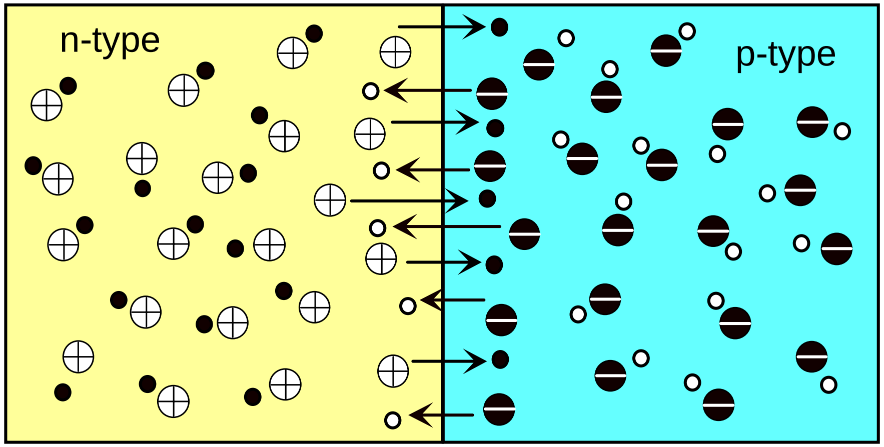
Pn Junction in Equilibrium

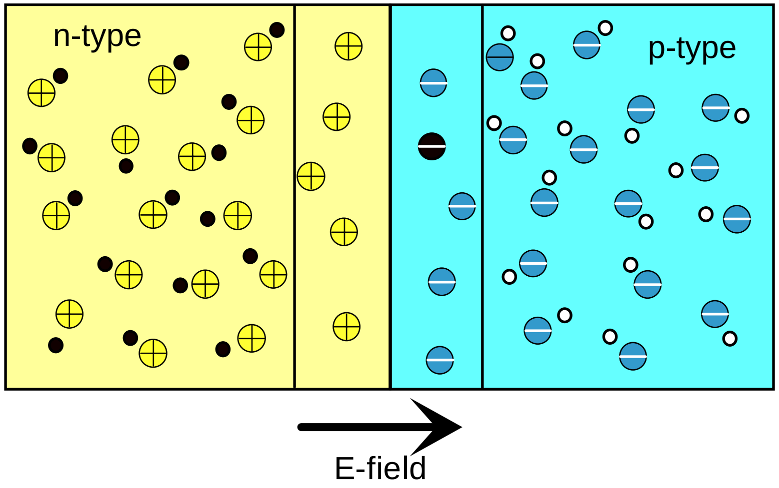
- Pn Junction consists of p-type and n-type material in contact with each other.
- Thermal equilibrium means that there are no inputs – no voltage, no extra heat, no light.
- When n and p-type material come into contact, electrons diffuse from n-type to type and vice versa (ie majority carriers cross junction).
- Once the majority carriers have crossed the junction, they become minority carriers, and have a limited lifetime.
- Dopant atoms are fixed and cannot diffuse.



Movement of electron and holes across junction

Pn Junction in Equilibrium

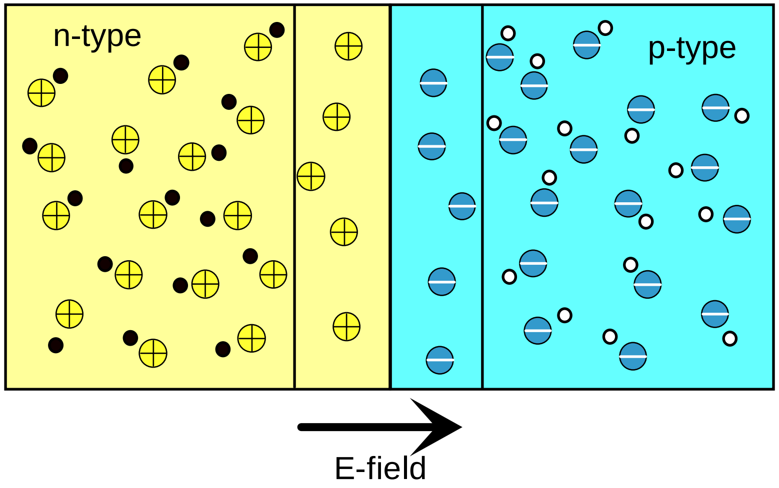
- **Space-Charge Region (or depletion region)**
  - **Movement of holes in p-type region (and electrons in n-type region) across junction leaves behind ionized dopant atoms.**
  - **Previously, the charge on these dopant atoms was not considered since the material was electrically neutral and the fixed dopant atoms do not contribute to current.**
  - **In a pn junction, these ionised donor cores set up an electric field near the interface between n and p-type material.**
  - **This region is called the space charge region (SCR) or the depletion region (since it depleted of carriers).**



Remember the direction of the arrow of the Electric field points in the direction a POSTIVE charge (ie a hole) would go

Pn Junction in Equilibrium

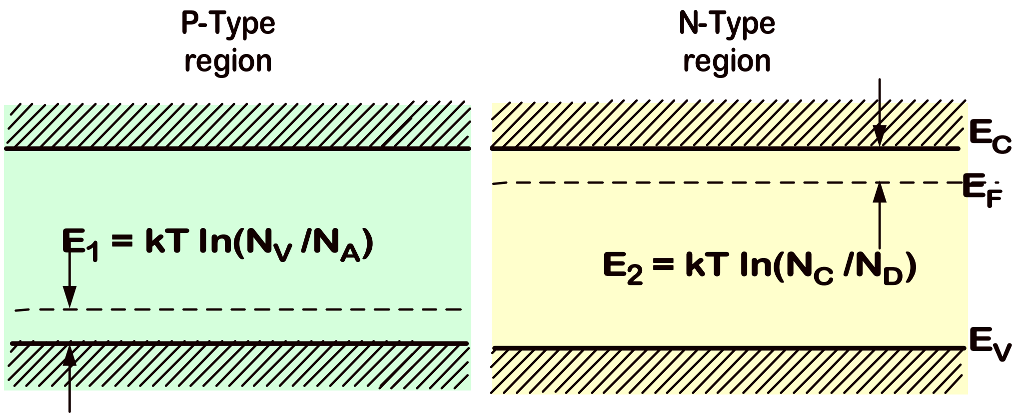
- **Drift Current**
  - **The electric field is in such a direction so as to sweep holes in the n-type material across to the p-type material and vice versa.**
  - **The electric field will sweep minority carriers in either n-type or p-type material across to the other side of the junction where they will be majority carriers.**
  - **Normally, unless the minority carrier concentration is increased in some fashion, this current remains very small.**
  - **Minority carrier concentration can be increased due to increased temperature, optical generation, or injection of carriers.**
  - **Under equilibrium conditions, the net current is 0. The diffusion current equals the drift current for both electron and holes currents.**

