A Simple Orthogonal Space-Time Coding Scheme for Asynchronous Cooperative Systems for Frequency Selective Fading Channels

Zheng Li, Xiang-Gen Xia, and Moon Ho Lee

***Abstract*—In this paper, we propose a simple orthogonal space time transmission scheme for asynchronous cooperative systems. In the proposed scheme, OFDM is implemented at the source node, some very simple operations, namely time reversion and complex conjugation, are implemented at the relay nodes, and a two-step of cyclic prefix (CP) removal is performed at the destination. The CP at the source node is used for combating the frequency selective fading channels and the timing errors. In this scheme, the received signals at the destination node have the orthogonal code structure on each subcarrier and thus it has the fast symbol-wise ML decoding and can achieve full spatial diversity when SNR is large without the requirement of symbol level synchronization. It should be emphasized that since no Add/Remove CP or IFFT/FFT operation is needed at the relay nodes, the relay nodes do not have to know any information about the channels and the timing errors, and the complexity of the relay nodes is very low.**

***Index Terms*—Alamouti code, asynchronous cooperative systems, OFDM, orthogonal codes.**

I. INTRODUCTION

SPACE-TIME coding is an effective technique to exploit spatial diversity not only for MIMO but also for cooperative communication systems [1]. However, in cooperative systems, since different relay nodes have different oscillators and different locations, there may exist timing errors, i.e., the signals transmitted from different relay nodes may arrive at the destination at different times. There have been some studies on space-time coding to achieve asynchronous cooperative diversity, see for example, [2]-[9].

In [2], a simple Alamouti scheme is proposed to achieve asynchronous cooperative diversity, where the relay nodes only need to perform a few very simple operations: time-reversion and complex conjugation, and the destination node has the Alamouti code structure. However, there are mainly two drawbacks of the scheme in [2]. First, the proposed scheme is only valid for the case of two relay nodes. Second, it is only valid for flat fading channels. In [3], it is extended to the case of any number of relay nodes but is still limited to flat fading channels. Moreover, the complexity of the four group decodable codes used in [3] is higher than the symbol-wise decoding of OSTBCs. Recently, there is a similar scheme in [4] to achieve cooperative diversity in asynchronous two-way relay networks. The scheme in [4] is valid for frequency selective fading channels for any number of relay nodes. However, the scheme in [4] requires Add/Remove CP at the relay nodes, which means that the relay nodes have to know the maximum path delay of the channels and the maximum value of the timing errors and therefore it may increase the overhead of the whole system. Since there are two kinds of timing errors in the two-way relay networks [4], Add/Remove CP at the relay nodes seems to be mandatory for the system studied in [4].

The proposed scheme in this paper is an extension and an improvement of the one in [2]. In this paper, we consider frequency selective fading channels and the proposed OSTBC scheme is valid for any number of relay nodes. We propose that the source node to implement OFDM with CP to combat frequency selective fading and the timing errors, the relay nodes also only to implement time-reversion and complex conjugation, and the destination node to implement a two-step of CP removal. By doing so, at the destination node, the received signals have the orthogonal code [10], [11] structure on each subcarrier and thus it has the fast symbol-wise ML decoding. It is also shown that the proposed simple scheme can achieve full spatial diversity when SNR is large. Since no Add/Remove CP or IFFT/FFT operation is needed at the relay nodes, the relay nodes do not have to know any information about the channels and the timing errors, and therefore the complexity of the relay nodes is very low. Comparing to the scheme in [4], it is simpler and reduces the overhead of the whole system. In order to achieve the multipath diversity, repeating the proposed OSTBC across subcarriers as space-time-frequency coding can be similarly done as in [4]. The validity of the proposed scheme is proved both mathematically and from simulations.

This paper is organized as follows. In Section II, the system model is described. The simple space-time transmission scheme is given and the validity of the scheme is proved in Section III. Simulation results are presented in Section IV. Finally the conclusions are given in Section V.

II. SYSTEM MODEL

Consider a cooperative system with one source node, one destination node and J relay nodes $R_i, 1 \leq i \leq J$, as shown in Fig. 1. Every node in the system is assumed to have only one antenna. We consider half-duplex mode in this paper. To transmit the information from the source node S to the destination node D, there undergo two phases. In the first phase, the source node S broadcasts the information to

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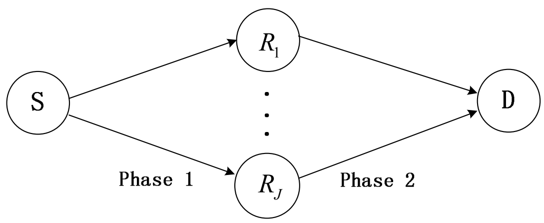

Fig. 1. Cooperative system architecture.

the J relay nodes. Meanwhile, the relay nodes receive the information. During the second phase, the source node S stops the transmission and the J relay nodes process and send the received signals to the destination node D. It is assumed that there is no direct transmission between S and D.

