CISC621 Algorithm Design and Analysis, Fall 2015
Homework set 5, due Friday, December 12, 4:30pm

As always, check the "homework sheet" from the syllabus for general homework details. Remember that individual problems are just that and that resort to someone else's solution (in person, found on the web, in textbooks, etc.) is not allowed.

Homework may be submitted to Saunders' mailbox in 101 Smith Hall. Staff leaves and door is locked at 4:30pm. Each homework solution is on an entirely separate (set of) sheet(s) of paper and is identified with your name(s). Do not staple solutions of two or more problems together.

13. [Individual Problem] The Actor Study Problem: As a budding actor, you have a list of famous actors whose technique you would like to study. You would like to find a small set of films such that each of your actors has a role in at least one of the films. Formally, the input is an integer k, a list of actors $A = (a_1, a_2, \ldots, a_n)$, and list of films $F = (f_1, f_2, \ldots, f_m)$, where each f_i is a list of the actors appearing in the i-th film. The problem is to determine if there is a collection of films $G \subset F$ of size k such that every actor in A is in at least one of the films of G. Show that the Actor Study Problem is NP-complete.

14. [Group Problem]

    a. Exercise 34.5-2 from CLRS (0-1 integer programming)

    b. Show that $\{0, 1, -1\}$ integer programming is NP-hard. This is the problem, given integer matrix A and integer vector b to find a $\{0, 1, -1\}$ vector x such that $Ax \leq b$.

15. [Group Problem] A Carpenter's Ruler consists of line segments of integral lengths, hinged together consecutively. The line segments (called links) can rotate freely about the hinges. Such a ruler models a robot arm. In moving a robot arm it is found convenient to fold the arm as compactly as possible. In such a folding, we allow the links to cross each other, but the angle at a hinge is restricted to be either 0 or 180 degrees. Specifically the problem is: Given a carpenter's ruler in which the successive links have lengths $\omega_1, \omega_2, \ldots, \omega_n$, and a bound D, determine whether the ruler can be folded into a length of at most D. It is known that the Carpenter's Ruler Problem is NP-complete. Be that as it may, when there is a specific relationship between D and the ω_i, the problem can be tractable. This exercise concerns such a situation.

    a. Suppose m is the largest of the ω_i's. Prove that the Carpenter's Ruler can always be folded into a length of no more than $2m$. Do this by giving an algorithm for folding the ruler and basing the result on the folding that your algorithm produces.

    b. Prove that the bound in part a is the best possible. That is, for any given $\epsilon > 0$, explain how to construct a ruler for which the optimal folded length is at least $2m - \epsilon$.

16. [Group Problem] "nearly true." In the k-3-CNF problem we are given a 3-CNF formula ϕ with n variables and m clauses. We wish to determine whether there exists a truth assignment to the variables of ϕ such that *at most* k clauses evaluate to 0 (i.e. false), and hence that at least $m-k$ clauses evaluate to true. Prove that the k-3-CNF problem is NP-complete for any positive constant integer k.

    Hint: See exercise 34.5-8. You may find it helpful to show the following problem is NP-complete: Given 3-CNF formula ϕ and integer j, determine if ϕ's variables have a truth assignment such that *exactly* j clauses evaluate to 1.