1.)

A x B is a vector perpendicular to the plane of A and B

$$A\ x\ B = \det\begin{Bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 9 & 2 & -5 \\ -3 & 6 & 4 \end{Bmatrix} = (8+30)\hat{x} - (36-15)\hat{y} + (54+6)\hat{z}$$

$$= 38\hat{x} - 21\hat{y} + 60\hat{z}$$

Unit Vector -> $\frac{A\ x\ B}{|A\ x\ B|} = \frac{38\hat{x}-21\hat{y}+60\hat{z}}{\sqrt{38^2+(-21)^2+60^2}} = \frac{38\hat{x}-21\hat{y}+60\hat{z}}{\pm 74.06}$

$$= \pm(.5131\hat{x} - .2836\hat{y} + .8101\hat{z}$$

2)

$V = (x+7z)\hat{x} + (-3y+3x)\hat{y} + (x-az)\hat{z}$

$$\nabla \cdot V = 0 = \frac{\partial}{\partial x}(x+7z) + \frac{\partial}{\partial y}(-3y+3x) + \frac{\partial}{\partial z}(x-az) = 1-3-a = 0$$

$a = -2$

3.)

$\overline{W} = (8z-2ay)\hat{x} + (4x+3bz)\hat{y} + (4cx+6y+1)\hat{z}$

Want a, b, c so that $\overline{\nabla}\times\overline{W} = 0$

$$\overline{\nabla}\times\overline{W} = \left[\frac{\partial(4cx+6y+1)}{\partial y} - \frac{\partial(4x+3bz)}{\partial z}\right]\hat{x} + \left[\frac{\partial(8z-2ay)}{\partial z} - \frac{\partial(4cx+6y+1)}{\partial x}\right]\hat{y} + \left[\frac{\partial(4x+3bz)}{\partial x} - \frac{\partial(8z-2ay)}{\partial y}\right]\hat{z}$$

$$\overline{\nabla}\times\overline{W} = (6-3b)\hat{x} + (8-4c)\hat{y} + (4+2a)\hat{z} = 0$$

$\boxed{b=2 \quad c=2 \quad a=-2}$

4.)

$\overline{A} = 22\cos\theta\hat{\theta}$

Want $\overline{\nabla} \times \overline{A}$ at the point $(3, \pi, 0)$

$$\overline{\nabla} \times \overline{A} = \frac{1}{r\sin\theta}\left[\frac{\partial A_\phi \sin\theta}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial rA_\phi}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial rA_\theta}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\hat{\phi}$$

$$\overline{\nabla} \times \overline{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(0)}{\partial\theta} - \frac{\partial(22\cos\theta)}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial(0)}{\partial\phi} - \frac{\partial(r\cdot 0)}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial(r22\cos\theta)}{\partial r} - \frac{\partial(0)}{\partial\theta}\right]\hat{\phi}$$

$$\overline{\nabla} \times \overline{A} = (0)\hat{r} + (0)\hat{\theta} + \left(\frac{1}{r}22\cos\theta\right)\hat{\phi}$$

At $(3, \pi, 0)$:

$$\boxed{\overline{\nabla} \times \overline{A} = \frac{1}{3}22(-1)\hat{\phi} = -\frac{22}{3}\hat{\phi}}$$

5.)

$$7\cos(\omega t)+12\sin\left(\omega t-\frac{3\pi}{4}\right)$$

$$=\mathrm{Re}\left[\left(7e^{j0}+12e^{j\left(-\frac{3\pi}{4}\right)}\right)e^{j\omega t}\right]$$

$$=\mathrm{Re}\left[\left(7-6\sqrt{2}-6\sqrt{2}\,j\right)e^{j\omega t}\right]$$

Magnitude: $A=\sqrt{\left(7-6\sqrt{2}\right)^2+\left(6\sqrt{2}\right)^2}=8.61$

Phase: $\varphi=\sin^{-1}\left(\frac{6\sqrt{2}}{8.614}\right)=1.40rad$

$$\therefore 7\cos(\omega t)+12\sin\left(\omega t-\frac{3\pi}{4}\right)$$

$$=\mathrm{Re}\left[8.61e^{j\left(\omega t-1.40-\frac{\pi}{2}\right)}\right]$$

$$=8.61\cos\left(\omega t-8.614-\frac{\pi}{2}\right)$$

* Note that the phase is negative and includes a negative factor of $\frac{\pi}{2}$ due to the real and imaginary components both being negative.

6.)