Assume the channel between any two terminals S $\rightarrow R_i$ or $R_i \rightarrow$ D is frequency selective Rayleigh fading with L independent propagation paths. We also assume that the channel is quasi-static, i.e., slow fading. The channel impulse response from the source node S to the ith relay node R_i is written as

$$h_{SR_i}(t) = \sum_{l=0}^{L-1} \alpha_{SR_i}(l)\delta(t - \tau_{l,SR_i}) \tag{1}$$

where $\alpha_{SR_i}(l)$ represents the channel coefficient of the l-th path of the channel from S to R_i, τ_{l,SR_i} is the corresponding path delay. Each channel coefficient $\alpha_{SR_i}(l)$ is modelled as zero mean complex Gaussian random variable with variance σ^2_{l,SR_i}. We also assume that $\alpha_{SR_i}(l)$ are i.i.d. random variables for any i, l. For convenience, the power of the L paths are normalized such that $\sum_{l=0}^{L-1} \sigma^2_{l,SR_i} = 1$.

Similarly, the channel impulse response from the ith relay node R_i to the destination D is written as

$$h_{R_iD}(t) = \sum_{l=0}^{L-1} \alpha_{R_iD}(l)\delta(t - \tau_{l,R_iD}) \tag{2}$$

where $\alpha_{R_iD}(l)$ represents the channel coefficient of the l-th path of the channel from R_i to D, τ_{l,R_iD} is the corresponding path delay. Each channel coefficient $\alpha_{R_iD}(l)$ is modelled as zero mean complex Gaussian random variable with variance σ^2_{l,R_iD}. $\alpha_{R_iD}(l)$ are i.i.d. random variables for any i, l and the power of the L paths are normalized to $\sum_{l=0}^{L-1} \sigma^2_{l,R_iD} = 1$.

III. A Simple Space-Time Coding Scheme

In this section, we design a simple space-time coding scheme for the asynchronous cooperative system to achieve full cooperative (spatial) diversity and fast ML decoding at the destination. Without loss of generality, we assume that the signals from R_i, $i > 1$, arrive at the destination later than the signals from R_1. First let us consider the case of two relay nodes in the system. Then we will show that the scheme is also valid when the relay nodes are more than two.

A. Implementation at the Source Node

At the source node, information bits are first modulated into complex symbols $X_{i,j}$, then each N modulated symbols as a block are fed to an OFDM modulator of N subcarriers. Denote two consecutive OFDM blocks as $\mathbf{X}_1$=$[X_{1,0}, X_{1,1}, \ldots, X_{1,N-1}]^T$ and $\mathbf{X}_2$=$[X_{2,0}, X_{2,1}, \ldots, X_{2,N-1}]^T$, where $(\cdot)^T$ represents the transpose operation. In the OFDM modulator, the two consecutive blocks are modulated by N-point FFT. Then each block is preceded by a cyclic prefix (CP) with length ℓ_{cp}. Thus, each OFDM symbol consists of $L_s \triangleq N + \ell_{cp}$ samples. Finally, the OFDM symbols are broadcasted to the two relays. Denote τ_{SR_2D} as the overall relative delay from the source to R_2 and then to the destination node, where "relative" means relative to relay node R_1. In order to combat both the frequency selective fading channels and the timing errors, we assume that ℓ_{cp} is larger than $\max_{i,l}\{\tau_{l,SR_i} + \tau_{l,R_iD} + \tau_{SR_2D}\}$. Note that, when the channels are flat fading, $\tau_{l,SR_i} = \tau_{l,R_iD} = 0$, and in this case, ℓ_{cp} only has to be larger than the overall timing delay τ_{SR_2D} as what has been used in [2].

Denote two consecutive OFDM symbols as $\overline{\mathbf{X}}_1$ and $\overline{\mathbf{X}}_2$, where $\overline{\mathbf{X}}_i$ consists of FFT($\mathbf{X}_i$) and the corresponding CP for $i = 1, 2$.

