

4.4.1 Formulas for n and p

The number of electrons/cm³ and holes/cm³ with energies between E and $E + dE$ has been established to be $g_c(E)f(E)dE$ and $g_v(E)[1 - f(E)]dE$, respectively. The total carrier concentration in a band is therefore obtained by simply integrating the appropriate distribution function over the energy band—that is,

$$n = \int_{E_c}^{E_{\text{top}}} g_c(E)f(E)dE \quad (4.48a)$$

$$p = \int_{E_{\text{bottom}}}^{E_v} g_v(E)[1 - f(E)]dE \quad (4.48b)$$

Substituting the density of states and Fermi function expressions into eqs. (4.48), noting that little error is introduced by letting $E_{\text{bottom}} \rightarrow -\infty$ and $E_{\text{top}} \rightarrow \infty$, and rearranging the result into a convenient form, one obtains

$$n = N_c \mathcal{F}_{1/2}(\eta_c) \quad (4.49a)$$

$$p = N_v \mathcal{F}_{1/2}(\eta_v) \quad (4.49b)$$

where

$$N_c = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{3/2} \quad \dots \text{effective density of conduction band states} \quad (4.50a)$$

$$N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{3/2} \quad \dots \text{effective density of valence band states} \quad (4.50b)$$

$$\mathcal{F}_{1/2}(\eta) = \frac{2}{\sqrt{\pi}} F_{1/2}(\eta) \quad (4.51)$$

$$F_{1/2}(\eta) = \int_0^\infty \frac{\xi^{1/2} d\xi}{1 + e^{\xi - \eta}} \quad \dots \text{Fermi-Dirac integral of order } 1/2 \quad (4.52)$$

and

$$\eta_c = (E_F - E_c)/kT \quad (4.53a)$$

$$\eta_v = (E_v - E_F)/kT \quad (4.53b)$$

The eq. (4.49) concentration relationships are valid for any conceivable positioning of the Fermi level. N_c and N_v , the effective density of states, are of course readily computed for a given material and temperature: the 300 °K values of these constants

Table 4.2 Concentration Parameters and Functions.

A. Effective density of states at 300 °K

Semiconductor	N_C (cm ⁻³)	N_V (cm ⁻³)
Ge	1.03×10^{19}	5.35×10^{18}
Si	3.22×10^{19}	1.83×10^{19}
GaAs	4.21×10^{17}	9.52×10^{18}

B. Selected properties of the $\mathcal{F}_j(\eta)$ functions^[4]

$$\mathcal{F}_j(\eta) \equiv \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{\xi^j d\xi}{1 + e^{\xi-\eta}}$$

$$\mathcal{F}_j(\eta) \rightarrow e^\eta \quad \text{as } \eta \rightarrow -\infty$$

$$\frac{d}{d\eta} \mathcal{F}_j(\eta) = \mathcal{F}_{j-1}(\eta)$$

$$\mathcal{F}_{1/2}(\eta) \approx [e^{-\eta} + \xi(\eta)]^{-1}$$

$$\text{where } \xi(\eta) = 3\sqrt{\pi/2} [(\eta + 2.13) + (|\eta - 2.13|^{2.4} + 9.6)^{5/12}]^{-3/2}$$

with a maximum error of $\sim \pm 0.5\%$

$$\eta \approx \frac{\ln u}{1 - u^2} + \frac{(3\sqrt{\pi} u/4)^{2/3}}{1 + [0.24 + 1.08(3\sqrt{\pi} u/4)^{2/3}]^{-2}}$$

$$\text{where } u \equiv \mathcal{F}_{1/2}(\eta)$$

$$\text{with a maximum error of } \sim \pm 0.5\%$$

for Ge, Si, and GaAs are listed in Table 4.2A. [At 300 °K, $N_{C,V} = (2.509 \times 10^{19}/\text{cm}^3) (m_{n,p}^*/m_0)^{3/2}$.] $\mathcal{F}_{1/2}(\eta)$, on the other hand, is obtained from literature tabulations, through direct computation, or by the use of analytical approximations. Selected properties of the $\mathcal{F}_j(\eta)$ family of modified Fermi-Dirac integrals is presented in Table 4.2B. Of the several analytical approximations for $\mathcal{F}_{1/2}(\eta)$ (known) and $\eta(\mathcal{F}_{1/2}$ known) suggested in the device literature, the entries in this table appear to provide the best combination of accuracy and convenience. The asymptotic approach of $\mathcal{F}_{1/2}(\eta)$ to $\exp(\eta)$ as η increases negatively (entry #2 in Table 4.2B) is examined in detail in Fig. 4.15. For additional information about the $\mathcal{F}_{1/2}(\eta)$ function and a summary of available $\mathcal{F}_{1/2}(\eta)$ versus η tabulations, the reader is referred to the excellent review paper by Blakemore.^[4]

As is evident from Fig. 4.15, $\mathcal{F}_{1/2}(\eta)$ is closely approximated by $\exp(\eta)$ when $\eta \leq -3$. Utilizing this approximation, one obtains

$$n = N_C e^{(E_F - E_C)/kT} \quad \dots E_C - E_F \geq 3kT \quad (\eta_C \leq -3) \quad (4.54a)$$

$$p = N_V e^{(E_V - E_F)/kT} \quad \dots E_F - E_V \geq 3kT \quad (\eta_V \leq -3) \quad (4.54b)$$