$\bar{V} = 3\hat{x} + 4\hat{y} + 5\hat{z}$

Spherical:

$r = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) = \tan^{-1}\left(\frac{\sqrt{25}}{5}\right) = \tan^{-1}(1) = \pi / 4\, rad$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 0.9273\, rad$$

$\boxed{\bar{V} = (5\sqrt{2}, \pi / 4, 0.9273)}$

$V_R = V_x \sin\theta \cos\phi + V_y \sin\theta \sin\phi + V_z \cos\theta$

$V_\theta = V_x \cos\theta \cos\phi + V_y \cos\theta \sin\phi - V_z \sin\theta$

$V_\phi = -V_x \sin\theta + V_y \cos\phi$

$V_R = 3 \sin.79 \cos.9273 + 4 \sin.79 \sin.93 + 5 \cos.79 = 7.07 = R$

$V_\theta = 3 \cos.79 \cos.93 + 4 \cos.79 \sin.93 - 5 \sin.79 = 0$

$V_\phi = -3 \sin.93 + 4 \cos.93 = 0$

7.)

Problem 3.5 Given vectors $\mathbf{A} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3$, $\mathbf{B} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}4$, and $\mathbf{C} = \hat{\mathbf{y}}2 - \hat{\mathbf{z}}4$, find

(a) A and $\hat{\mathbf{a}}$,
(b) the component of $\mathbf{B}$ along $\mathbf{C}$,
(c) θ_{AC},
(d) $\mathbf{A} \times \mathbf{C}$,
(e) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$,
(f) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$,
(g) $\hat{\mathbf{x}} \times \mathbf{B}$, and
(h) $(\mathbf{A} \times \hat{\mathbf{y}}) \cdot \hat{\mathbf{z}}$.

Solution:

(a) From Eq. (3.4),

$$A = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14},$$

and, from Eq. (3.5),

$$\hat{\mathbf{a}}_A = \frac{\hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3}{\sqrt{14}}.$$

(**b**) The component of **B** along **C** (see Section 3-1.4) is given by

$$B\cos\theta_{BC} = \frac{\mathbf{B}\cdot\mathbf{C}}{C} = \frac{-8}{\sqrt{20}} = -1.8\,.$$

(**c**) From Eq. (3.21),

$$\theta_{AC} = \cos^{-1}\frac{\mathbf{A}\cdot\mathbf{C}}{AC} = \cos^{-1}\frac{4+12}{\sqrt{14}\sqrt{20}} = \cos^{-1}\frac{16}{\sqrt{280}} = 17.0^\circ.$$

(**d**) From Eq. (3.27),

$$\mathbf{A}\times\mathbf{C} = \hat{\mathbf{x}}(2(-4)-(-3)2)+\hat{\mathbf{y}}((-3)0-1(-4))+\hat{\mathbf{z}}(1(2)-2(0)) = -\hat{\mathbf{x}}2+\hat{\mathbf{y}}4+\hat{\mathbf{z}}2.$$

(**e**) From Eq. (3.27) and Eq. (3.17),

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{A}\cdot(\hat{\mathbf{x}}16+\hat{\mathbf{y}}8+\hat{\mathbf{z}}4) = 1(16)+2(8)+(-3)4 = 20.$$

Eq. (3.30) could also have been used in the solution. Also, Eq. (3.29) could be used in conjunction with the result of part (d).

(**f**) By repeated application of Eq. (3.27),

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{A}\times(\hat{\mathbf{x}}16+\hat{\mathbf{y}}8+\hat{\mathbf{z}}4) = \hat{\mathbf{x}}32-\hat{\mathbf{y}}52-\hat{\mathbf{z}}24.$$

Eq. (3.33) could also have been used.

(**g**) From Eq. (3.27),

$$\hat{\mathbf{x}}\times\mathbf{B} = -\hat{\mathbf{z}}4.$$

(**h**) From Eq. (3.27) and Eq. (3.17),

$$(\mathbf{A}\times\hat{\mathbf{y}})\cdot\hat{\mathbf{z}} = (\hat{\mathbf{x}}3+\hat{\mathbf{z}})\cdot\hat{\mathbf{z}} = 1.$$

Eq. (3.29) and Eq. (3.25) could also have been used in the solution.

8.)

Problem 3.16 Given $\mathbf{B} = \hat{\mathbf{x}}(z-3y) + \hat{\mathbf{y}}(2x-3z) - \hat{\mathbf{z}}(x+y)$, find a unit vector parallel to $\mathbf{B}$ at point $P(1,0,-1)$.