TABLE I
Processing at the Relay Nodes (J=2)

	R_1	R_2
OFDM 1	$\zeta(\overline{\mathbf{Y}}_{11})$	$-\overline{\mathbf{Y}}^*_{22}$
OFDM 2	$\zeta(\overline{\mathbf{Y}}_{12})$	$\overline{\mathbf{Y}}^*_{21}$

B. Implementation at the Relay Nodes

At the relay nodes, the received noisy signals will be simply processed and forwarded to the destination node as follows. Assume the channel coefficients are constant during two OFDM symbol intervals. Then, the received signals at relay i, i=1, 2, for two successive OFDM symbol durations can be written as:

$$\overline{\mathbf{Y}}_{i1} = \sqrt{P_1}\overline{\mathbf{X}}_1 * h_{SR_i} + \overline{\mathbf{n}}_{i1}, \tag{3}$$

$$\overline{\mathbf{Y}}_{i2} = \sqrt{P_1}\overline{\mathbf{X}}_2 * h_{SR_i} + \overline{\mathbf{n}}_{i2} \tag{4}$$

where $\sqrt{P_1}$ is the transmission power at the source node, h_{SR_i} is an $L \times 1$ vector defined as $h_{SR_i} = [\alpha_{SR_i}(0), \ldots, \alpha_{SR_i}(L-1)]^T$, h_{R_iD} is defined similarly, and $*$ denotes the linear convolution. $\overline{\mathbf{n}}_{i1}$ and $\overline{\mathbf{n}}_{i2}$ are the corresponding additive white Gaussian noise (AWGN) at relay node i with zero-mean and unit-variance, in two successive OFDM symbol durations, respectively.

Then, the two relay nodes will process and transmit the received noisy signals as shown in Table I, where $(\cdot)^*$ denotes the complex conjugation and $\zeta(\cdot)$ represents the time-reversal of the signal, i.e., $\zeta(\overline{\mathbf{Y}}(n)) \triangleq \overline{\mathbf{Y}}(L_s - n)$, $n = 0, 1, ..., L_s - 1$, and $\overline{\mathbf{Y}}(L_s) \triangleq \overline{\mathbf{Y}}(0)$. After performing the simple processing, the relay nodes amplify the signals with a scalar λ=$\sqrt{\frac{P_2}{P_1+1}}$ in order to maintain the average transmission power of any relay node to be P_2. Note that, although the above processing has the discrete form, it can be implemented simply in the analog domain.

Also note that, here the processing at the relay nodes is different from that in [2]. The processing in [2] is accidentally (only) valid for flat fading channels and cannot be applied to frequency selective fading channels. As explained in [4], in order to make the scheme valid for frequency selective fading channels, it is required that for any relay R_i, it can only implement time reversal on the received OFDM symbols or only implement complex conjugation on the received OFDM symbols, i.e., the operations of time reversal and complex conjugation cannot be implemented on the same relay node.

C. Implementation at the Destination Node

At the destination node, the CP is removed first for each OFDM symbol. Note that relay node R_1 implements the time reversions of the noisy signals including both information symbols and CP: $\zeta(\overline{\mathbf{Y}}(n)) = \overline{\mathbf{Y}}(L_s - n)$, $n = 0, 1, ..., L_s - 1$. What we want is, however, that after the CP removal, we want to obtain the time reversal version of only the information symbols, i.e., $\zeta(\mathrm{FFT}(\mathbf{X}_1))$ and $\zeta(\mathrm{FFT}(\mathbf{X}_2))$. Then by using some properties of FFT/IFFT, we can construct the Alamouti code structure on each subcarrier at the destination as we shall see later. For this purpose, we claim the following result.

Claim: We can obtain $\zeta(h'_{SR_1}) \circledast \zeta(\mathrm{FFT}(\mathbf{X}_i))$ at the destination by performing the following two-step of CP removal for the two consecutive OFDM symbols:

1) Remove the CP as in a conventional OFDM system and get an N-point vector;
2) Shift the last $\tau_1' = \ell_{cp} - (\tau_1 - 1)$ samples of the N-point vector as the first τ_1' samples.

In the above, h'_{SR_i} is an $N \times 1$ vector which is defined as $h'_{SR_i} = [\alpha_{SR_i}(0), \ldots, \alpha_{SR_i}(L-1), 0, \ldots, 0]^T$, h'_{R_iD} is defined similarly, and $\circledast$ denotes the circular convolution, and τ_1 denotes the maximum path delay of the channel S $\rightarrow R_1$, i.e., τ_1=$\max_l\{\tau_{l,SR_1}\}$.

Note that in [2], the CP is also removed by two steps, however, in [2], the two steps are only performed on the second OFDM symbol in two consecutive OFDM symbols, while here the two steps are performed on both of the two consecutive OFDM symbols.

