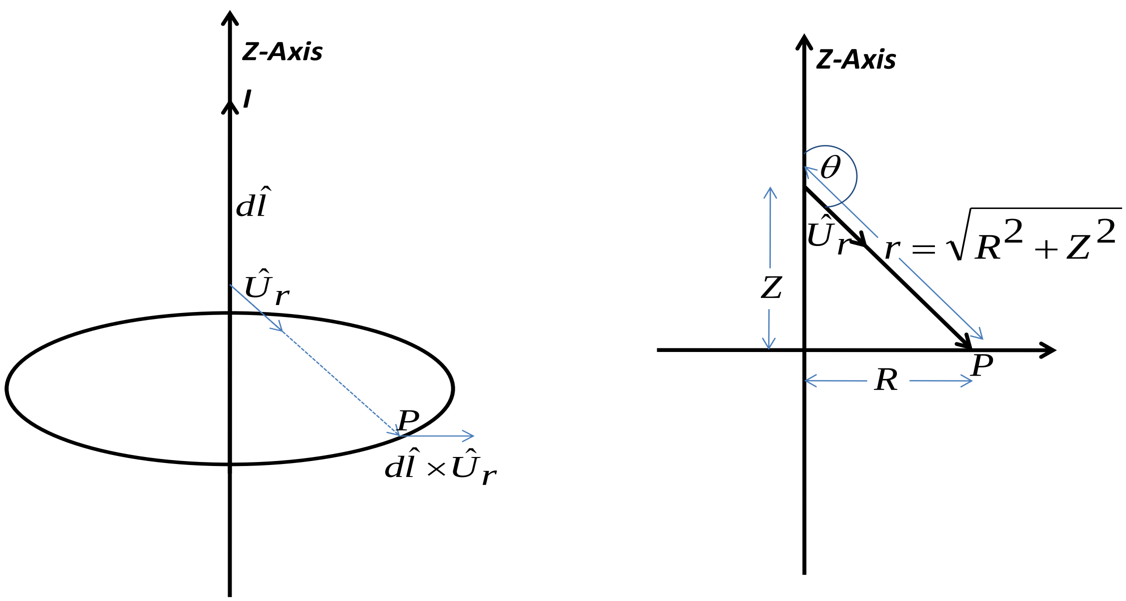
1.

The Biot – Savart Law:

$$\bar{B} = \frac{\mu_0 I}{4\pi} \oint \frac{dl \times \hat{\mathrm{U}}_{\mathrm{r}}}{r^2}$$



(a) (b)

Fig 1(a). Calculating B at a point P in the vicinity of a current carrying wire of infinite length lying along the z-axis . (b) A 2D rendering of Fig 1(a).

$\left|dl \times \hat{U}_r\right| = d\bar{z}\sin\theta$, as seen in Fig.1 (a).

$\sin\theta = \sin(180-\theta) = \frac{R}{r}$, as evident from Fig. 1(b).

$$\bar{B} = \frac{(\mu_0 I)}{4\pi}\int_{-\infty}^{\infty}\frac{d\bar{l}\times\hat{U}_r}{r^2} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{R}{r^3}d\bar{z}$$

$$= \mu_0\frac{I}{4\pi}\,R\int_{-\infty}^{\infty}\frac{d\bar{z}}{(R^2+z^2)^{\frac{3}{2}}}$$

$Let\ z = R\tan\theta$

Therefore, $d\bar{z} = R\sec^2\theta\, d\theta$

$(R^2+z^2)^{\frac{3}{2}} = [(R\tan\theta)^2 + R^2]^{\frac{3}{2}}$

$= R^3\sec^3\theta$

$$\int \left(\frac{d\bar{z}}{(R^2+z^2)^{\frac{3}{2}}}\right) = \int (\frac{R\sec^2\theta\, d\theta}{R^3\sec^3\theta} = \frac{1}{R^2}\int \left(\frac{d\theta}{sec\theta}\right) \quad = \frac{1}{R^2}\int\ \cos\theta\, d\theta = \frac{\sin\theta}{R^2}$$