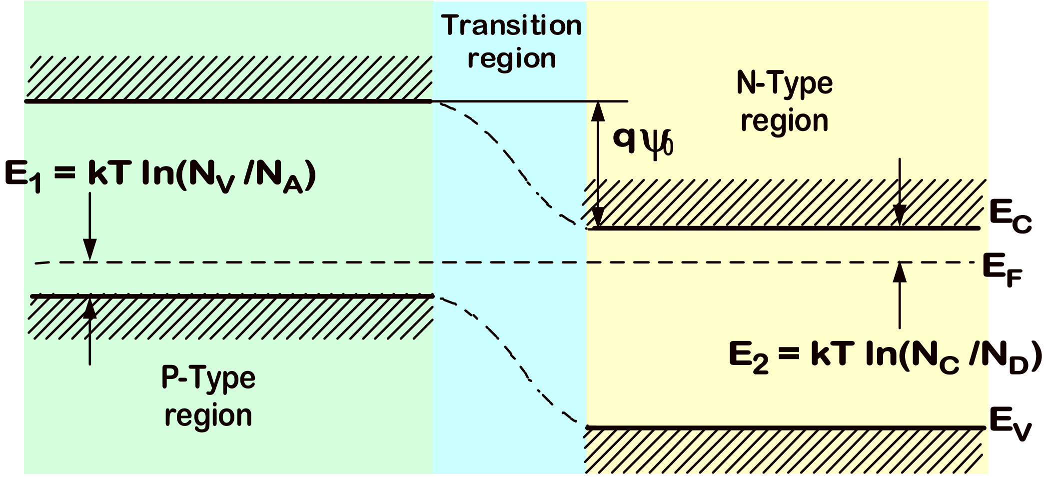


Remember the direction of the arrow of the Electric field points in the direction a POSTIVE charge (ie a hole) would go

Band Diagrams in Equilibrium

- The Fermi-level in the entire system under thermal equilibrium must be constant. For a system in equilibrium, the average energy must be constant and so the Fermi-level must be constant.
- Well away from the junction, bulk conditions dominate such that the Fermi level is at its bulk value.
- At the junction, the slope in the band diagram indicated the presence of the electric feils





Built-in Voltage

- The electric field across the interface of a pn junction gives rise to a voltage across the interface, called the built-in voltage, V_0.
- The built-in voltage cannot be measured by externally connecting probes to the device.
- V_0 is due to the difference between the Fermi levels of the joined materials, and can be calculated from this.

$$q\Psi_0 = E_g - E_1 - E_2$$

$$= E_g - kT\ln\left(\frac{N_V}{N_A}\right) - kT\ln\left(\frac{N_C}{N_D}\right)$$

$$= E_g - kT\ln\left(\frac{N_V N_C}{N_A N_D}\right)$$

using

$$n_i^2 = N_C N_V \exp\left(-E_G \big/ kT\right)$$

gives

$$q\Psi_0 = kT\ln\left(\frac{N_C N_V}{n_i^2}\right) - kT\ln\left(\frac{N_C N_V}{N_A N_D}\right)$$

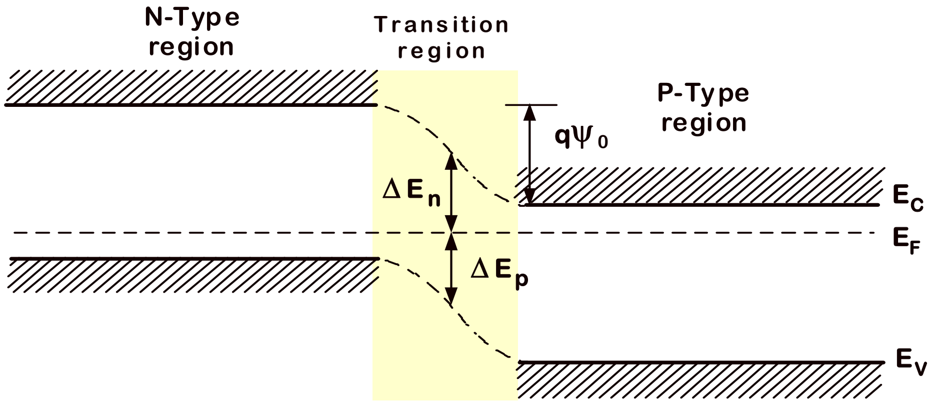
$$\Psi_0 = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

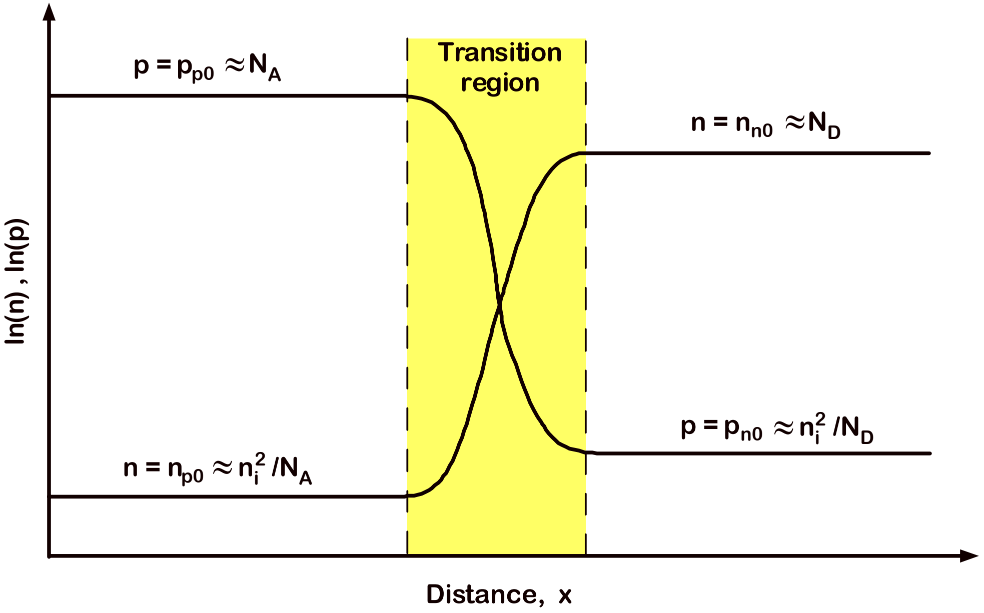
Carrier Concentration in Equilibrium

- **The difference between the Fermi-level and the conduction band (valence band) gives the electron (hole) concentration.**
- **From the band diagram, we can sketch the carrier concentration.**
- **Outside of the depletion region, carriers retain their equilibrium values.**
- **Since the built-in voltage depends on the difference between the doping on either side of the junction, the carrier concentrations are related to each other by the built-in voltage**

$$p_{n0} = p_{p0} \exp\left({}^{qV}\!/_{kT}\right) = N_A \exp\left({}^{qV}\!/_{kT}\right) = {}^{n_i^2}\!/_{N_D}$$