The proof of this claim is in Appendix. With the above claimed result, after the CP removal, the received signals for two successive OFDM symbol durations can be written as:

$$\begin{aligned}\mathbf{z}_1 \;=\;& \lambda((\sqrt{P_1}\zeta(\mathrm{FFT}(\mathbf{X}_1)) \circledast \zeta(h'_{SR_1}) + \mathbf{n}_{11}) \circledast h'_{R_1D} \\ & -(\sqrt{P_1}(\mathrm{FFT}(\mathbf{X}_2))^* \circledast h'^*_{SR_2} \\ & +\mathbf{n}_{22}) \circledast \Gamma_{SR_2D} \circledast \Gamma'_1 \circledast h'_{R_2D}) + \mathbf{w}_1 \end{aligned} \tag{5}$$

$$\begin{aligned}\mathbf{z}_2 \;=\;& \lambda((\sqrt{P_1}\zeta(\mathrm{FFT}(\mathbf{X}_2)) \circledast \zeta(h'_{SR_1}) + \mathbf{n}_{12}) \circledast h'_{R_1D} \\ & +(\sqrt{P_1}(\mathrm{FFT}(\mathbf{X}_1))^* \circledast h'^*_{SR_2} \\ & +\mathbf{n}_{21}) \circledast \Gamma_{SR_2D} \circledast \Gamma'_1 \circledast h'_{R_2D}) + \mathbf{w}_2 \end{aligned} \tag{6}$$

where Γ_{SR_2D} is an $N \times 1$ vector that represents the timing errors in the time domain which is defined as $\Gamma_{SR_2D} = [\mathbf{0}_{\tau_{SR_2D}}, 1, 0, \ldots, 0]^T$, where $\mathbf{0}_{\tau_{SR_2D}}$ is a $1 \times \tau_{SR_2D}$ vector of all zeros, and Γ'_1 denotes the shift of τ'_1 samples in the time domain which can be similarly defined as $\Gamma'_1 = [\mathbf{0}_{\tau'_1}, 1, 0, \ldots, 0]^T$, here $\mathbf{0}_{\tau'_1}$ is a $1\times\tau'_1$ vector of all zeros. Since the signals transmitted from R_2 will arrive at the destination τ_{SR_2D} samples later and after the CP removal, the signals are further shifted by τ'_1 samples, the total number of shifted samples is denoted by $\tau_2 = \tau_{SR_2D} + \tau'_1$. $\mathbf{n}_{i1}$ and $\mathbf{n}_{i2}$ are the AWGN at the relay nodes after the CP removal, $\mathbf{w}_1$ and $\mathbf{w}_2$ are the corresponding AWGN at the destination node.

Then, the received signals are transformed by the N-point FFT. As mentioned before, because of the timing errors, the signals from relay node R_2 arrive at the destination node τ_{SR_2D} samples later than the signals from relay node R_1. Since ℓ_{cp} is long enough, we can still maintain the orthogonality between the subcarriers. The delay in the time domain corresponds to a phase change in the frequency domain:

$$\mathbf{f}^{\tau_{SR_2D}} = [1, e^{-j2\pi\tau_{SR_2D}/N}, \ldots, e^{-j2\pi\tau_{SR_2D}(N-1)/N}]^T$$

with $\mathbf{f} = [1, e^{-j2\pi/N}, \ldots, e^{-j2\pi(N-1)/N}]^T$. Similarly, the shift of τ'_1 samples in the time domain also corresponds to a phase change $\mathbf{f}^{\tau'_1}$. Thus, the total phase change is $\mathbf{f}^{\tau_2}$.

Let $\mathbf{Z}_1$=$[Z_{1,0}, Z_{1,1}, \ldots, Z_{1,N-1}]^T$ and $\mathbf{Z}_2$=$[Z_{2,0}, Z_{2,1}, \ldots, Z_{2,N-1}]^T$ be the received signals for two consecutive OFDM blocks at the destination node after the CP removal and the FFT transformations. Then, $\mathbf{Z}_1$ and $\mathbf{Z}_2$ can be written as:

$$\begin{aligned}\mathbf{Z}_1 \;=\;& \lambda[\sqrt{P_1}\mathrm{FFT}(\zeta(\mathrm{FFT}(\mathbf{X}_1))) \circ H_{SR_1} \circ H_{R_1D} \\ & +\sqrt{P_1}\mathrm{FFT}(-(\mathrm{FFT}(\mathbf{X}_2))^*) \circ \mathbf{f}^{\tau_2} \circ H_{SR_2} \circ H_{R_2D} \\ & +\mathbf{N}_{11} \circ H_{R_1D} - \mathbf{N}_{22} \circ \mathbf{f}^{\tau_2} \circ H_{R_2D}] + \mathbf{W}_1, \end{aligned} \tag{7}$$