$since\ z = R\tan\theta$

$$\sin\theta = \frac{z}{\sqrt{z^2+R^2}}$$

$$\int \left(\frac{dz}{(R^2+z^2)^{\frac{3}{2}}}\right) = \frac{\sin\theta}{R^2} = \frac{z}{R^2\sqrt{z^2+R^2}}\Big|_{-\infty}^{\infty}$$

$$= \frac{2}{R^2}$$

$$B = \frac{\mu_0 I}{4\pi}\ R\int_{-\infty}^{\infty}\frac{dz}{(R^2+z^2\ )^{\frac{3}{2}}} = \frac{\mu_0 I}{2\pi R}$$

$$\bar{B} = \frac{\mu_0 I}{2\pi R}\ (tangent\ to\ the\ loop)$$

Now, integrating B along a loop of radius R,

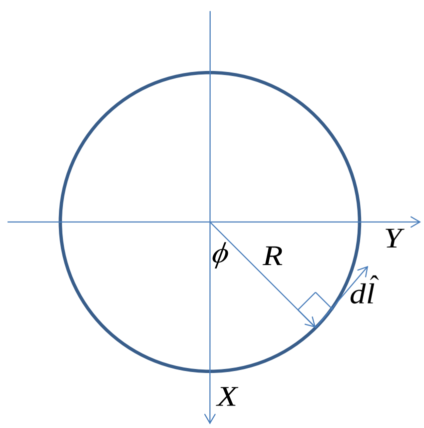


Fig. 2.The current carrying infinite long wire is perpendicular to the XY plane at the origin of the loop depicted above.

$\left|d\hat{l}\right| = Rd\phi$, as one can observe from Fig. 2.

$$\oint \bar{B}\cdot d\bar{l} = \int_0^{2\pi} B\cdot Rd\phi = \int_0^{2\pi}\frac{\mu_0 I}{2\pi R}Rd\phi$$

$$= \mu_0 I$$

The above can be rewritten as follows,

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 \int \bar{J} \cdot \hat{r} d\bar{A}$$

$$Where\ J\ is\ current\ density\ \left(\frac{A}{m^2}\right) and\ A\ is\ Area\ (m^2)$$

Stokes Theorem,

$$\int_S (\nabla \times\ A) ds\ = \oint A \cdot d\bar{l}$$

Therefore,

$$\oint \bar{B} \cdot d\bar{l} = \int_S (\nabla \times \bar{B}) = \mu_0 \int J \cdot \hat{r} d\bar{A}$$

$$\nabla \times B = \mu \bar{J}, Ampere's\ Law$$

2.

$$\Phi_M = Net\ Magnetic\ Flux = \oint \bar{B} \cdot \overline{ds} = Q_m\ (total\ magnetic\ charge)$$

$$\int \zeta_{mv} dv$$

$$in\ accordance\ with\ the\ divergence\ theorem \int \nabla \cdot \bar{B} dv = \oint \bar{B} \cdot \overline{ds}$$

$$= \nabla \cdot \bar{B} = \zeta_{mv}$$

$$There\ are\ no\ magnetic\ monopoles$$

$$\nabla \cdot \bar{B} = 0$$

3.

Problem 4.30 Show that the electric potential difference V_{12} between two points in air at radial distances r_1 and r_2 from an infinite line of charge with density ρ_l along the z-axis is $V_{12} = (\rho_l/2\pi\varepsilon_0)\ln(r_2/r_1)$.

Solution: From Eq. (4.33), the electric field due to an infinite line of charge is

$$\mathbf{E} = \hat{\mathbf{r}}E_r = \hat{\mathbf{r}}\frac{\rho_l}{2\pi\varepsilon_0 r}.$$

Hence, the potential difference is

$$V_{12} = -\int_{r_2}^{r_1} \mathbf{E}\cdot d\mathbf{l} = -\int_{r_2}^{r_1} \frac{\hat{\mathbf{r}}\rho_l}{2\pi\varepsilon_0 r}\cdot\hat{\mathbf{r}}\,dr = \frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right).$$

4.