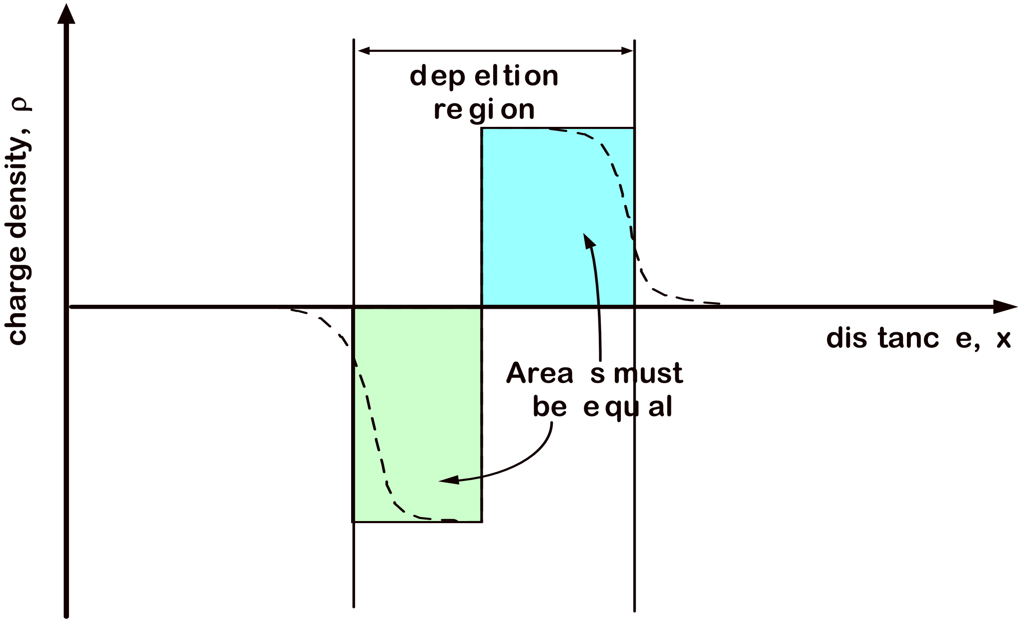
$$n_{p0} = n_{n0} \exp\left({}^{qV}\!/_{kT}\right) = N_D \exp\left({}^{qV}\!/_{kT}\right) = {}^{n_i^2}\!/_{N_A}$$





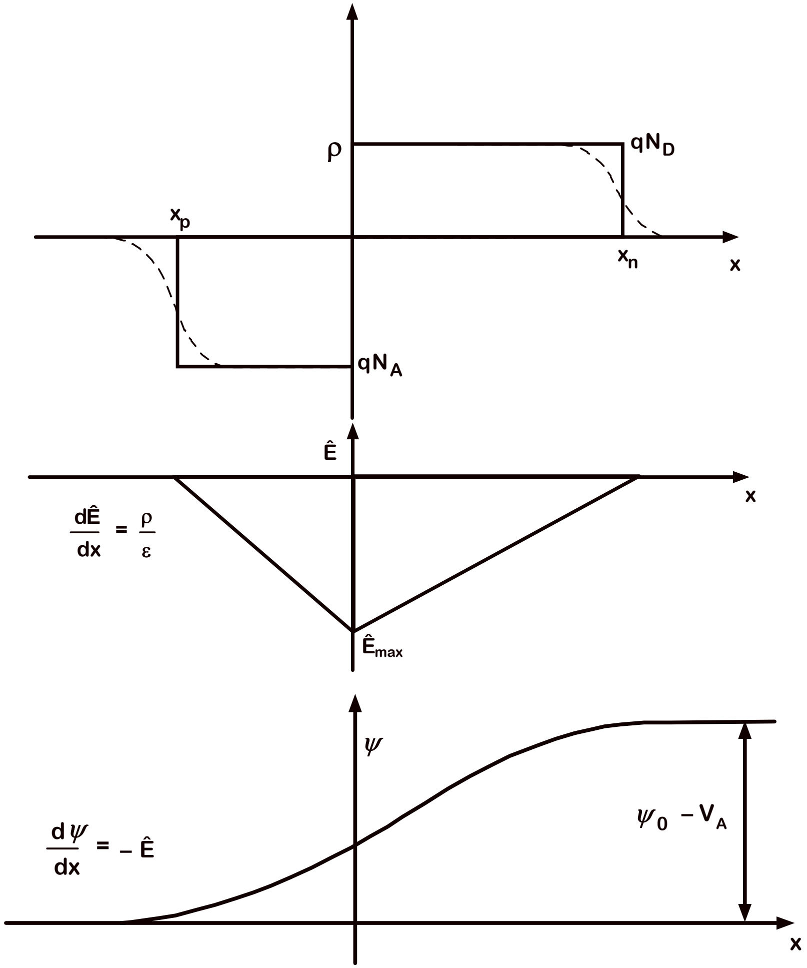
Depletion Region Properties

- The depletion region consists of a region of fixed charge corresponding to the ionized dopant atoms cores that “lost” their electrons or holes due to the diffusion current.
- The depletion region tails off exponentially away from the junction edge.
- Assuming that the depletion region is zero a certain distance away from the junction edge (called the depletion region width) greatly simplified analysis.
- Above assumption is called depletion region approximation: The depletion approximation assumes that the electric field is confined to a finite region.
- For constant doping it approximates the charge density as constant in the transition region and zero everywhere else.
- The amount of charge on the two sides of the depletion region must be equal.



Depletion Region Width

- The width of the depletion region can be calculated by integrating the charge density in the depletion region to get the electric field, and then integrating again to get an expression for the built-on voltage, whose value we already know from the difference in the Fermi levels.



$$\frac{d\hat{E}}{dx} = \rho \qquad \rho = \begin{cases} \dfrac{qN_A}{\varepsilon_s} & when\; x_p \le x < 0 \\ \dfrac{qN_D}{\varepsilon_s} & when\; 0 \le x < x_n \end{cases} \qquad N_A x_p = N_D x_n$$

Integrating twice and setting this equal to the built in voltage, V_0, allows us to find the maximum value of the electric field

$$\hat{E}_{\max} = -\sqrt{\frac{2q}{\varepsilon}\frac{\psi_o}{\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

