512 and 1024 bits respectively. Furthermore, when k = 5, we observed that around 69.7% of cost in SkNN$_m$ is accounted due to SMIN$_n$ which is initiated k times in SkNN$_m$ (once in each iteration). Also, the cost incurred due to SMIN$_n$ increases from 69.7% to at least 75% when k is increased from 5 to 25.

In addition, by fixing $n = 2000, m = 6, l = 6$ and $K = 512$, we compared the running times of both protocols for varying values of k. As shown in Figure 2(f), the running time of SkNN$_b$ remains to be constant at 0.73 minutes since it is almost independent of k. However, the running time of SkNN$_m$ changes from 11.93 to 55.65 minutes as we increase k from 5 to 25.

Based on the above results, it is clear that the computation costs of SkNN$_m$ are significantly higher than that of SkNN$_b$. However, we emphasize that SkNN$_m$ is more secure than SkNN$_b$; therefore, the two protocols act as a trade-off between security and efficiency. Also, it is important to note that Bob's computation cost is mainly due to the encryption of his input query record. As an example, for $m = 6$, Bob's computation costs are 4 and 17 milliseconds when K is 512 and 1024 bits respectively. This further shows that our protocols are very efficient from end-user's perspective. In SkNN$_b$, SSED is the bottleneck whereas in SkNN$_m$ the bottleneck is SMIN$_n$.

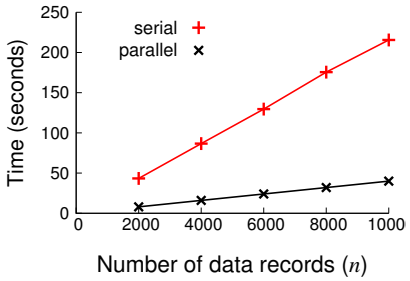


Fig. 3. Parallel vs. serial versions of SkNN$_{\text{b}}$ for $m = 6, k = 5$ and $K = 512$

C. Towards Performance Improvement

At first, it seems that the proposed protocols are costly and may not scale well for large datasets. However, in both protocols, we emphasize that the computations involved on each data record are independent of others. Therefore, we can parallelize the operations on data records for efficiency purpose. To further justify this claim, we implemented a parallel version of our SkNN$_{\text{b}}$ protocol using OpenMP programming and compared its computation costs with its serial version. As mentioned earlier, our machine has 6 cores which can be used to perform parallel operations on 6 threads. For $m = 6, k = 5$ and $K = 512$ bits, the comparison results are shown in Figure 3. We observe that the parallel version of SkNN$_{\text{b}}$ is roughly 6 times more efficient than its serial version. This is because of the fact that the parallel version can execute operations on 6 data records at a time (i.e., on 6 threads in parallel). E.g., when $n = 10000$, the running times of parallel and serial versions of SkNN$_{\text{b}}$ are 40 and 215.59 seconds respectively.

We believe that similar efficiency gains can be achieved by parallelizing the operations in SkNN$_{\text{m}}$. Based on the above discussions, especially in a cloud computing environment where high performance parallel processing can easily be achieved, we claim that the scalability issue of the proposed protocols can be eliminated or mitigated. In addition, using the existing map-reduce techniques, we can drastically improve the performance further by executing parallel operations on multiple nodes. We will leave this analysis to future work.

VI. Conclusion

The k-nearest neighbors is one of the commonly used query in many data mining applications. Under an outsourced database environment, where encrypted data are stored in the cloud, secure query processing over encrypted data becomes challenging. The existing SkNN techniques over encrypted data are not secure. In this paper, we proposed two novel SkNN protocols over encrypted data in the cloud. The first protocol, which acts as a basic solution, leaks some information to the cloud. On the other hand, our second protocol is fully secure, that is, it protects the confidentiality of the data, user's input query, and also hides the data access patterns. However, the second protocol is more expensive compared to the basic protocol. Also, we evaluated the performance of our protocols under different parameter settings. As a future work, we will investigate and extend our research to other complex conjunctive queries over encrypted data.

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