Problem 4.40 A coaxial resistor of length l consists of two concentric cylinders. The inner cylinder has radius a and is made of a material with conductivity σ_1, and the outer cylinder, extending between $r = a$ and $r = b$, is made of a material with conductivity σ_2. If the two ends of the resistor are capped with conducting plates, show that the resistance between the two ends is $R = l/[\pi(\sigma_1 a^2 + \sigma_2(b^2 - a^2))]$.

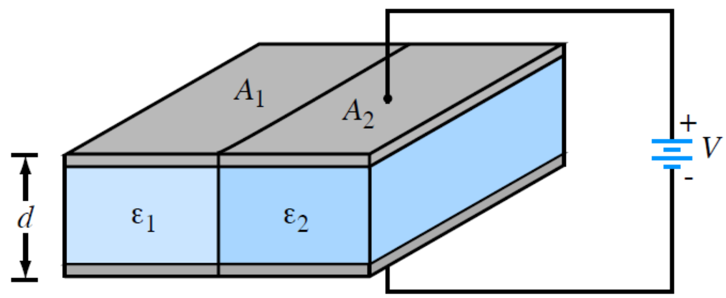
Solution: Due to the conducting plates, the ends of the coaxial resistor are each uniform at the same potential. Hence, the electric field everywhere in the resistor will be parallel to the axis of the resistor, in which case the two cylinders can be considered to be two separate resistors in parallel. Then, from Eq. (4.70),

$$\frac{1}{R} = \frac{1}{R_{\text{inner}}} + \frac{1}{R_{\text{outer}}} = \frac{\sigma_1 A_1}{l_1} + \frac{\sigma_2 A_2}{l_2} = \frac{\sigma_1 \pi a^2}{l} + \frac{\sigma_2 \pi (b^2 - a^2)}{l},$$

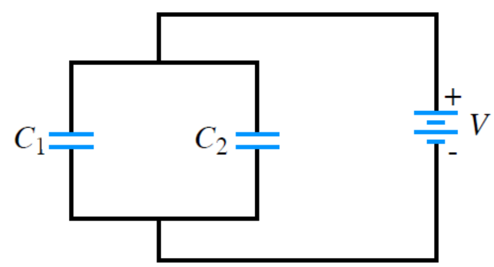
or

$$R = \frac{l}{\pi(\sigma_1 a^2 + \sigma_2(b^2 - a^2))} \quad (\Omega).$$

Problem 4.52 Figure 4-34a (P4.52(a)) depicts a capacitor consisting of two parallel, conducting plates separated by a distance d. The space between the plates



(a)



(b)

Figure P4.52: (a) Capacitor with parallel dielectric section, and (b) equivalent circuit.

contains two adjacent dielectrics, one with permittivity ε_1 and surface area A_1 and another with ε_2 and A_2. The objective of this problem is to show that the capacitance C of the configuration shown in Fig. 4-34a (P4.52(a)) is equivalent to two capacitances in parallel, as illustrated in Fig. 4-34b (P4.52(b)), with

$$C = C_1 + C_2, \tag{4.132}$$

where

$$C_1 = \frac{\varepsilon_1 A_1}{d}, \tag{4.133}$$

$$C_2 = \frac{\varepsilon_2 A_2}{d}. \tag{4.134}$$

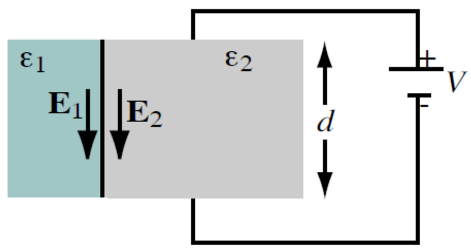
To this end, you are asked to proceed as follows:

(a) Find the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ in the two dielectric layers.

(b) Calculate the energy stored in each section and use the result to calculate C_1 and C_2.

(c) Use the total energy stored in the capacitor to obtain an expression for C. Show that Eq. (4.132) is indeed a valid result.

Solution:



(c)

Figure P4.52: (c) Electric field inside of capacitor.