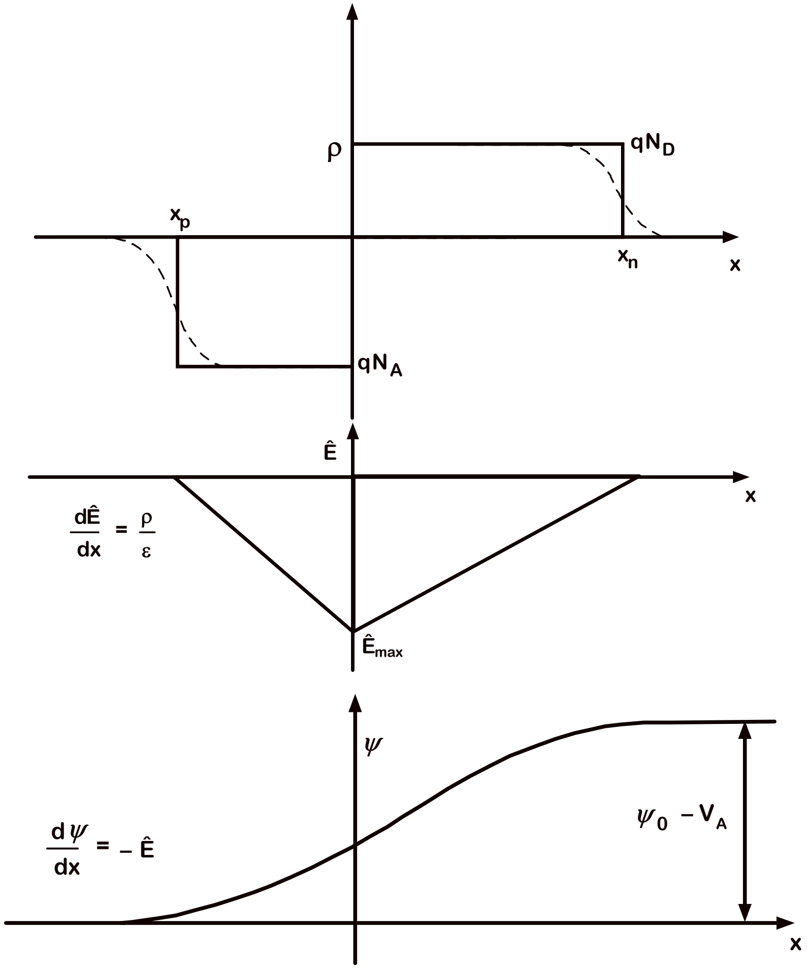
Depletion Region Width

- **The depletion region width is given by:**

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon}{q}\frac{\psi_o}{\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

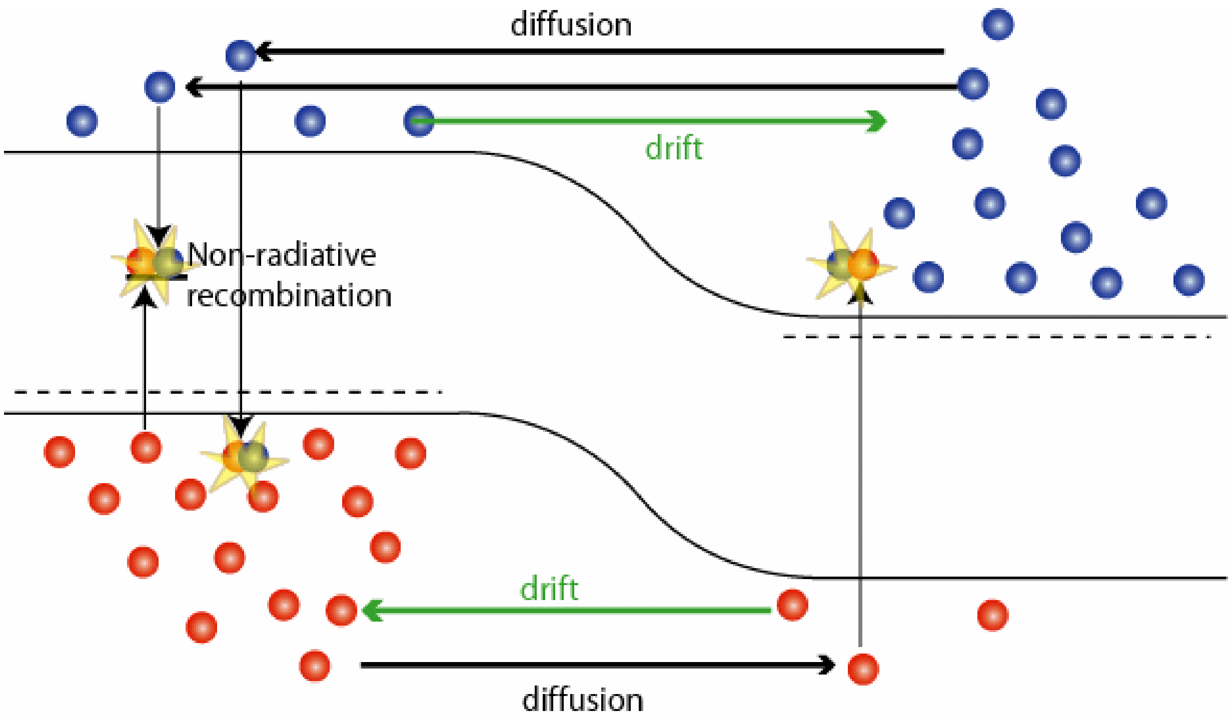
$$x_p = W\frac{N_D}{N_A + N_D} \quad \text{and} \quad x_n = W\frac{N_A}{N_A + N_D}$$

- **The maximum electric field increases as the doping increases and is controlled by the doping of the more *lightly* doped side**
- **The depletion region width is also controlled by the more *lightly* doped side.**