$$\begin{aligned}\mathbf{Z}_2 \;=\;& \lambda[\sqrt{P_1}\mathrm{FFT}(\zeta(\mathrm{FFT}(\mathbf{X}_2))) \circ H_{SR_1} \circ H_{R_1D} \\ & +\sqrt{P_1}\mathrm{FFT}((\mathrm{FFT}(\mathbf{X}_1))^*) \circ \mathbf{f}^{\tau_2} \circ H_{SR_2} \circ H_{R_2D} \\ & +\mathbf{N}_{12} \circ H_{R_1D} + \mathbf{N}_{21} \circ \mathbf{f}^{\tau_2} \circ H_{R_2D}] + \mathbf{W}_2 \end{aligned} \tag{8}$$

where $\circ$ is the Hadamard product, i.e., the component-wise product, and $H_{SR_1} = \mathrm{FFT}(\zeta(h'_{SR_1})), H_{R_1D} = \mathrm{FFT}(h'_{R_1D}), H_{SR_2} = \mathrm{FFT}(h'^*_{SR_2}), H_{R_2D} = \mathrm{FFT}(h'_{R_2D})$, $\mathbf{N}_{i1} = \mathrm{FFT}(\mathbf{n}_{i1})$, $\mathbf{N}_{i2} = \mathrm{FFT}(\mathbf{n}_{i2})$ $\mathbf{W}_1 = \mathrm{FFT}(\mathbf{w}_1)$, $\mathbf{W}_2 = \mathrm{FFT}(\mathbf{w}_2)$.

We will make use of the following properties to simplify (7) and (8):

1) For an $N \times 1$ point vector $\mathbf{X}$, $(\mathrm{FFT}(\mathbf{X}))^* = \mathrm{IFFT}(\mathbf{X}^*)$;
2) For an $N \times 1$ point vector $\mathbf{X}$, $\mathrm{FFT}(\zeta(\mathrm{FFT}(\mathbf{X})))$=$\mathrm{IFFT}(\mathrm{FFT}(\mathbf{X})) = \mathbf{X}$.

By using the above two properties, (7) and (8) can be written in the following Alamouti code form on each subcarrier k, $0 \le k \le N-1$:

$$\begin{aligned}\begin{bmatrix} Z_{1,k} \\ Z_{2,k} \end{bmatrix} =\;& \lambda\sqrt{P_1}\begin{bmatrix} X_{1,k} & -X^*_{2,k} \\ X_{2,k} & X^*_{1,k} \end{bmatrix}\begin{bmatrix} H_{SR_1,k}H_{R_1D,k} \\ f_k^{\tau_2}H_{SR_2,k}H_{R_2D,k} \end{bmatrix} \\ & +\lambda\begin{bmatrix} N_{11,k}H_{R_1D,k} - N_{22,k}f_k^{\tau_2}H_{R_2D,k} \\ N_{12,k}H_{R_1D,k} + N_{21,k}f_k^{\tau_2}H_{R_2D,k} \end{bmatrix} + \begin{bmatrix} W_{1,k} \\ W_{2,k} \end{bmatrix} \end{aligned} \tag{9}$$

where $f_k^{\tau_2} = \exp(-j2\pi k\tau_2/N)$, $H_{SR_i,k}$ is the kth element of H_{SR_i}, $H_{R_iD,k}$ is the kth element of H_{R_iD}, $N_{i1,k}$ and $N_{i2,k}$ are the kth elements of $\mathbf{N}_{i1}$ and $\mathbf{N}_{i2}$, respectively, $W_{1,k}$ and $W_{2,k}$ are the kth elements of $\mathbf{W}_1$ and $\mathbf{W}_2$, respectively.The Alamouti code form in (9) tells us that the Alamouti fast symbol-wise ML decoding can be applied at the destination.

D. The Scheme for Multiple Relay Nodes

When there are more than two relay nodes, we can also construct OSTBC structure on each subcarrier if the length of

TABLE II
PROCESSING AT THE RELAY NODES (J=4)

	R_1	R_2	R_3	R_4
OFDM 1	$\zeta(\bar{\mathbf{Y}}_{11})$	$-\bar{\mathbf{Y}}_{22}^*$		
OFDM 2	$\zeta(\bar{\mathbf{Y}}_{12})$	$\bar{\mathbf{Y}}_{21}^*$		
OFDM 3			$\zeta(\bar{\mathbf{Y}}_{31})$	$-\bar{\mathbf{Y}}_{42}^*$
OFDM 4			$\zeta(\bar{\mathbf{Y}}_{32})$	$\bar{\mathbf{Y}}_{41}^*$

CP ℓ_{cp} is larger than $\max_{i,l}\{\tau_{l,SR_i} + \tau_{l,R_iD} + \tau_{SR_iD}\}$. For example, when there are four relay nodes, we can perform the processing at the relay nodes as in Table II.

