

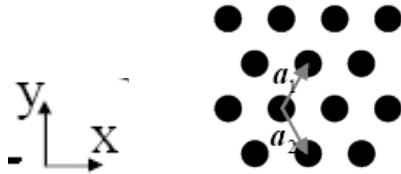
**Homework #9 - due Monday, 12 December 2005, in ECE office, 140 Evans Hall**

1. **Photonic Crystal Reciprocal Structure:** Consider the triangular lattice in 2-dimensional real space shown below. Calculate the reciprocal lattice vectors,  $b_1, b_2$ . (b) sketch some points of the reciprocal lattice. (c) sketch the first Brillouin zone and label the important symmetry points, G, X, M.

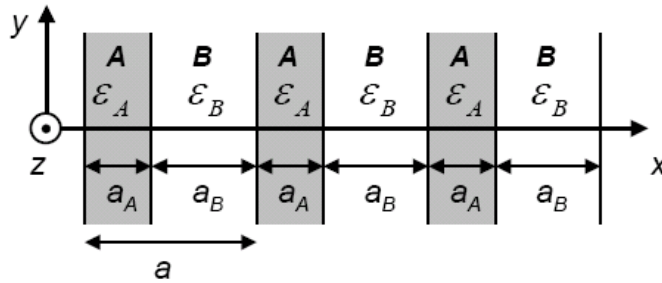
Hint, consider the lattice below.

$$a_1 = a(\hat{x}\sqrt{3} + \hat{y}) / 2$$

$$a_2 = a(-\hat{x}\sqrt{3} + \hat{y}) / 2$$



2. **One dimensional photonic crystal:** Consider an alternating dielectric stack comprising media A



and B in the x direction shown below.

The reciprocal lattice vectors are given by  $b = 2\pi p/a$  where  $p$  is an integer. The electric field for the wave equation is:

$$\frac{\partial^2}{\partial z^2} E(z,t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \epsilon(z) E(z,t) = 0$$

The periodic function for the dielectric constant  $\epsilon(z)$  may be expanded as a Fourier series:

$$\epsilon(z) = \sum_{p=-\infty}^{\infty} \epsilon_p e^{j\frac{2\pi p}{a}z}$$

(a) calculate the Fourier coefficients:

$$\epsilon_p = \frac{1}{a} \int_{z=0}^a \epsilon(z) e^{-j\frac{2\pi p}{a}z} dz$$

and write them down for the cases  $p = 0$  (hint  $\epsilon$  is real) and  $p \neq 0$ .

Use phasor notation for

$$E(z,t) = e^{-j\omega t} E(z)$$

where

$$E(z) = \sum_{p=-\infty}^{\infty} c_p e^{j(k+p\frac{2\pi}{a})z}$$

which has the Bloch form since

$$\left\{ e^{j\frac{2\pi p}{a}z} \right\}_{p=-\infty}^{\infty}$$

has the periodicity of the lattice.

(b) Substitute E and  $\varepsilon$  into the wave equation and set the prefactor of

$$e^{j(k+p\frac{2\pi}{a})z}$$

equal to zero to obtain the eigenvalue equation for  $\omega$  and k in terms of  $c_p$ , and  $\varepsilon$ . (Hint: careful to distinguish between the index parameter p, and the dummy summation index, (e.g. p').

**3. Electronic Paper Design:** Consider an electronic paper system of a thin sheet containing white nanospheres in a blue dye. Select the physical parameters, such as layer thickness and particle size. One figure of merit for the performance is the switching speed. Assuming that video refresh rates (e.g. 60 Hz) are needed, calculate the speed that a particle must reach to make a one way pass through the thickness in the refresh period, starting from rest. What force is required? Using values of viscosity and mobility from handbooks, discuss the electric charge on a nanosphere and the required applied voltage. As these values reasonable and attainable today?

Homework assignments will appear on the web at:

[http://www.ece.udel.edu/~kolodzey/courses/eleg667\\_016f05.html](http://www.ece.udel.edu/~kolodzey/courses/eleg667_016f05.html)

Note: On each submission, give your name, due date, assignment number and course number.