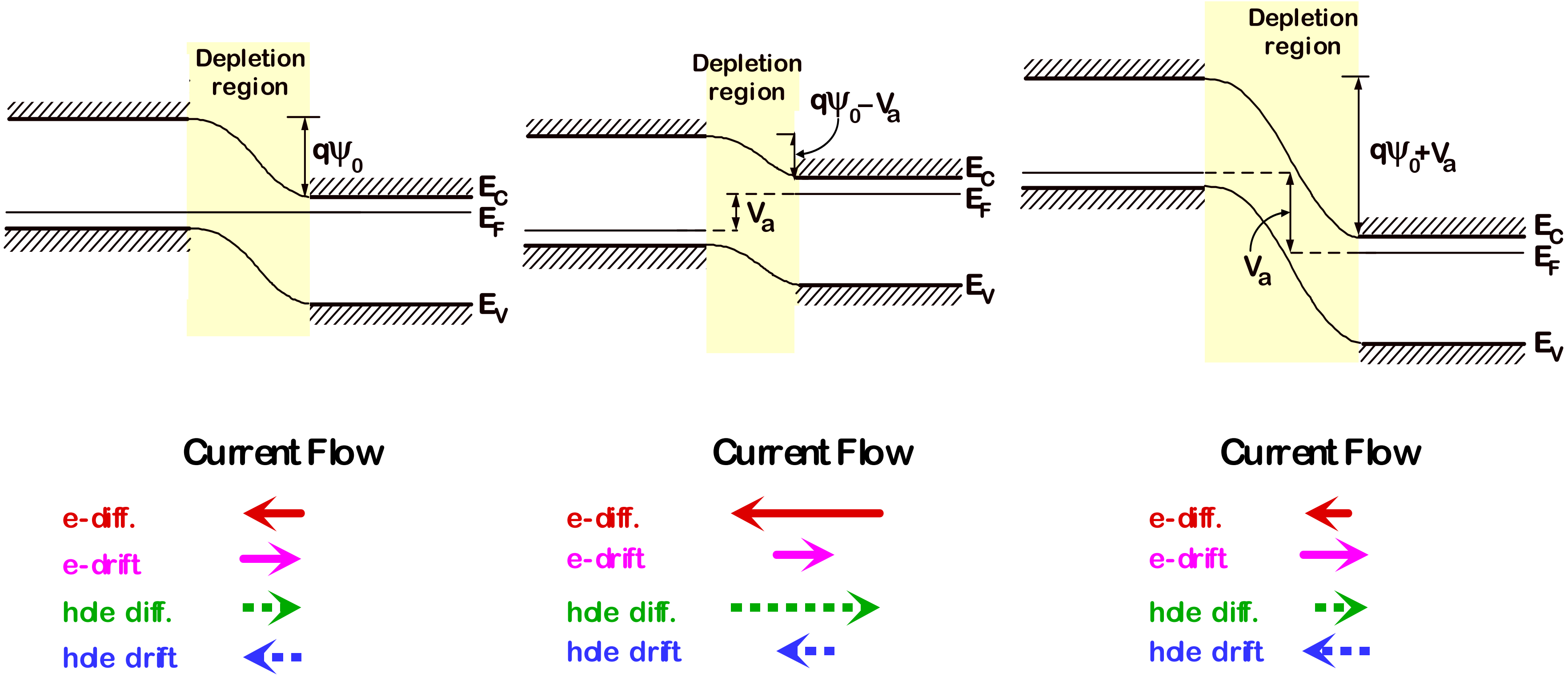
Pn junction under bias

- Forward bias corresponds to applying a voltage which REDUCES the electric field at the interface. Reverse bias INCREASES the electric field at the interface.
- The application of voltage disturbs the equilibrium between the diffusion current and drift current.
- Reducing the electric field (ie, forward bias) reduces the barrier to diffusion current, thereby allowing an increased diffusion current flow. The drift current stays the same, and a net current exists in the device.
- In reverse bias the barrier to diffusion current is increased, reducing the diffusion current while the drift current remains the same. Again a (very small!) net current exists.



Pn junction under bias

- Summary of currents and band diagrams.
- The applied voltage on the band diagram is indicated by the difference between the Fermi-levels (quasi-Fermi levels)



Depletion width under bias

- The application of an applied voltage changes the built-in voltage at the interface. Assuming that the applied voltage is all dropped across the depletion region, the above equations are just modified by Ψ_0 changing to Ψ_0 - Va where Va is the applied voltage.
- The equations then become:

$$\hat{E}_{\max} = -\sqrt{\frac{2\varepsilon}{q}\frac{\psi_o - V_a}{\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

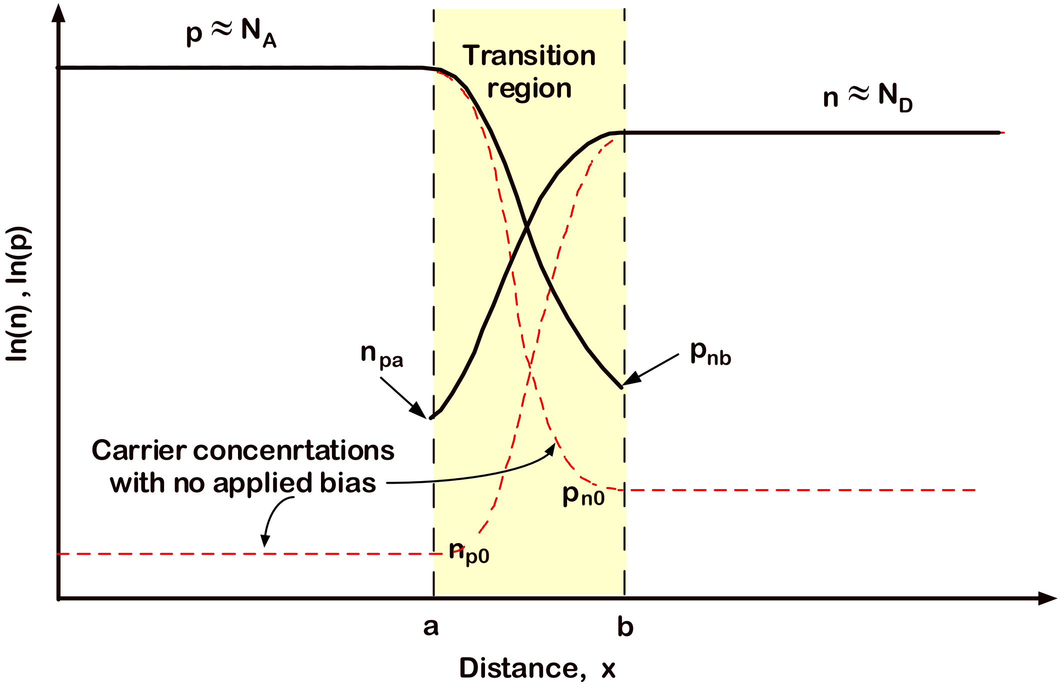
with the other variables as defined before.

- Forward bias reduces the depletion region width, reverse bias increases it.

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon}{q}\frac{\psi_o - V_a}{\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

Carrier injection in forward bias

- The application of forward bias to a pn junction increases the number of carriers that diffuse from one side of the junction to the other.
- This causes the number of minority carriers to increase from its equilibrium value at the depletion region edge, with the number of excess carriers depending on the applied bias.
- The “injection” of excess minority carriers at the edge of the depletion region due to forward bias is called carrier injection.
- With no generation, the number of carriers decreases farther away from the junction as the minority carriers recombine.
- More than a few diffusion lengths away from the junction, the carrier concentrations are back to equilibrium values.



Carrier injection in forward bias

- The number of minority carriers at the edges of the depletion region can be calculated assuming that

$$J_p = q\mu_p p\hat{E} - qD_p \frac{dp}{dx} \approx 0$$

- Such an assumption is valid since at positive biases, both the drift and diffusion terms are large and opposing in the depletion region, such that their magnitude is large compared to the total current. This then gives:

$$\hat{E} \approx \frac{kT}{q}\frac{1}{n}\frac{dn}{dx}$$

integrating

$$\psi_o - V_a = -\frac{kT}{q}\ln n\Big|_{x_p}^{x_n}$$

$$= \frac{kT}{q}\ln\left(\frac{n_{nb}}{n_{pa}}\right)$$

- Using charge neutrality at the edge of the depletion region we get:
- Combining eqns gives:

$$n_{nb} = N_D + p_{nb} \approx n_{n0} = n_{p0}\exp\left({}^{q\psi_0}\!/_{kT}\right)$$

$$n_{pa} = n_{nb}\exp\left({}^{-q\psi}\!/_{kT}\right)\exp\left({}^{qV_a}\!/_{kT}\right)$$

$$n_{pa} = n_{p0}\exp\left({}^{qV_a}\!/_{kT}\right) = \frac{n_i^2}{N_A}\exp\left({}^{qV_a}\!/_{kT}\right) \text{ and}$$

$$p_{nb} = p_{n0}\exp\left({}^{qV_a}\!/_{kT}\right) = \frac{n_i^2}{N_D}\exp\left({}^{qV_a}\!/_{kT}\right)$$