Solution: At $P(1,0,-1)$,

$$\mathbf{B} = \hat{\mathbf{x}}(-1) + \hat{\mathbf{y}}(2+3) - \hat{\mathbf{z}}(1) = -\hat{\mathbf{x}} + \hat{\mathbf{y}}5 - \hat{\mathbf{z}},$$

$$\hat{\mathbf{b}} = \frac{\mathbf{B}}{|\mathbf{B}|} = \frac{-\hat{\mathbf{x}} + \hat{\mathbf{y}}5 - \hat{\mathbf{z}}}{\sqrt{1+25+1}} = \frac{-\hat{\mathbf{x}} + \hat{\mathbf{y}}5 - \hat{\mathbf{z}}}{\sqrt{27}}.$$

9.)

Sections 3-4 to 3-7: Gradient, Divergence, and Curl Operators

Problem 3.32 Find the gradient of the following scalar functions:

(a) $T = 3/(x^2 + z^2)$,

(b) $V = xy^2z^4$,

(c) $U = z\cos\phi/(1+r^2)$,
(d) $W = e^{-R}\sin\theta$,
(e) $S = 4x^2e^{-z} + y^3$,
(f) $N = r^2\cos^2\phi$,
(g) $M = R\cos\theta\sin\phi$.

Solution:

(a) From Eq. (3.72),

$$\nabla T = -\hat{\mathbf{x}}\frac{6x}{(x^2+z^2)^2} - \hat{\mathbf{z}}\frac{6z}{(x^2+z^2)^2}.$$

(b) From Eq. (3.72),

$$\nabla V = \hat{\mathbf{x}}y^2z^4 + \hat{\mathbf{y}}2xyz^4 + \hat{\mathbf{z}}4xy^2z^3.$$

(c) From Eq. (3.82),

$$\nabla U = -\hat{\mathbf{r}}\frac{2rz\cos\phi}{(1+r^2)^2} - \hat{\boldsymbol{\phi}}\frac{z\sin\phi}{r(1+r^2)} + \hat{\mathbf{z}}\frac{\cos\phi}{1+r^2}.$$

(d) From Eq. (3.83),

$$\nabla W = -\hat{\mathbf{R}}e^{-R}\sin\theta + \hat{\boldsymbol{\theta}}(e^{-R}/R)\cos\theta.$$

(e) From Eq. (3.72),

$$S = 4x^2e^{-z} + y^3,$$
$$\nabla S = \hat{\mathbf{x}}\frac{\partial S}{\partial x} + \hat{\mathbf{y}}\frac{\partial S}{\partial y} + \hat{\mathbf{z}}\frac{\partial S}{\partial z} = \hat{\mathbf{x}}8xe^{-z} + \hat{\mathbf{y}}3y^2 - \hat{\mathbf{z}}4x^2e^{-z}.$$

(f) From Eq. (3.82),

$$N = r^2\cos^2\phi,$$
$$\nabla N = \hat{\mathbf{r}}\frac{\partial N}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial N}{\partial\phi} + \hat{\mathbf{z}}\frac{\partial N}{\partial z} = \hat{\mathbf{r}}2r\cos^2\phi - \hat{\boldsymbol{\phi}}2r\sin\phi\cos\phi.$$

(g) From Eq. (3.83),

$$M = R\cos\theta\sin\phi,$$
$$\nabla M = \hat{\mathbf{R}}\frac{\partial M}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial M}{\partial\theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial M}{\partial\phi} = \hat{\mathbf{R}}\cos\theta\sin\phi - \hat{\boldsymbol{\theta}}\sin\theta\sin\phi + \hat{\boldsymbol{\phi}}\frac{\cos\phi}{\tan\theta}.$$

10.)

Problem 3.39 For the vector field $\mathbf{E} = \hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy$, verify the divergence theorem by computing:

(a) the total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each and parallel to the Cartesian axes, and

(b) the integral of $\nabla \cdot \mathbf{E}$ over the cube's volume.

