Solving parametric linear systems

Mark W. Giesbrecht
Depatment of Computer Science
University of Manitoba
Winnipeg, Manitoba, R3T 2N2, Canada

B. David Saunders
Department of Computer and Information Science
University of Delaware
Newark, DE 19716, USA

Speaker: David Saunders: saunders@udel.edu

Key identifying words: parametric linear system, computer algebra, polynomial matrix.

When solving the linear system $Ax = b$, where the entries of the matrix A and vector b are polynomial expressions in several parameters, computer algebra systems such as Maple and Mathematica will give the "generic" solution. Often more is desired. For example, at values of the parameters which are not zeroes of the determinant of A, the solution is easily given. But it is also desirable to know the rank and nature of the solution for points in parameter space at which the matrix is singular.

The problem of succinctly describing all solutions is computationally intractable (it is P-space hard). Even the size of the output can be exponential in the size of the inputs. Previously described solution methods have required exponential time in the size of the matrix. In this paper we present two algorithms and provide an analysis to show the computation time is bounded by a function which is polynomial in the matrix size and in the degrees of the entries as polynomials in the parameters. It is exponential only in the number of parameters. Thus, for any fixed number of parameters, the algorithms run in time which is a polynomial function of the size of the input matrix and right hand side vector (or matrix).