IV equation of a pn junction

- **We solve the 4 basic equation sets to find the current in a pn junction.**

*Summarising Basic Device Equations:

•Poisson's Equation:

$$\frac{d\hat{E}}{dx} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}\left(p - n + N_D^+ - N_A^-\right)$$

•Transport equations:

$$J_n = q\mu_n n\hat{E} + qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p p\hat{E} - qD_p \frac{dp}{dx}$$

•Continuity Equation:

$$\tfrac{1}{q}\frac{dJ_n}{dx} = G_n - U_n$$

$$-\tfrac{1}{q}\frac{dJ_p}{dx} = G_p - U_p$$

The following simplifications will be made:

1) 1 dimensional device
2) Thermal Equilibrium
3) Steady-state
4) Depletion approximation

IV equation of a pn junction

- **Several further assumptions can be made to simplify the above equations.**
  1. **If solving for current only in quasi-neutral region (ie., everywhere but depletion region), the charge density is zero, as is the electric field.**
  2. **Assumption (1) also implies that minority carrier current will be controlled by diffusion.**
  3. **No light generation, Gn= Gp = 0**
  4. **Recombination is SRH**
  5. **Low level injection**

$$J_n = qD_n \frac{dn(x')}{dx} = \text{ minority carrier current on p-side}$$

$$J_p = -qD_p \frac{dp(x)}{dx} = \text{minority carrier current on n-side}$$

where n(x') is the electron concentration in the p-type side and
p(x) is the hole concentration in the n-type side.

Since the current will be calculated independently on each side of the junction, we introduce a change in variable such the x' is the distance from the junction into the p-doped side and x is the distance from the junction into the n-doped side

IV equation of a pn junction

- **From the continuity equation we get:**

$$\frac{1}{q}\frac{dJ_n}{dx'} = \frac{\Delta n}{\tau_n} = \frac{n(x') - n_{p0}}{\tau_n}$$

$$-\frac{1}{q}\frac{dJ_p}{dx} = \frac{\Delta p}{\tau_p} = \frac{p(x) - p_{n0}}{\tau_p}$$

- **Combing the transport and continuity we get:**

$$D_n \frac{d^2 n(x)}{dx^2} = \frac{n(x) - n_{p0}}{\tau_n}$$ and $$D_p \frac{d^2 p(x)}{dx^2} = \frac{p(x) - p_{n0}}{\tau_p}$$

- **Using**

$L_n = \sqrt{D_n \tau_n}$ and $L_p = \sqrt{D_p \tau_p}$

$$\frac{d^2 \Delta n}{dx^2} = \frac{\Delta n}{L_n^2}$$ and $$\frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{L_p^2}$$

The general solution to the above equations are:

$$\Delta n = Ae^{x/L_n} + Be^{-x/L_n}$$

$$\Delta p = Ae^{x/L_p} + Be^{-x/L_p}$$

where A and B are constants determined by the boundary conditions

IV equation of a pn junction

- **For the case of a pn junction in which the surfaces are far away**

(1) Boundary condition of pn junction (x=0) $n(0) = \frac{n_i^2}{N_D} \exp\left(qV_a/kT\right)$

(2) Boundary at "surface": Carrier concentration must be finite as x→∞ (ie A =0)

- **Notes:**
  - **For most devices, at least one side of the material will have a boundary condition as in (1).**
  - **If the material does not contain a on junction, both boundary conditions will be "surface" boundary conditions.**
  - **The boundary condition at the surface may be a boundary condition on the carrier concentration, on the generation, or on the current.**
- **For the above boundary conditions we get:**

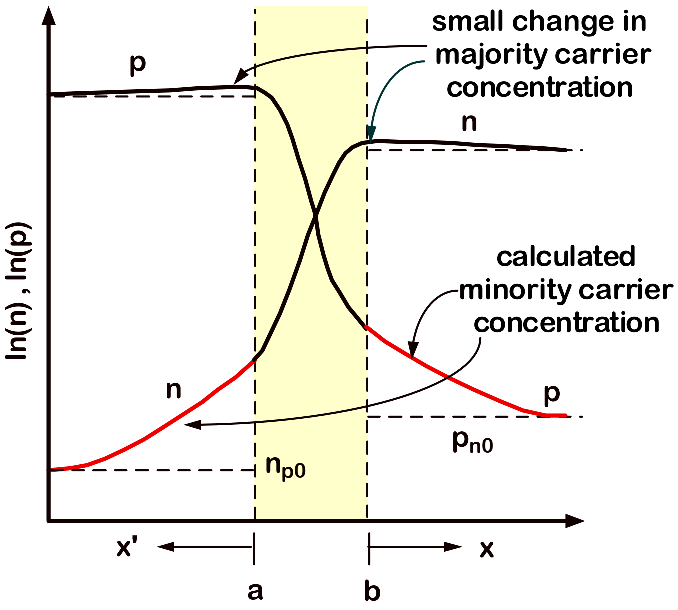
$$p_n(x) = p_{n0} + p_{n0}\left[\exp\left(qV/kT\right) - 1\right] e^{-x/L_p}$$

$$n_p(x) = n_{p0} + n_{p0}\left[\exp\left(qV/kT\right) - 1\right] e^{-x/L_n}$$

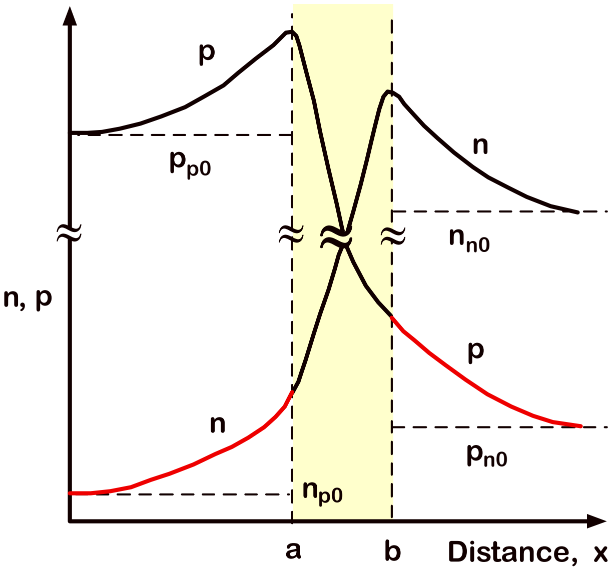
IV equation of a pn junction

- The excess carrier concentration decays away exponentially. The factor controlling the decay is the lifetime of the minority carriers.
- Since we are assuming charge neutrality outside the depletion region, then an increase in the minority carrier concentration means a corresponding increase in the majority carrier concentration. However, the majority carrier concentration is much higher than the minority carrier concentration and so does not change much as long as the device is in low injection.
- The current will depend on the of the minority carrier concentration., and so on the lifetime of the minority carriers.

