Homework 2 Solutions
15 points

1. In class we calculated the contact resistance when a narrow conductor with M modes is connected to two very wide contacts. If the number of modes in the contacts is not infinite, but some finite number, N, then the left-moving and right moving carriers inside the contacts have different electrochemical potentials, as shown in the figure below. Show that the contact resistance taking this into account is given by

$$R_c = (h/2e^2)[1/M – 1/N]$$

For further discussions on the nature of the contact resistance at different types of interfaces see Landauer (1989) *J. Phys. Cond. Matter*, **1**, 8099 and M. C. Payne (1989) *J. Phys. Cond. Matter*, **1**, 4931.

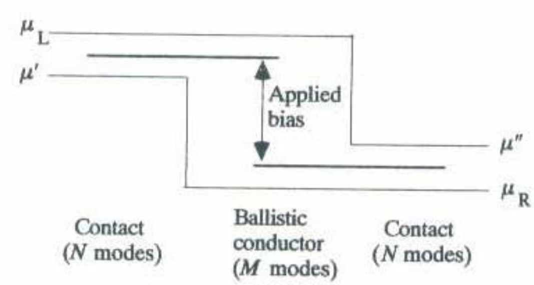


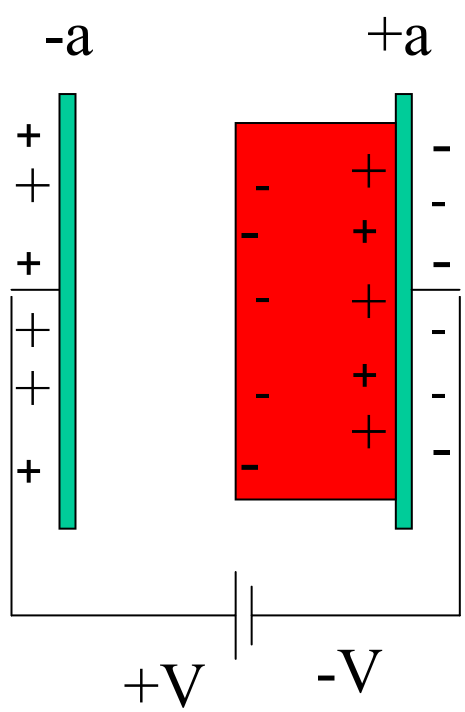
Fig. E.2.1. Spatial variation of the electrochemical potential for a ballistic conductor with *M* modes connected between two contacts having a finite number of modes (*N*).

In class the number of conduction modes in each contact was infinite and the number of conduction the conductor was M. There was a contact resistance of $R_c = (h/2e^2)[1/M]$. This contact resistance was caused by the limitation on the number of available levels (modes) in the conductor. Now, in one direction (+*k*) there will be no limitation on the density of states (every charge carrier from the contact can find a mode to carry it) but there will be a conduction limitation at the (-k) end given by the limited number of modes in the contact. The difference in the currents will be $I^+ = (2e/h)\, M\, (\mu_L - \mu_R) = (2e/h)(M – N)\, (\mu_L - \mu'') + (2e/h)\, N\, (\mu'' - \mu_R)$, $I^- = (2e/h)\, N\, (\mu'' - \mu_R)$. Therefore $I^+ - I^- = (2e^2/h)(M – N)\, (\mu_L - \mu'')/e = (2e^2/h)(M – N)V_{app}$. $I = G\, V_{app}$ and $G = (2e^2/h)(M – N)$. Then $R_c = (h/2e^2)(M – N)^{-1}$. While this value is not equal to the requested, the requested value is negative because M > N so 1/M – 1/N, which is physically impossible.

2. Pure water has a dielectric constant of 80 in static electric fields but its index of refraction for visible light is 1.33. Calculate the ratio of the static to this high-frequency dielectric constant and account qualitatively for the discrepancy.

At high frequency $n \simeq (\varepsilon_R)^{1/2}$, or $n \simeq 1.33$. At high frequencies (visible light) the dielectric constant is reduced by more than a factor of 40. The oscillating electric field cannot couple to the molecules rotations or vibrations, leaving only polarization of the electron cloud around the molecule.

3. A large plane parallel capacitor is half filled with a uniform and homogeneous dielectric having the dielectric constant K. The conducting surfaces $x = -a$ and $x = a$ have potential V and $-V$ respectively, and $\varepsilon = \varepsilon_0$ where $-a < x < 0$, and $\varepsilon = K\varepsilon_0$ where $0 < x < a$.
    a. Find E and D where $-a < x < 0$.
    b. Find E and D where $0 < x < a$.
    c. Locate all charges and specify if they are real or polarization charges.



$-a < x < 0$ $\mathbf{E} = -V/a$
$\mathbf{D} = \varepsilon_0\varepsilon_R\mathbf{E} = -\ \varepsilon_0\ V/a$, $(\varepsilon_R = 1)$

$0 < x < a$ $E = -V/a$
The normal vector displacements are equal at interfaces, *i.e.,* $D_{1n} = D_{2n}$
Thus $\mathbf{D} = \varepsilon_0\varepsilon_R\mathbf{E} = -\ \varepsilon_0\ V/a$,
$\varepsilon_R = k\varepsilon_0$, $\mathbf{E} = -\ V/ka$,

There are real charges on each of the plates. Between the plates there are no real charges, but there are polarization charges.