CISC 320 Midterm Exam

Friday, March 26, 2010

Name: ________________________________

There are 23 questions. The first 18 questions count 3 points each.
True or False: write T or F in the blank.
Multiple Choice: write the letter of the best choice in the blank.
Very Short Answer: write the very short answer in the blank.
Then there is a 6 point question and four 10 point questions, for a total of 100 points.

1. ____ True or false: There cannot be a comparison based algorithm for finding the maximum and minimum of an array of n numbers using fewer than $5n/4$ comparisons.

2. ____ For the problem of finding the maximum and the second largest of an array of n numbers, the best algorithm we discussed uses about ____ comparisons.

   (a) $n - 1$
   (b) $n\lg(n)$
   (c) $n + \lg(n)$
   (d) 3n/2

3. ____ Algorithm A manipulates an array of n items. It requires $\Theta(n^2)$ time. For arrays of 1000 items algorithm A takes about 1 millisecond. On an array of 1,000,000 items how long do you expect algorithm A to take?

   (a) about 17 milliseconds
   (b) about 17 seconds
   (c) about 17 minutes.
   (d) about 17 hours.

4. ____ True or False: Nobody knows whether or not there might be a comparison based sorting method that uses $\Omega(n)$ comparisons to sort an array of n numbers.

5. ____ True or False: Nobody knows whether or not there might be a comparison based sorting method that uses $O(n)$ comparisons to sort an array of n numbers.

6. ____ True or False: The recurrence $T(n) = T(n/2) + O(1)$, for $n \geq 2$ with $T(1) = 1$ has the solution $T(n) = O(\lg(n))$.

7. ____ Professor Moriarty has four algorithms whose runtime functions (call them T_2, T_3, T_4, T_5) satisfy the four recurrences

$$T_k(n) = \begin{cases} 1, & \text{for } n = 1, \\ kT_k(n/k) + O(n), & \text{for } n > 1, \end{cases} \text{, for } k = 2, 3, 4, 5.$$

Which best describes the situation?

(a) $T_k(n) = \mathrm{O}(n)$, for each k in 2..5.
(b) $T_k(n) = \mathrm{O}(n\lg(n))$, for each k in 2..5.
(c) $T_k(n) = \mathrm{O}(n^{\lg(5)})$, for some k in 2..5.
(d) none of the above

8. ____ Devious Professor Moriarty has developed four additional algorithms whose runtime functions (we'll also call them T_2, T_3, T_4, T_5, satisfy the four recurrences,

$$T_k(n) = \begin{cases} 1, & \text{for } n = 1, \\ kT_k(n/2) + O(n), & \text{for } n > 1, \end{cases} \text{, for } k = 2, 3, 4, 5.$$

Which best describes the situation?

(a) $T_k(n) = \mathrm{O}(n)$, for each k in 2..5.
(b) $T_k(n) = \mathrm{O}(n\lg(n))$, for each k in 2..5.
(c) $T_k(n) = \mathrm{O}(n^{\lg(5)})$, for some k in 2..5.
(d) none of the above

9. ____ Which is true of the recurrence

$$T(n) = \begin{cases} 1, & \text{for } n = 1, \\ 7T(n/2) + O(n^2), & \text{for } n > 1 \end{cases} ?$$

(a) $T(n)$ describes the runtime of a matrix multiplication algorithm.
(b) $T(n) = \mathrm{O}(n^{\lg(7)})$.
(c) $T(n) = \mathrm{O}(n^{2.81})$.
(d) all of the above

10. ____ Which recurrence relation describes the runtime of FFT (the fast Fourier transform).

(a) $T(n) = 2T(n/2) + \mathrm{O}(n)$
(b) $T(n) = 2T(n/2) + n/2T(2)$
(c) both of the above
(d) none of the above

11. ____ When the FFT is called on $n = 2^k$ points, a primitive root of unity, ω, is used. This ω must be a primitive mth root of unity, for which m?

(a) $m = k$
(b) $m = n$
(c) $m = n/2$

(d) none of the above

12. ____ When the FFT is called on n points a primitive root of unity, ω, is used. For certain X and Y, the two recursive calls work with X of the points and with the Y-th power of ω.

(a) $X = n/2$ points, $Y = 2$ is exponent of ω
(b) $X = n$ points, $Y = 1/2$ is exponent of ω
(c) $X = n/2$ points, $Y = 1/2$ is exponent of ω
(d) $X = n$ points, $Y = 2$ is exponent of ω

13. ____ True or false: The recurrence $T(n) = 2T(n/2) + \mathrm{O}(n^2)$ describes the worst case number of comparisons used in mergesort.

14. ____ True or false: The leaves of a full binary tree are all on the same level.

15. ____ A graph is a DAG if, with respect to it's depth first search forest, it has no ____ edges.

(a) tree
(b) forward
(c) back
(d) cross

16. ____ RSA public key encryption is hard to crack, because

(a) Modular arithmetic is a total mystery
(b) testing for primality is difficult
(c) factoring large composite numbers is difficult
(d) The Fermat test is fooled by some non-primes called Carmichael numbers.

17. ____ Let n be the number of days since the beginning of the earth's rotation until your date of birth. What is $3^n \bmod 2$?

18. ____ What is $2^{2^{2^2}} \bmod 7$?

19. (6 points)

(a) A directed acyclic graph can be linearized. Explain what this means.

(b) Draw a linearization of this graph: $V = \{A, B, C, D\}$, nbr[A] = C, D; nbr[B] = A, D; nbr[C] = D; nbr[D] = null.

(c) How may a linearization of a DAG be constructed from the results of depth first search that computes previsit and postvisit times for each node?

20. (10 points)

   (a) Using `modmul(a, b, m)` which computes $a \times b \bmod m$, write `modexp(a, e, m)`, which computes $a^e \bmod m$, for n bit numbers a, e, m. Your program should use no more than $2n$ calls to modmul, that is, no more than $2n$ multiplications or squarings mod m.

   (b) Up to big-O, What is the runtime of your `modexp`?

21. (5 points)

   (a) Write `mergesort(a[0..n-1])`. You may use (without defining it) `merge(a[0..i-1], b[0..j-1])`, which merges the two sorted array segments a and b, putting the result in $c[0..i+j-1]$.

   (b)

22. (10 points)

(a) Let $f(x) = \sum_{i+0}^{n-1} f_i x^i$ be a n term polynomial. Explain how $f(a)$ and $f(-a)$ can be computed by combining $f_e(a^2)$ and $f_o(a^2)$, where f_e and f_o are the even part and the odd part of f, respectively.

(b) When ω is an n-th root of unity, what power of ω is equal to $-\omega$?

23. (10 points)

(a) Write `explore(G, v)`, which does a depth first search of the graph $G = (V, E)$ beginning at vertex v. You may use the adjacency list representation, with `nbr[u]` being the list of neighbors of u in G.

(b) Up to big-O, what is the run time of `explore`?

24. Have a good spring break.