Carrier concentration on log scale



Carrier concentration on linear scale



IV equation of a pn junction

- **Minority Carrier Current Flow**
  - **The currents can be calculated from the gradient of the carrier concentration.**
  - **Note: the current that will be calculated are due to minority carriers only! We deal with majority carriers later and also check the validity of the assumption that the current flow is due to minority carriers.**
- **On n-type doped side of junction**

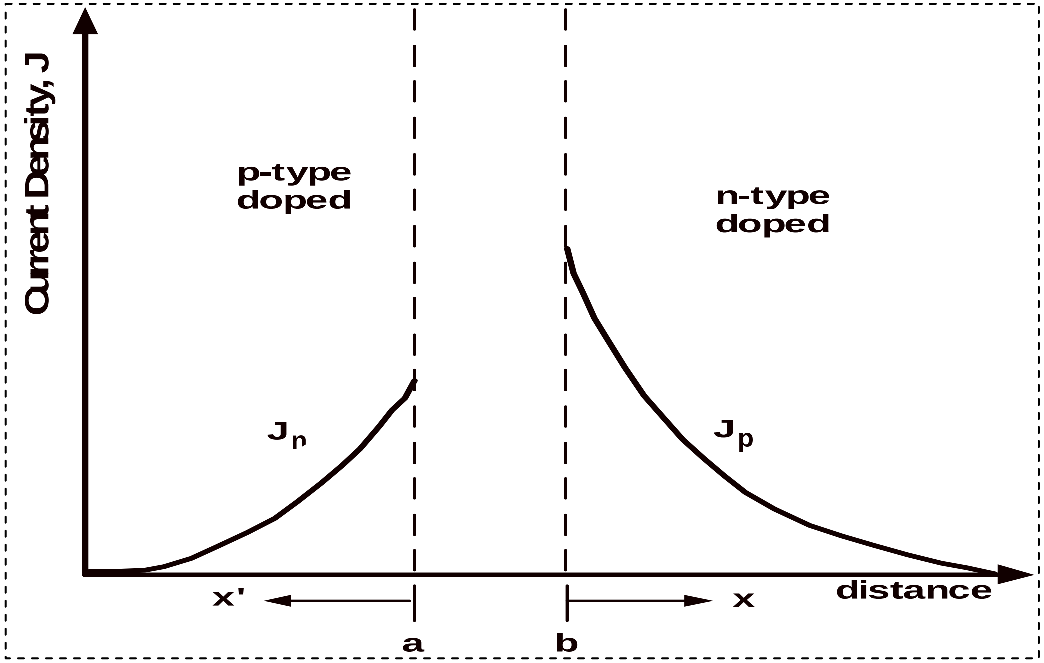
$$J_p = -qD_p \frac{dp}{dx}$$

and plugging in p(x) from above gives:

$$J_p = q\frac{D_n n_{p0}}{L_n}\left[\exp\left({}^{qV}\!/_{kT}\right)-1\right]e^{-{}^{x}\!/_{L_n}}$$

•Similarly for the p-type side:

$$J_n = -q\frac{D_p p_{n0}}{L_p}\left[\exp\left({}^{qV}\!/_{kT}\right)-1\right]e^{-{}^{x}\!/_{L_p}}$$



IV equation of a pn junction

- **Current Flow in Depletion Region**
  - **The current in the depletion region is difficult to calculate exactly since the electric field cannot be assumed to go to zero.**
  - **However, the recombination in the depletion region is usually assumed to be zero since the depletion width is generally much less than the diffusion length of minority carriers and the carriers are swept through the depletion region by the electric field.**
  - **In non-idealities, we will derive an approximation for the recombination in the depletion region.**
  - **If no recombination or generation:**

$$\frac{1}{q}\frac{dJ_n}{dx} = G_n - U_n = 0 \quad \text{therfore } J_n = \text{constant}$$

$$-\frac{1}{q}\frac{dJ_p}{dx} = G_p - U_p = 0 \quad \text{therfore } J_p = \text{constant}$$

IV equation of a pn junction

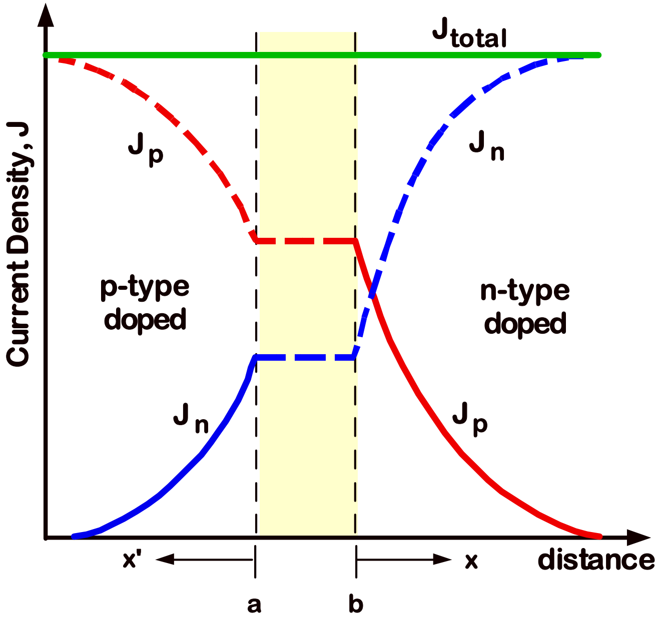
- **Total Current**
  - **The previous currents calculated are the minority carrier currents and the currents in the depletion region. The majority carrier current is difficult to calculate explicitly, but can, like the depletion region current, be easily calculated based on the minority carrier current. .**
  - **The total current on either side of the junction must be constant since otherwise a net charge density would gradually build up over time.**
  - **Mathematically on one side of the junction .**

$$\frac{1}{q}\frac{dJ_T}{dx} = \frac{1}{q}\frac{dJ_n}{dx} + \frac{1}{q}\frac{dJ_p}{dx} = U_n - U_p = 0$$

$$J_{total} = J_n(x'=0) + J_p(x=0)$$

$$= \left[q\frac{D_n}{L_n}n_{p0} + q\frac{D_p}{L_p}p_{n0}\right]\left[e^{qV/kT}-1\right]$$

$$= \left[q\frac{D_n n_i^2}{L_n N_A} + q\frac{D_p n_i^2}{L_p N_D}\right]\left[e^{qV/kT}-1\right]$$

