excerpts from CISC 320 Final Exam

Given Friday, May. 26, 2000

May 23, 2001

These multiple choice questions treat many of the topics we have covered since the midterm exam. They may be used as a review aid. Also consult the "latest info" section of the course web site to review the topics covered and assigned reading. The final exam will cover material discussed since the midterm exam. It begins with section 6.6 on dynamic equivalence relations (union-find).

1. ____ A tree with N edges has
   (a) N-1 vertices.
   (b) N vertices.
   (c) N+1 vertices.
   (d) It varies. The number of vertices in a tree is not determined by the number of edges.

2. ____ Consider Dijkstra's algorithm to determine single source shortest paths in a graph. The vertices of the graph are considered one by one. At each stage the next vertex considered is
   (a) one nearest the source that has not yet been considered.
   (b) one nearest the set of previously processed vertices that has not yet been considered.
   (c) Both are true. The same vertex meets (vertices meet) these two conditions.
   (d) None of the above.

3. ____ In Dijkstra's algorithm, to be able to get the next nearest vertex a priority queue is used. The priority queue functions most important to this use are
   (a) insert and delete.
   (b) insert and extractmin.
   (c) extractmin and decreasekey.
   (d) sort and delete.

4. ____ A heap is a priority queue represented as a tree which is ingeniously stored in an array. Which of the following are true.
   (a) The tree root is in the the first position in the array.
   (b) If a tree node is in position i of the array and has two children, then one of it's children is in position $2i$.
   (c) Both (a) and (b) are true.
   (d) None of the above.

5. ____ Just now gigahertz (10^9 cycles per second) cpu processors are being produced. Chip design experts are predicting that we can expect processor speeds to increase by another factor of 1 million in the next thirty years (continuing Moore's law of speed doubling every 1.5 years). Such machines will be petahertz processors, executing 10^{15} instructions per second! Suppose I take a current gigahertz processor and run it on a problem continuously for the next 30 years. Then, 30 years in the future, I buy a petahertz machine and run it on the same problem. How long will it take the petahertz processor to catch up with the gigahertz processor that has been running for 30 years? Hint: 30 years is just a little under 10^9 seconds.

   (a) A few minutes.
   (b) A few days.
   (c) A few months.
   (d) Never catch up (Zeno's paradox).

6. ____ Consider the recurrence relation T(1) = 0, and T(N) = 2T(N/2) + N, for N ¿= 2. Then T(N) is

   (a) O(N).
   (b) O($N \lg(N)$).
   (c) O(N^2).
   (d) O($(1/2)N^2$).

7. ____ Algorithms A and B each manipulate an array of n items. Algorithm A requires n steps. Algorithm B requires n*lg(n) steps. For each algorithm, a step requires a constant amount of time, but the steps of algorithm A are twice as long as those of algorithm B. Suppose algorithm A requires about 6 seconds to process an array of 1,000,000 items. Hint: how long does a single step for Algorithm A take? for Algorithm B?

   (a) Algorithm B will take about 3 seconds.
   (b) Algorithm B will take about 1 minute.
   (c) Algorithm B will take about 10 days.
   (d) Algorithm B will take more time than the age of the universe.

8. ____ Which of the following is true about minimal spanning trees in weighted graphs?

   (a) If the graph is connected there is a unique minimal spanning tree.
   (b) If the graph is not connected, there is no minimal spanning tree.
   (c) Both (a) and (b) are true.
   (d) None of the above.

9. ____ Which of the following is true about spanning trees in a graph?

   (a) Every connected graph has a unique depth first spanning tree.
   (b) Every connected graph has a unique breadth first spanning tree.
   (c) Every connected weighted graph has a unique minimal spanning tree.
   (d) None of the above.

10. ____ Which class of computational problems is being described? This class is the set of problems for which there is an algorithm which runs in O(N^k) time in the worst case for inputs of size N, and where k is a constant independent of input size.

   (a) P
   (b) NP
   (c) NP-complete
   (d) None of the above.

11. ____ Which class of computational problems is being described? This class is the set of problems for which there is an algorithm which runs in $O(N^k)$ time, where N is the size of the input and k is a constant independent of input size, and what this algorithm does is check an answer (more generally, a certificate). That is, this algorithm takes both an instance of input for the problem and a proposed result for that input. It then determines if that answer is correct or not.

(a) P
(b) NP
(c) NP-complete
(d) None of the above.