Solution:

(a) For a cube, the closed surface integral has 6 sides:

$$\oint \mathbf{E} \cdot d\mathbf{s} = F_{\text{top}} + F_{\text{bottom}} + F_{\text{right}} + F_{\text{left}} + F_{\text{front}} + F_{\text{back}},$$

$$F_{\text{top}} = \int_{x=-1}^{1}\int_{y=-1}^{1} (\hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy)\big|_{z=1} \cdot (\hat{\mathbf{z}}\,dy\,dx)$$

$$= -\int_{x=-1}^{1}\int_{y=-1}^{1} xy\,dy\,dx = \left(\left(\frac{x^2y^2}{4}\right)\Bigg|_{y=-1}^{1}\right)\Bigg|_{x=-1}^{1} = 0,$$

$$F_{\text{bottom}} = \int_{x=-1}^{1}\int_{y=-1}^{1} (\hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy)\big|_{z=-1} \cdot (-\hat{\mathbf{z}}\,dy\,dx)$$

$$= \int_{x=-1}^{1}\int_{y=-1}^{1} xy\,dy\,dx = \left(\left(\frac{x^2y^2}{4}\right)\Bigg|_{y=-1}^{1}\right)\Bigg|_{x=-1}^{1} = 0,$$

$$F_{\text{right}} = \int_{x=-1}^{1}\int_{z=-1}^{1} (\hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy)\big|_{y=1} \cdot (\hat{\mathbf{y}}\,dz\,dx)$$

$$= -\int_{x=-1}^{1}\int_{z=-1}^{1} z^2\,dz\,dx = -\left(\left(\frac{xz^3}{3}\right)\Bigg|_{z=-1}^{1}\right)\Bigg|_{x=-1}^{1} = \frac{-4}{3},$$

$$F_{\text{left}} = \int_{x=-1}^{1}\int_{z=-1}^{1} (\hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy)\big|_{y=-1} \cdot (-\hat{\mathbf{y}}\,dz\,dx)$$

$$= -\int_{x=-1}^{1}\int_{z=-1}^{1} z^2\,dz\,dx = -\left(\left(\frac{xz^3}{3}\right)\Bigg|_{z=-1}^{1}\right)\Bigg|_{x=-1}^{1} = \frac{-4}{3},$$

$$F_{\text{front}} = \int_{y=-1}^{1}\int_{z=-1}^{1} (\hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy)\big|_{x=1} \cdot (\hat{\mathbf{x}}\,dz\,dy)$$

$$= \int_{y=-1}^{1}\int_{z=-1}^{1} z\,dz\,dy = \left(\left(\frac{yz^2}{2}\right)\Bigg|_{z=-1}^{1}\right)\Bigg|_{y=-1}^{1} = 0,$$

$$F_{\text{back}} = \int_{y=-1}^{1}\int_{z=-1}^{1} (\hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy)\big|_{x=-1} \cdot (-\hat{\mathbf{x}}\,dz\,dy)$$

$$= \int_{y=-1}^{1}\int_{z=-1}^{1} z\,dz\,dy = \left(\left(\frac{yz^2}{2}\right)\Bigg|_{z=-1}^{1}\right)\Bigg|_{y=-1}^{1} = 0,$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = 0 + 0 + \frac{-4}{3} + \frac{-4}{3} + 0 + 0 = \frac{-8}{3}.$$

(b)

$$\iiint \nabla\cdot\mathbf{E}\,dv = \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} \nabla\cdot(\hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy)\,dz\,dy\,dx$$
$$= \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} (z-z^2)\,dz\,dy\,dx$$
$$= \left(\left(\left(xy\left(\frac{z^2}{2}-\frac{z^3}{3}\right)\right)\Bigg|_{z=-1}^{1}\right)\Bigg|_{y=-1}^{1}\right)\Bigg|_{x=-1}^{1} = \frac{-8}{3}.$$

11.)

Problem 3.41 A vector field $\mathbf{D} = \hat{\mathbf{r}}r^3$ exists in the region between two concentric cylindrical surfaces defined by $r=1$ and $r=2$, with both cylinders extending between $z=0$ and $z=5$. Verify the divergence theorem by evaluating:

(a) $\oint_S \mathbf{D}\cdot d\mathbf{s}$,

(b) $\int_{\mathcal{V}} \nabla\cdot\mathbf{D}\,d\mathcal{V}$.

Solution:

(a)

$$\iint \mathbf{D}\cdot d\mathbf{s} = F_{\text{inner}} + F_{\text{outer}} + F_{\text{bottom}} + F_{\text{top}},$$
$$F_{\text{inner}} = \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left((\hat{\mathbf{r}}r^3)\cdot(-\hat{\mathbf{r}}r\,dz\,d\phi)\right)\Big|_{r=1}$$
$$= \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left(-r^4\,dz\,d\phi\right)\Big|_{r=1} = -10\pi,$$
$$F_{\text{outer}} = \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left((\hat{\mathbf{r}}r^3)\cdot(\hat{\mathbf{r}}r\,dz\,d\phi)\right)\Big|_{r=2}$$
$$= \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left(r^4\,dz\,d\phi\right)\Big|_{r=2} = 160\pi,$$
$$F_{\text{bottom}} = \int_{r=1}^{2}\int_{\phi=0}^{2\pi} \left((\hat{\mathbf{r}}r^3)\cdot(-\hat{\mathbf{z}}r\,d\phi\,dr)\right)\Big|_{z=0} = 0,$$

$$F_{\text{top}} = \int_{r=1}^{2}\int_{\phi=0}^{2\pi} \left((\hat{\mathbf{r}}r^3)\cdot(\hat{\mathbf{z}}r\,d\phi\,dr)\right)\Big|_{z=5} = 0.$$