Note that in Table II, R_1 and R_2 process and transmit the received signals in the first and the second OFDM symbol durations while R_3 and R_4 wait and do nothing during this period, and in the third and the fourth OFDM symbol durations, R_3 and R_4 process and transmit the received signals while R_1 and R_2 wait and do nothing. Based on this observation, we can give the way of CP removal as follows.

For the signals transmitted from R_1 and R_2, we can still perform the two steps as mentioned above in the case of two relay nodes. Thus the total number of shifted samples for R_2 is $\tau_{SR_2D} + \tau_1'$ and the total phase change for the signals transmitted from R_2 is $\mathbf{f}^{\tau_2}$. For the signals transmitted from R_3 and R_4, in the second step of the CP removal, we shift $\tau_3' = \ell_{cp} - (\tau_3 - 1)$ samples, where τ_3=$\max_l\{\tau_{l,SR_3}\}$. Thus the total number of shifted samples for R_4 is $\tau_{SR_4D} + \tau_3'$ and the total phase change for the signals transmitted from R_4 is $\mathbf{f}^{\tau_4}$, where $\tau_4 = \tau_{SR_4D} + \tau_3'$.

After the CP removal and the FFT, we can construct the following OSTBC $\mathbf{G}_{4\times4}$ on each subcarrier:

$$\mathbf{G}_{4\times4} = \sqrt{2}\begin{bmatrix} X_1 & -X_2^* & 0 & 0 \\ X_2 & X_1^* & 0 & 0 \\ 0 & 0 & X_1 & -X_2^* \\ 0 & 0 & X_2 & X_1^* \end{bmatrix}. \tag{10}$$

the constructed $\mathbf{G}_{4\times4}$ has rate $1/2$ and the scalar $\sqrt{2}$ ensures that the average transmission power of the relay nodes is P_2, i.e., the amplifying scalar at the relay nodes is $\sqrt{\frac{2P_2}{P_1+1}}$. The reason why the above code is constructed is as follows. As explained in [4], if the requirement that each relay node can only implement time reversal or only implement complex conjugation is satisfied, all the constructed OSTBCs must have a special property: each column of the code has all its elements either complex conjugated or non-conjugated. It was proved in [4] that when J is even, the rate of such an OSTBC is upper bounded by $2/J$. Clearly, when J is even, a block diagonal structure as the above $\mathbf{G}_{4\times4}$ with Alamouti codes in the blocks of diagonals has already reached the rate upper bound.

We adopt the power allocation strategy in [12] in our proposed scheme as in [2]. Denote P as the total transmission power in the whole scheme. By following the power allocation in [12], we have:

$$P_1 = JP_2 = \frac{P}{2} \tag{11}$$

where J is the number of the relay nodes. With the above power allocation and using the results in [12], the destination node can achieve diversity order J when the SNR is large enough.

Note that the available multi-path diversity in the frequency selective fading channels can not be exploited through the constructed OSTBC structure. In order to achieve the multi-path diversity, we can repeat the transmitted symbols across the subcarriers to construct space-time-frequency block codes which can be similarly done as in [4]. For example, if we repeat the transmitted symbols twice, we can construct the repeated Alamouti structure as follows:

$$\mathbf{G}_{4\times2} = \begin{bmatrix} X_1 & -X_2^* \\ X_1 & -X_2^* \\ X_2 & X_1^* \\ X_2 & X_1^* \end{bmatrix}, \tag{12}$$

which can achieve not only the full spatial diversity but also multipath diversity order 2 when $L \geq 2$.

IV. Simulation Results

In this section, we show some simulation results for our proposed scheme. In the simulation, we assume that the OFDM has $N = 64$ subcarriers with the total bandwidth of 10MHz, thus the corresponding OFDM symbol duration is $T_s = 6.4\mu s$. The length of cyclic prefix $\ell_{cp} = 16$, i.e, $1.6\mu s$. For simplicity, we assume that all the channels have a simple two-ray ($L = 2$) equal power delay profile with a delay of $0.3\mu s$ between the two rays. The timing errors τ_{SR_iD} are randomly chosen from 0 to $0.6\mu s$ with the uniform distribution. We further assume that the channel state information (CSI) is perfectly known at the destination. The information bit rate is assumed to be 1bit/s/Hz in the simulations. We use the power allocation strategy in (11).