(a) Within each dielectric section, $\mathbf{E}$ will point from the plate with positive voltage to the plate with negative voltage, as shown in Fig. P4-52(c). From $V = Ed$,

$$E_1 = E_2 = \frac{V}{d}.$$

(b)

$$W_{e_1} = \frac{1}{2}\varepsilon_1 E_1^2 \cdot \mathcal{V} = \frac{1}{2}\varepsilon_1 \frac{V^2}{d^2} \cdot A_1 d = \frac{1}{2}\varepsilon_1 V^2 \frac{A_1}{d}.$$

But, from Eq. (4.121),

$$W_{e_1} = \frac{1}{2} C_1 V^2.$$

Hence $C_1 = \varepsilon_1 \frac{A_1}{d}$. Similarly, $C_2 = \varepsilon_2 \frac{A_2}{d}$.

(c) Total energy is

$$W_e = W_{e_1} + W_{e_2} = \frac{1}{2}\frac{V^2}{d}(\varepsilon_1 A_1 + \varepsilon_2 A_2) = \frac{1}{2} C V^2.$$

6.

$$\boldsymbol{E} = E_0 \sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\cos\omega t\, \boldsymbol{a_y}$$

$$\boldsymbol{H} = H_{01}\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}\sin\omega t\, a_x - H_{02}\cos\frac{\pi x}{a}\sin\frac{\pi z}{d}\sin\omega t\, \boldsymbol{a_z}$$

$$Using\ \rho_S = \boldsymbol{a_n}\cdot\boldsymbol{D} = \boldsymbol{a_n}\cdot 4\epsilon_0\boldsymbol{E_0}, we\ obtain$$

$$[\rho_S]_{x=0} = \boldsymbol{a_x}\cdot 4\epsilon_0[\boldsymbol{E}]_{\boldsymbol{x=0}} = \boldsymbol{0}$$

$$[\rho_S]_{x=a} = -\boldsymbol{a_x}\cdot 4\epsilon_0[\boldsymbol{E}]_{\boldsymbol{x=a}} = \boldsymbol{0}$$

$$[\rho_S]_{y=0} = \boldsymbol{a_y}\cdot 4\epsilon_0[\boldsymbol{E}]_{\boldsymbol{y=0}} = 4\epsilon_0 E_0\sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\cos\omega t$$

$$[\rho_S]_{y=b} = -\boldsymbol{a_y}\cdot 4\epsilon_0[\boldsymbol{E}]_{\boldsymbol{y=b}} = -4\epsilon_0 E_0\sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\cos\omega t$$

$$[\rho_S]_{z=0} = \boldsymbol{a_z}\cdot 4\epsilon_0[\boldsymbol{E}]_{\boldsymbol{z=0}} = \boldsymbol{0}$$

$$[\rho_S]_{z=d} = -\boldsymbol{a_z}\cdot 4\epsilon_0[\boldsymbol{E}]_{\boldsymbol{z=d}} = \boldsymbol{0}$$

$$\boldsymbol{Using\ J_s = a_n \times H, we\ obtain}$$

$$[J_S]_{x=0} = \boldsymbol{a_x}\times[\boldsymbol{H}]_{\boldsymbol{x=0}} = H_{02}\sin\frac{\pi z}{d}\sin\omega t\ \ \boldsymbol{a_y}$$

$$[J_S]_{x=a} = -\boldsymbol{a_x}\times[\boldsymbol{H}]_{\boldsymbol{x=a}} = H_{02}\sin\frac{\pi z}{d}\sin\omega t\ \ \boldsymbol{a_y}$$

$$[J_S]_{y=0} = \boldsymbol{a_y}\times[\boldsymbol{H}]_{\boldsymbol{y=0}} = -H_{02}\cos\frac{\pi x}{a}\sin\frac{\pi \mathrm{z}}{\mathrm{d}}\sin\omega t\ \ \boldsymbol{a_x} - H_{01}\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}\sin\omega t\ \ \boldsymbol{a_Z}$$