Therefore, $\iint \mathbf{D}\cdot d\mathbf{s} = 150\pi$.

(**b**) From the back cover, $\nabla\cdot\mathbf{D} = (1/r)(\partial/\partial r)(rr^3) = 4r^2$. Therefore,

$$\iiint \nabla\cdot\mathbf{D}\,d\nu = \int_{z=0}^{5}\int_{\phi=0}^{2\pi}\int_{r=1}^{2} 4r^2 r\,dr\,d\phi\,dz = \left(\left(\left(r^4\right)\Big|_{r=1}^{2}\right)\Big|_{\phi=0}^{2\pi}\right)\Big|_{z=0}^{5} = 150\pi.$$

12.)

Problem 3.44 Repeat Problem 3.43 for the contour shown in Fig. P3.43(b).

Solution: In addition to the independent condition that $z = 0$, the three lines of the triangle are represented by the equations $y = 0$, $y = 2 - x$, and $y = x$, respectively.

(**a**)

$$\oint \mathbf{E}\cdot d\mathbf{l} = L_1 + L_2 + L_3,$$

$$\begin{aligned}
L_1 &= \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2+2y^2))\cdot(\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz) \\
&= \int_{x=0}^{2} (xy)\big|_{y=0,z=0}\,dx - \int_{y=0}^{0} (x^2+2y^2)\big|_{z=0}\,dy + \int_{z=0}^{0} (0)\big|_{y=0}\,dz = 0, \\
L_2 &= \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2+2y^2))\cdot(\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz) \\
&= \int_{x=2}^{1} (xy)\big|_{z=0,y=2-x}\,dx - \int_{y=0}^{1} (x^2+2y^2)\big|_{x=2-y,z=0}\,dy + \int_{z=0}^{0} (0)\big|_{y=2-x}\,dz \\
&= \left(x^2 - \frac{x^3}{3}\right)\Big|_{x=2}^{1} - (4y - 2y^2 + y^3)\big|_{y=0}^{1} + 0 = \frac{-11}{3},
\end{aligned}$$

$$\begin{aligned}
L_3 &= \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2+2y^2))\cdot(\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz) \\
&= \int_{x=1}^{0} (xy)\big|_{y=x,z=0}\,dx - \int_{y=1}^{0} (x^2+2y^2)\big|_{x=y,z=0}\,dy + \int_{z=0}^{0} (0)\big|_{y=x}\,dz \\
&= \left(\frac{x^3}{3}\right)\Big|_{x=1}^{0} - (y^3)\big|_{y=1}^{0} + 0 = \frac{2}{3}.
\end{aligned}$$

Therefore,

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 - \frac{11}{3} + \frac{2}{3} = -3.$$

(**b**) From Eq. (3.105), $\nabla \times \mathbf{E} = -\hat{\mathbf{z}}3x$, so that

$$\begin{aligned}
\iint \nabla \times \mathbf{E} \cdot d\mathrm{s} &= \int_{x=0}^{1} \int_{y=0}^{x} ((-\hat{\mathbf{z}}3x) \cdot (\hat{\mathbf{z}}\, dy\, dx))|_{z=0} \\
&\quad + \int_{x=1}^{2} \int_{y=0}^{2-x} ((-\hat{\mathbf{z}}3x) \cdot (\hat{\mathbf{z}}\, dy\, dx))|_{z=0} \\
&= -\int_{x=0}^{1} \int_{y=0}^{x} 3x\, dy\, dx - \int_{x=1}^{2} \int_{y=0}^{2-x} 3x\, dy\, dx \\
&= -\int_{x=0}^{1} 3x(x-0)\, dx - \int_{x=1}^{2} 3x((2-x)-0)\, dx \\
&= -\left(x^3\right)\Big|_0^1 - \left(3x^2 - x^3\right)\Big|_{x=1}^{2} = -3.
\end{aligned}$$