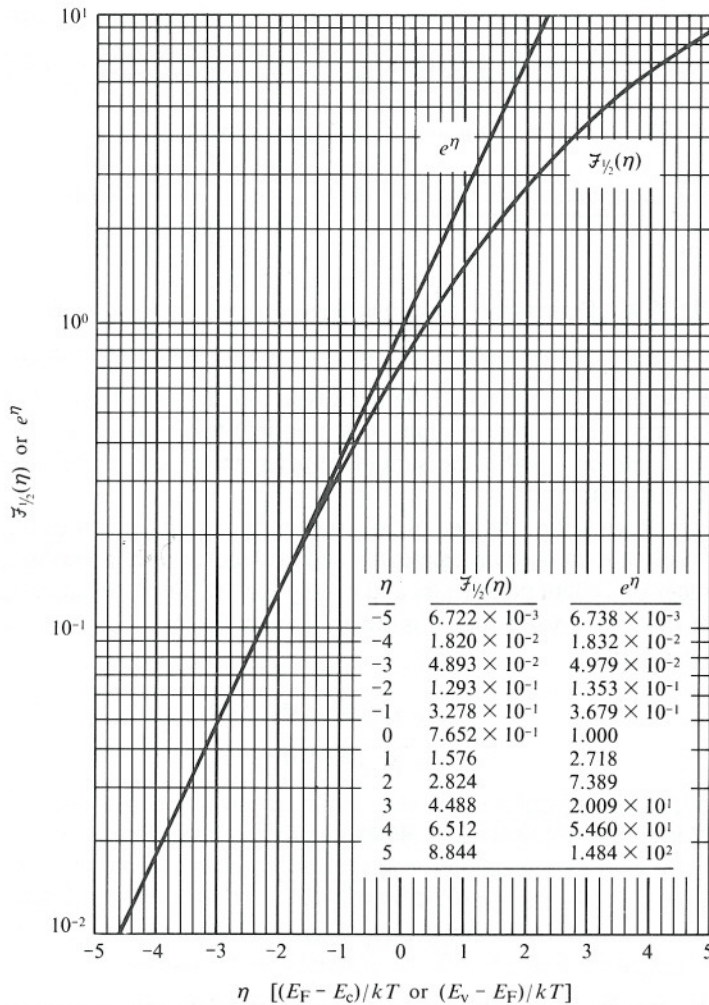


Fig. 4.15 Comparison of the modified Fermi-Dirac integral and e^η for η near zero (corresponding to Fermi level positionings near the band edges).

The inequalities adjacent to eqs. (4.54) are simultaneously satisfied if the Fermi level lies in the band gap more than $3kT$ from either band edge. For the cited positioning of the Fermi level (also see Fig. 4.16), the semiconductor is said to be *nondegenerate* and eqs. (4.54) are referred to as nondegenerate relationships. Conversely, if the Fermi level is within $3kT$ of either band edge or lies inside a band, the semiconductor is said to be *degenerate*. It should be noted that a nondegenerate positioning of the Fermi level makes $f(E) \approx \exp[-(E - E_F)/kT]$ for all conduction band energies and $1 - f(E) \approx \exp[(E - E_F)/kT]$ for all valence band energies. The simplified form of

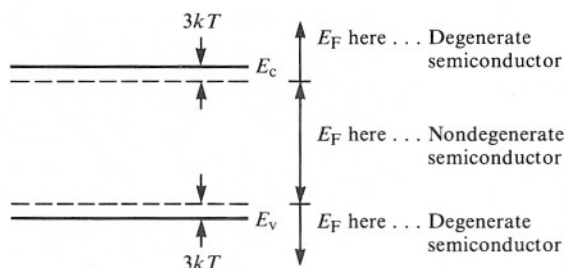


Fig. 4.16 Definition of degenerate/nondegenerate semiconductors.

the occupancy factors is a Maxwell-Boltzmann type function which also describes, for example, the energy distribution of molecules in a high-temperature, low-density gas. When substituted into eq. (4.48), the simplified occupancy factors lead directly to the nondegenerate relationships.

Although in closed form, the eq. (4.54) relationships find limited usage in device analyses. More often than not one employs an equivalent set of relationships involving a reduced number of system parameters and energy levels. Since the nondegenerate relationships are obviously valid for an intrinsic semiconductor where $n = p = n_i$ and $E_F = E_i$, one can write

$$n_i = N_C e^{(E_i - E_c)/kT} \quad (4.55a)$$

$$n_i = N_V e^{(E_i - E_v)/kT} \quad (4.55b)$$

or, solving for the effective density of states,

$$N_C = n_i e^{(E_c - E_i)/kT} \quad (4.56a)$$

$$N_V = n_i e^{(E_i - E_v)/kT} \quad (4.56b)$$

Eliminating N_C and N_V in the original nondegenerate relationships using eqs. (4.56) then yields

$$n = n_i e^{(E_F - E_i)/kT} \quad (4.57a)$$

$$p = n_i e^{(E_i - E_F)/kT} \quad (4.57b)$$

Like eqs. (4.54), the more convenient (4.57) expressions are valid for any semiconductor in equilibrium whose doping is such as to give rise to a nondegenerate positioning of the Fermi level.