CISC621 Algorithm Design and Analysis, Spring 2016
Homework set V, problems 14, 15, 16,17, due Friday, May, 4pm

Check the "homework sheet" from the syllabus for general homework details. In particular, each homework solution is on an entirely separate (set of) sheet(s) of paper and is identified with your name(s). Do not staple solutions to two or more problems together.

14. [Individual Problem — island evacuation]

    The Delmarva peninsula is a water surrounded body of land including parts of the states of Delaware, Maryland, and Virginia. It is technically an island, having the Delaware Bay and Atlantic Ociean on the east, the Chesapeake Bay on the south and west and the C&D (Chesapeake and Delaware) Canal on the north. There are only a few bridges (6 or 7) from this island to the mainland. Similarly large, well populated islands include Manhattan and Staten Island in New York.

    In preparation for the need to evacuate the island in advance of a large storm such as a Hurricane, you are charged with calculating how rapidly the island can be evacuated. To simplify the model we will assume each person has a designated car they will use and that the cars are in designated locations. The function to be computed is EvTime(G,w,c,O) which returns the number of hours needed to evacuate the island. The input information is a graph, G = (V,E), depicting the road network along with a weight function, w(u,v), which records the capacity, in cars per hour, of the road segment from intersection u to intersection v. The c is a function on V we'll write as v.c for v in V. This v.c is the number of cars located at intersection v. For example a parking garage may be modeled as a vertex, g, with a single edge connecting to an adjacent road (the garage entrance), the road/entrance vertex being r. Then w(g,r) is the max rate that cars can flow out the entrance to the road, and g.c is the number of cars in the garage. Finally O is a set of vertices representing "off the island". Each vertex x in O has a single neighbor y not in O, the edge (y,x) being a bridge (or tunnel or ferry) from island to mainland. A car is considered evacuated when it arrives at a vertex in O.

    Design and analyze an efficient algorithm for determining how long it will take, given G,w,c,O, to evacuate the island so described, moving all cars at vertices in V $\setminus$ O to vertices in O without exceeding any of the road segment capacities designated by w.

15. [Individual Problem — Space efficient Floyd-Warshall]

    Let G be a weighted graph given by weight matrix W. G may have negative weights but no negative cycles (including no negative self loops, no negative $w_{i,i}$). Show that the following version of Floyd-Warshall is correct. It uses $\Theta(n^2)$ memory rather than $\Theta(n^3)$. This is similar to exercise 25.2-4.

    FLOYD-WARSHALL"(W)

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n = W.rows.
D = W
for k = 1 to n
     for i = 1 to n if (i ≠ k) then
          for j = 1 to n if (j ≠ k) then
               d_{i,j} = min(d_{i,j}, d_{i,k} + d_{k,j})
Return D.
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16. [Group Problem — fat graphs] Let V' be a subset of the vertices V of a graph G = (V,E). The subgraph *induced* by V' is the graph G' = (V',E'), where E' is the subset of E consisting of all edges with both ends in V'. For instance a k-*clique* of G is a subgraph induced by k vertices that turns out to be a complete graph and an *independent set* of vertices has no edges in its induced subgraph. Considering graphs to have many edges to be "fat" and graphs with few edges "thin", we could say that a clique is fat and an independent set is thin. To quantify the degree of fatness, call an induced subgraph (k,l)-fat if it has at most k vertices and at least l edges. The problem FAT(G,k,l)

is to decide, given G,k,l, whether G has a (k,l)-fat subgraph. Show that FAT is NP Complete. Make clear your argument for each part of the recipe of NP Completeness proofs.

17. [Group Problem — very independent] A *very independent* set F of G = (V,E) is a subset of V such that the neighborhoods of the nodes in F are pairwise disjoint. Thus for u and v in F, not only is (u,v) not an edge, but for every vertex w, at least one of (u,w) and (v,w) is not an edge also. Problem VIS (Very-Independent-Set) is: Given undirected graph G = (V,E) and integer k, decide if there exists in G a very independent set of size k

    Show that VIS is NP Complete.