$$[J_S]_{y=b} = -\boldsymbol{a_y}\times[\boldsymbol{H}]_{\boldsymbol{y=b}} = H_{02}\cos\frac{\pi x}{a}\sin\frac{\pi \mathrm{z}}{\mathrm{d}}\sin\omega t\ \ \boldsymbol{a_x} + H_{01}\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}\sin\omega t\ \ \boldsymbol{a_Z}$$

$$[J_S]_{z=0} = \boldsymbol{a_z}\times[\boldsymbol{H}]_{\boldsymbol{z=0}} = H_{01}\sin\frac{\pi x}{a}\sin\omega t\ \ \boldsymbol{a_y}$$

$$[J_S]_{z=d} = -\boldsymbol{a_z}\times[\boldsymbol{H}]_{\boldsymbol{z=d}} = H_{01}\sin\frac{\pi x}{a}\sin\omega t\ \ \boldsymbol{a_y}$$

7.

Problem 5.16 In the arrangement shown in Fig. 5-41 (P5.16), each of the two long, parallel conductors carries a current I, is supported by 8-cm-long strings, and has a mass per unit length of 1.2 g/cm. Due to the repulsive force acting on the conductors, the angle θ between the supporting strings is $10°$. Determine the magnitude of I and the relative directions of the currents in the two conductors.

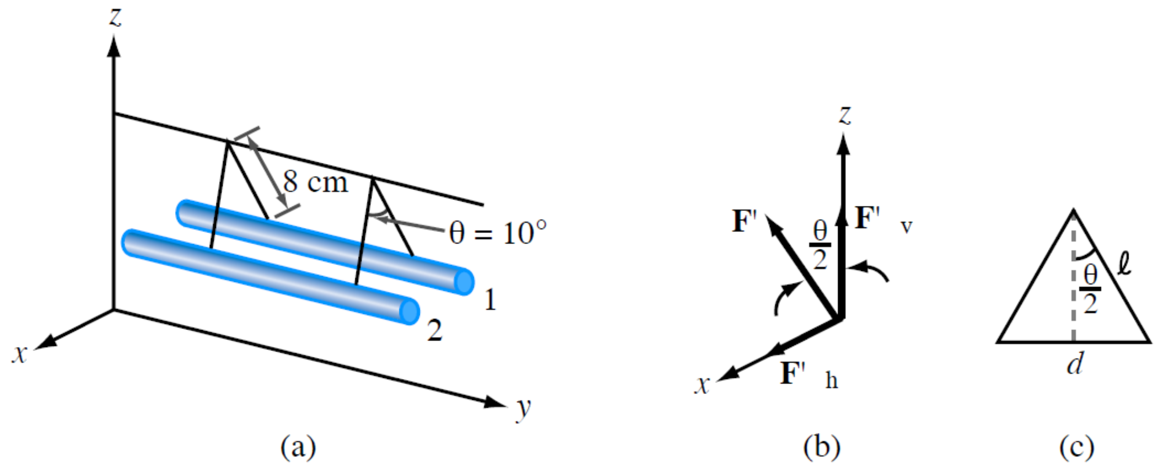


Figure P5.16: Parallel conductors supported by strings (Problem 5.16).

Solution: While the vertical component of the tension in the strings is counteracting the force of gravity on the wires, the horizontal component of the tension in the strings is counteracting the magnetic force, which is pushing the wires apart. According to Section 5-3, the magnetic force is repulsive when the currents are in opposite directions.

Figure P5.16(b) shows forces on wire 1 of part (a). The quantity $\mathbf{F}'$ is the tension force per unit length of wire due to the mass per unit length $m' = 1.2$ g/cm $= 0.12$

kg/m. The vertical component of $\mathbf{F}'$ balances out the gravitational force,

$$F'_{\mathrm{v}} = m'g, \tag{19}$$

where $g = 9.8$ (m/s^2). But

$$F'_{\mathrm{v}} = F' \cos(\theta/2). \tag{20}$$

Hence,

$$F' = \frac{m'g}{\cos(\theta/2)}. \tag{21}$$

The horizontal component of $\mathbf{F}'$ must be equal to the repulsion magnitude force given by Eq. (5.42):