In Fig. 2, we show the BER performance of destination node D when there are two relay nodes. We can construct the Alamouti code on each subcarrier at D, thus we can use the fast ML symbol-wise decoding, where the data symbols are drawn from BPSK. We give the BER curves of the Alamouti code for 2×1, i.e., 2 transmit and 1 receive antennas, and 2×2, i.e., 2 transmit and 2 receive antennas, MIMO BPSK systems with transmission power P_1 for reference. From Fig. 2, we can see that the slope of the BER curve of the constructed Alamouti scheme approaches the slope of the Alamouti MIMO 2×1 curve when P_1 increases. It implies that the receiver can achieve diversity order 2 when P_1 is large which verifies our analysis of the diversity order. In order to obtain the multi-path diversity of the frequency selective fading channels, we construct the repeated Alamouti $\mathbf{G}_{4\times2}$ as in (12). In order to maintain the same information bit rate, the data symbols are drawn from QPSK in this case. We can see that the slope of the curve for the space-time-frequency code $\mathbf{G}_{4\times2}$ is the same as the one for the 2×2 Alamouti MIMO system, i.e., the code $\mathbf{G}_{4\times2}$ can achieve full (both spatial and multipath) diversity (diversity order 4) when P_1 is large while the decoding is still the fast ML symbol-wise decoding.

When the relay nodes are four, we can construct OSTBC $\mathbf{G}_{4\times4}$ with QPSK to achieve full spatial diversity at the same bit rate and also with symbol-wise decoding. We can see from Fig. 3 that when P_1 is large, the slope of the BER curve of the constructed $\mathbf{G}_{4\times4}$ approaches the slope of the 2×2 Alamouti MIMO system. It implies that the receiver can achieve full spatial diversity (diversity order 4) through

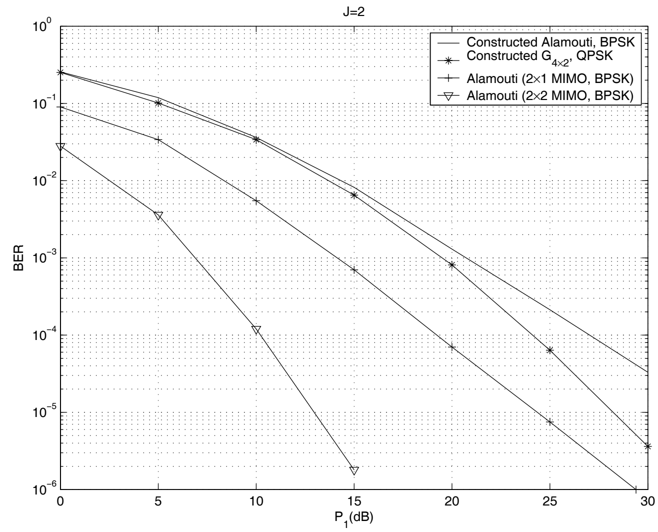

Fig. 2. BER comparison vs. P_1 with two relay nodes.

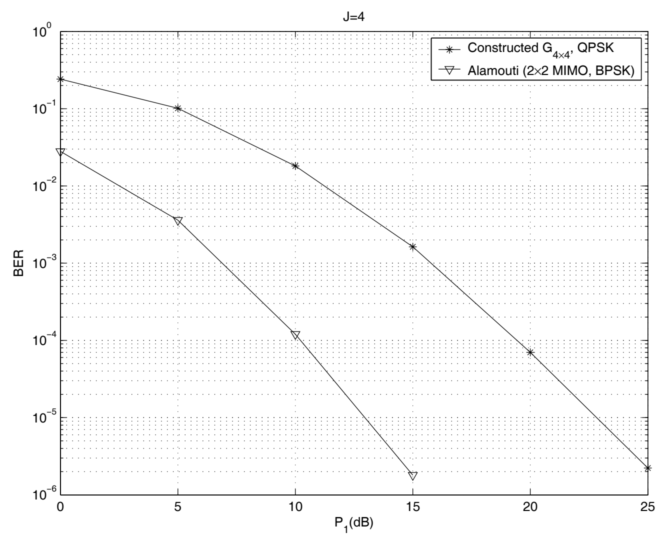

Fig. 3. BER comparison vs. P_1 with four relay nodes.

the proposed scheme and also verifies our analysis of the achievable diversity order when there are multiple relay nodes.