12. ____ To which of the following classes does the Traveling Salesman Problem belong?

(a) P
(b) NP
(c) NP-complete
(d) All of the above.

13. ____ What are the two principal ways to represent graphs? (circle two answers)

(a) adjacency heap,
(b) array of adjacency lists,
(c) adjacency matrix.
(d) stack of adjacency queues.

14. ____ How long does depth first search take on a graph given by adjacency lists for a graph with n nodes and m edges?

(a) $O(n)$.
(b) $O(n \lg(m))$.
(c) $O(n + m)$.
(d) $O(nm)$.

15. ____ Which of the following may be determined using depth first search of a graph.
(1) A topological sort of the vertices of the graph.
(2) The strongly connected components
(3) The presence or absence of cycles.

(a) None of the three.
(b) Exactly one of the three.
(c) Exactly two of the three.
(d) All three.

16. ____ The transitive closure of a graph $G = (V, E)$ is a graph with this property:

(a) It has (u, v) as an edge exactly when (u, v) is *not* an edge of G.
(b) It has (u, v) as an edge exactly when (v, u) is an edge of G.
(c) It has (u, v) as an edge exactly when there is a path from u to v in G.
(d) None of the above.

17. ____ The transitive closure of a graph could be computed using the AllPairsShortestPath algorithm for a weighted graph, by first assigning the edges the weight ____, then running the AllPairsShortestPath algorithm, then determining that (u, v) is an edge in the transitive closure if and only if the shortest path from u to v is less than ____.

(a) initial weight 0, shortest path length less than 1.
(b) initial weight 1, shortest path length less than $|V|$.
(c) initial weight inf, shortest path length less than $|E|$.
(d) None of the above. There is no connection between transitive closure and short paths.

18. ____ Using the Fast Fourier Transform, polynomials of degree n can be multiplied in time

(a) $O(n)$.
(b) $O(n \lg(n))$.
(c) $O(n^2)$.
(d) $O(n^2.81)$.

19. ____ Which two of these are equal?
(1) $n^{\lg(3)}$,
(2) $3^{\lg(n)}$,
(3) $2^{\lg(3)\lg(n)}$.

(a) (1) and (2).
(b) (2) and (3).
(c) (1) and (3).
(d) All three are equal.
(e) No two are equal.

20. ____ Strassen's matrix multiplication method for 2 by 2 matrices involves 7 multiplications and 18 addition/subtractions. This leads to an algorithm for n by n matrices whose run time $T(n)$ satisfies the recurrence:

(a) $T(n) = 2T(n/7) + O(n^2)$.
(b) $T(n) = 7T(n/7) + O(n^{1.59})$.
(c) $T(n) = 7T(n/2) + O(n^2)$.
(d) $T(n) = 7T(n/2) + O(n^{2.81})$.

21. ____ Karatsuba's integer multiplication method for 2 digit numbers involves 3 multiplications and 5 additions/subtractions of digits. This leads to an algorithm for n digit numbers whose run time $T(n)$ satisfies the recurrence:

(a) $T(n) = 2T(n/3) + O(n)$.
(b) $T(n) = 3T(n/2) + O(n)$.
(c) $T(n) = 3T(n/3) + O(n^1.59)$.
(d) $T(n) = 2T(n/2) + O(n^2.81)$.

22. ____ Consider the classical method of multiplication of many-digit numbers that we learned in school. For multiplication of two n digit numbers (positive integers) it requires how many single digit operations in the worst case?

(a) $\Theta(n)$.
(b) $\Theta(n \lg(n))$.
(c) $\Theta(n^{\lg(3)})$.
(d) $\Theta(n^2)$.

23. ____ Consider the divide and conquer "Karatsuba" method of multiplication of n-digit numbers, using three multiplications of $n/2$ digit numbers. For multiplication of two n digit numbers (positive integers) it requires how many single digit operations in the worst case?

(a) $\Theta(n)$.
(b) $\Theta(n \lg(n))$.
(c) $\Theta(n^{\lg(3)})$.
(d) $\Theta(n^2)$.

24. ____ Consider the FFT (Fast Fourier Transform) method of multiplication of polynomials. For multiplication of two polynomials of degree bounded above by n, this method requires how many arithmetic operations on coefficients in the worst case?

(a) $\Theta(n)$.
(b) $\Theta(n \lg(n))$.
(c) $\Theta(n^{\lg(3)})$.
(d) $\Theta(n^2)$.