$$F'_{\mathrm{h}} = \frac{\mu_0 I^2}{2\pi d} = \frac{\mu_0 I^2}{2\pi[2\ell \sin(\theta/2)]}, \tag{22}$$

where d is the spacing between the wires and ℓ is the length of the string, as shown in Fig. P5.16(c). From Fig. 5.16(b),

$$F'_{\mathrm{h}} = F' \sin(\theta/2) = \frac{m'g}{\cos(\theta/2)} \sin(\theta/2) = m'g \tan(\theta/2). \tag{23}$$

Equating Eqs. (22) and (23) and then solving for I, we have

$$I = \sin(\theta/2) \sqrt{\frac{4\pi \ell m' g}{\mu_0 \cos(\theta/2)}} = \sin 5^\circ \sqrt{\frac{4\pi \times 0.08 \times 0.12 \times 9.8}{4\pi \times 10^{-7} \cos 5^\circ}} = 84.8 \quad \text{(A)}.$$

8.

Problem 5.22 Repeat Problem 5.21 for a current density $\mathbf{J} = \hat{\mathbf{z}} J_0 e^{-r}$.

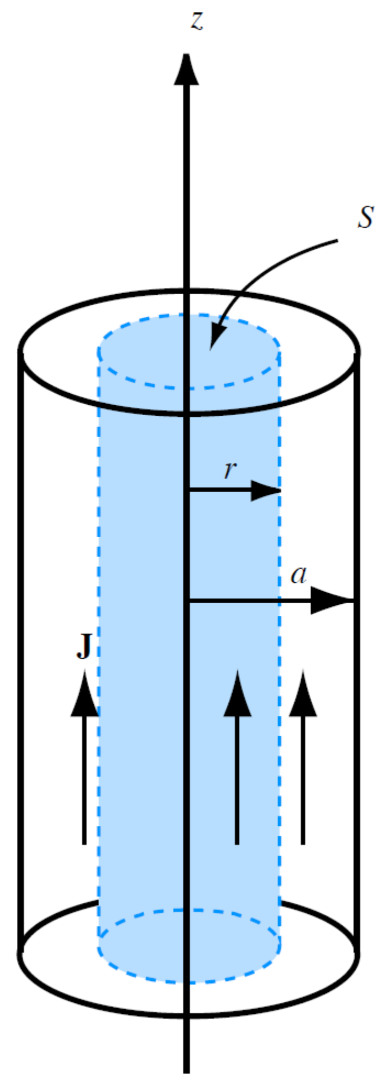


Figure P5.22: Cylindrical current.

Solution:

(a) For $r \leq a$, Ampère's law is

$$\oint_c \mathbf{H} \cdot d\mathbf{l} = I = \int_S \mathbf{J} \cdot d\mathbf{s},$$

$$\hat{\boldsymbol{\phi}} H \cdot \hat{\boldsymbol{\phi}} 2\pi r = \int_0^r \mathbf{J} \cdot d\mathbf{s} = \int_0^r \hat{\mathbf{z}} J_0 e^{-r} \cdot \hat{\mathbf{z}} 2\pi r \, dr,$$

$$2\pi r H = 2\pi J_0 \int_0^r r e^{-r} \, dr$$

$$= 2\pi J_0 [-e^{-r}(r+1)]_0^r = 2\pi J_0 [1 - e^{-r}(r+1)].$$

Hence,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} H = \hat{\boldsymbol{\phi}} \frac{J_0}{r} [1 - e^{-r}(r+1)], \qquad \text{for } r \leq a.$$

(b) For $r \geq a$,

$$2\pi r H = 2\pi J_0 [-e^{-r}(r+1)]_0^a = 2\pi J_0 [1 - e^{-a}(a+1)],$$

$$\mathbf{H} = \hat{\boldsymbol{\phi}} H = \hat{\boldsymbol{\phi}} \frac{J_0}{r} \left[1 - e^{-a}(a+1)\right], \qquad r \geq a.$$