V. Conclusion

In this paper, we proposed a simple space-time transmission scheme for asynchronous cooperative systems for frequency selective fading channels. OFDM is implemented at the source node, and very simple operations, namely time reversion and complex conjugation, are implemented at the relay nodes, a two-step of CP removal is performed at the destination. With this simple scheme, the received signals at the destination node have the orthogonal code form and therefore has the fast ML decoding and can achieve full spatial diversity when SNR is large. Unlike the previously studied schemes for frequency selective fading channels, no Add/Remove CP or FFT/IFFT operation is needed at the relay nodes. In order to achieve multipath diversity, repeating across subcarriers as space-time-frequency coding can be similarly done as in [4].

Appendix

Proof of the Claim

Denote three vectors $h = [h_1, h_2, ..., h_m]^T$, $\mathbf{x} = [x_1, ..., x_N]^T$, and $\overline{\mathbf{x}} = [x_{N-(\ell_{cp}-1)}, ..., x_N, x_1, ..., x_N]^T$, $m < \ell_{cp} < N$. From the definition of linear convolution, we have

$$h * \overline{\mathbf{x}} = \begin{bmatrix} x_{N-(\ell_{cp}-1)} & 0 & \cdots & 0 \\ \vdots & x_{N-(\ell_{cp}-1)} & \ddots & \vdots \\ x_N & \vdots & \ddots & 0 \\ x_1 & x_N & \ddots & x_{N-(\ell_{cp}-1)} \\ \vdots & x_1 & \ddots & \vdots \\ x_N & \vdots & \ddots & x_N \\ 0 & x_N & \ddots & x_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_N \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}$$

where the matrix on the right hand side has size $(m + N + \ell_{cp} - 1) \times m$.

Then we can write $\mathbf{S} = \zeta(h * \overline{\mathbf{x}})$ as

$$\mathbf{S} = \begin{bmatrix} x_{N-(\ell_{cp}-1)} & 0 & \cdots & 0 \\ 0 & \vdots & \cdots & x_N \\ \vdots & x_N & \ddots & \vdots \\ x_N & \vdots & \ddots & x_1 \\ \vdots & x_1 & \ddots & x_N \\ x_1 & x_N & \ddots & \vdots \\ x_N & \vdots & \ddots & x_{N-(\ell_{cp}-1)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-(\ell_{cp}-2)} & x_{N-(\ell_{cp}-1)} & \cdots & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}$$

In the following we will perform the two steps of CP removal on $\zeta(h * \overline{\mathbf{x}})$. The first step is equivalent to remove the first ℓ_{cp} rows of the $(m + N + \ell_{cp} - 1) \times m$ matrix above and choose the $(\ell_{cp} + 1)$-th to the $(\ell_{cp} + 1 + N)$-th rows to construct an $N \times m$ sub-matrix $\mathbf{S}_{step1}$, which can be written as

$$\begin{bmatrix} x_{N-(\ell_{cp}-m)} & x_{N-(\ell_{cp}-(m-1))} & \cdots & x_{N-(\ell_{cp}-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & x_1 \\ \vdots & x_1 & \ddots & x_N \\ x_1 & x_N & \ddots & \vdots \\ x_N & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-(\ell_{cp}-(m+1))} & x_{N-(\ell_{cp}-m)} & \cdots & x_{N-(\ell_{cp}-2)} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}$$

where the matrix on the right hand side has size $N \times m$.

The second step is equivalent to shift the bottom $\ell_{cp} - (m - 1)$ rows of the above $N \times m$ matrix to the top, $\mathbf{S}_{step2}$ can be written as

$$\begin{bmatrix} x_1 & x_N & \cdots & x_{N-(m-2)} \\ x_N & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-(\ell_{cp}-(m+1))} & x_{N-(\ell_{cp}-m)} & \ddots & x_1 \\ x_{N-(\ell_{cp}-m)} & x_{N-(\ell_{cp}-(m-1))} & \ddots & x_N \\ \vdots & \vdots & \ddots & \vdots \\ x_2 & x_1 & \cdots & x_{N-(m-3)} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix}.$$

Denote an $N \times 1$ vector $h' = [h_1, h_2, \ldots, h_m, 0, \ldots, 0]^T$. From the definition of circular convolution [4], $\mathbf{S}_{circ} = \zeta(h') \circledast \zeta(\mathbf{x})$ can be written as

$$\mathbf{S}_{circ} = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \\ x_N & x_1 & \cdots & x_{N-1} \\ \vdots & x_N & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_2 & x_3 & \cdots & x_1 \end{bmatrix} \begin{bmatrix} h_1 \\ 0 \\ \vdots \\ 0 \\ h_m \\ \vdots \\ h_2 \end{bmatrix}$$

It is not difficult to check that $\mathbf{S}_{circ} = \mathbf{S}_{step2}$, which implies the claim, i.e., $\zeta(h') \circledast \zeta(\mathbf{x})$ can be obtained after Step 2.

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