25. ____ Consider the addition of polynomials. For addition of two polynomials of degree bounded above by n, how many arithmetic operations on coefficients are required in the worst case?

(a) $\Theta(n)$.
(b) $\Theta(n \lg(n))$.
(c) $\Theta(n^{\lg(3)})$.
(d) $\Theta(n^2)$.

26. ____ Consider the addition of many-digit numbers, For addition of two n digit numbers (positive integers) it requires how many single digit operations in the worst case?

(a) $\Theta(n)$.
(b) $\Theta(n \lg(n))$.
(c) $\Theta(n^{\lg(3)})$.
(d) $\Theta(n^2)$.

27. ____ Consider the evaluation of a polynomial function at a specified value a. Evaluation of $f(a)$, for a given polynomial $f(x) = f_{n-1}x^{n-1} + f_{n-2}x^{n-2} + \ldots + f_1 x + f_0$ of degree bounded by n, may be done most efficiently by a method known as

(a) Horner's rule.
(b) Discrete Fourier Transform.
(c) Karatsuba's method.
(d) Fermat's formula.

28. ____ Consider the evaluation of a polynomial function at a specified value a. Evaluation of f(a), for a given polynomial $f(x) = f_{n-1}x^{n-1} + f_{n-2}x^{n-2} + \ldots + f_1 x + f_0$ of degree bounded by n, may be done in how many arithmetic operations on coefficients in the worst case? (choose sharpest correct answer)

(a) $O(n)$.
(b) $O(n \lg(n))$.
(c) $O(n^{1.59})$.
(d) $O(n^2)$.

29. ____ Consider the evaluation of a polynomial function at specified values a_i. For n values $a_1, a_2, \ldots, a_n$, evaluation of $f(a_1), f(a_2), \ldots, f(a_n)$, for a given polynomial $f(x) = f_{n-1}x^{n-1} + f_{n-2}x^{n-2} + \ldots + f_1 x + f_0$ of degree bounded by n, may be done in a number of coefficient operations which is

(a) $O(n^2)$.

(b) $O(n \lg(n))$, but only if the a_i's are specially chosen values such as the n-th roots of unity.

(c) Both (a) and (b) are true.

(d) None of the above.

30. ____ Disjoint sets may be represented by nodes containing a pointer to a parent node, with set roots having parent pointer referencing themselves (or null may be used). The FindSet(x) function then follows parent pointers to the root node and returns it as representative of the set. The Union(x,y) function joins two sets by making the root of one parent of the root of the other. Two heuristics may be used to improve the run time of a sequence of MakeSet, Union, and FindSet operations.

Which of the following is not a heuristic to improve disjoint set computing time?

(a) findset cascading.

(b) path compression.

(c) union by rank.

(d) more than one of the above is not a union-find heuristic.

31. ____ With the two heuristics implemented for the disjoint set operations, a sequence of n operations takes $O(n \lg^*(n))$ time. Let $\lg^*(n)$ be defined by $\lg^*(n) = 1$, if $n \leq 2$ and $\lg^*(n) = 1 + \lg^*(\lg(n))$, if $n > 2$. Write in the blank the value of $\lg^*(\text{gogol})$, where gogol is 10^{100}.

32. ____ The satisfiability problem is

(a) to determine if a graph's minimal spanning tree is smaller than its depth-first spanning tree.

(b) to evaluate a polynomial faster than by Horner's rule.

(c) to verify the true identity of the sender of a public key encrypted message.

(d) to determine if value of a boolean formula can be made "true" by a suitable assignment of values to the variables in the formula.

33. ____ To show a computational problem A is NP Complete, it would suffice to show that A is in NP and that

(a) problem A can be polynomially reduced to the Traveling Salesman Problem.

(b) the Traveling Salesman Problem can be polynomially reduced to problem A .

(c) either (a) or (b) would suffice.

(d) Both (a) and (b) are necessary.

34. ____ Which statement is false?

(a) The problem class P is contained in the class NP.

(b) The class of NP Complete problems is contained in the class NP.

(c) It *is not known* whether the class NP is contained in the class P.

(d) It *is known* that the class of NP Complete problems is contained in P.

Suppose number A is less than C^2, so that $A = aC + b$, where a and b are less than C. Then $A^2 = a^2C^2 + 2abC + b^2$. Explain how this formula can be exploited to computational advantage.