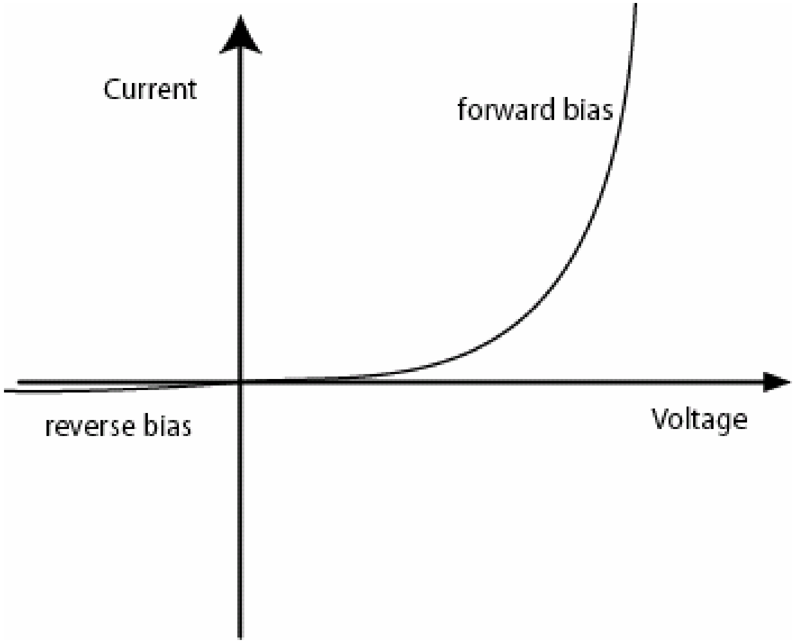


IV equation of a pn junction

- **Ideal Diode Equation**

$$I = I_0\left[\exp\left(qV/kT\right) - 1\right]$$ where I_0 is $$I_0 = qA\left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D}\right)$$

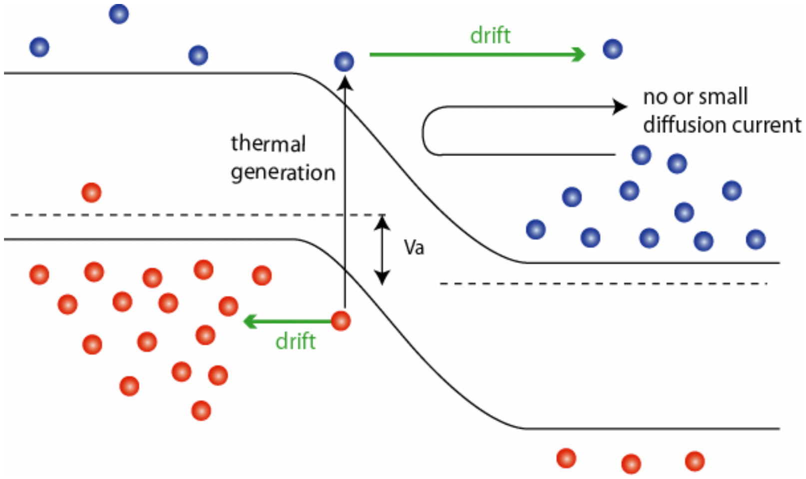
- **Any forward bias current flow in a pn junction is a recombination current. If there is no recombination, there is no steady-state current flow.**
- **Forward bias current depends on how many carriers there are near the edge of the depletion region. The background minority carrier concentration, recombination and diffusivity all determine this carrier concentration.**
- **Increasing the recombination rate or increasing the diffusivity (mobility) increases the forward bias current.**
- **more heavily doped side will dominate the current flow.**
- **Higher band gap pn junction give lower forward bias currents.**

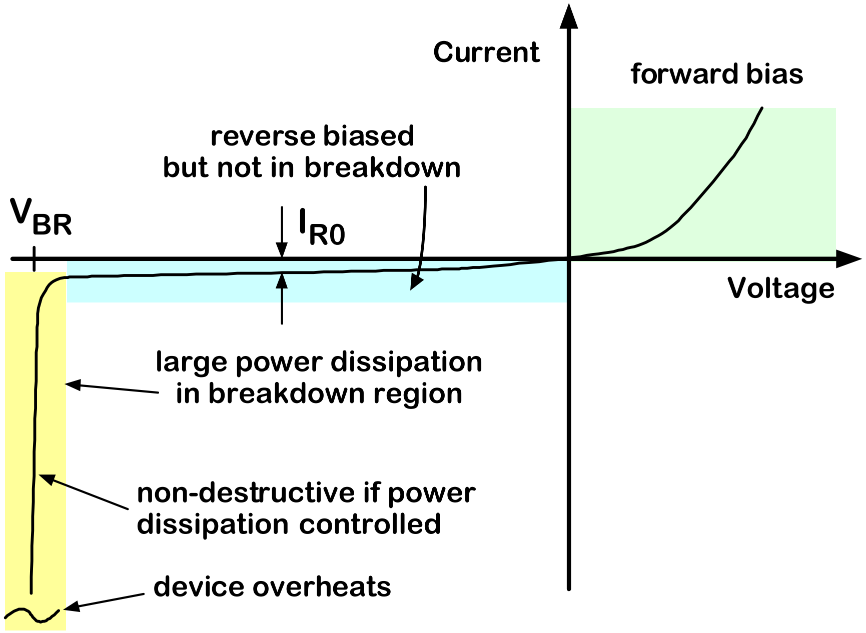


Reverse Breakdown

- **Reverse bias increases the electric field at the junction and increases the depletion width.**
- **The increased electric field increases the barrier to the diffusion current, hence reducing it.**
- **Drift current remains unchanged, as it is controlled by the *number* of carriers crossing the junction, not their speed.**
- **The reverse current increases slightly since the W increases, and hence the**

and hence the number of carriers thermally generated in the depletion region increases, hence the number swept across by the electric field increased.





reverse biased
but not in breakdown

large power dissipation
in breakdown region

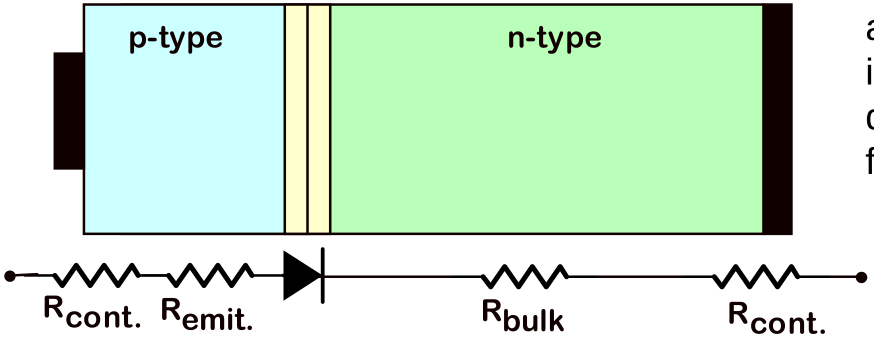
non-destructive if power
dissipation controlled

device overheats

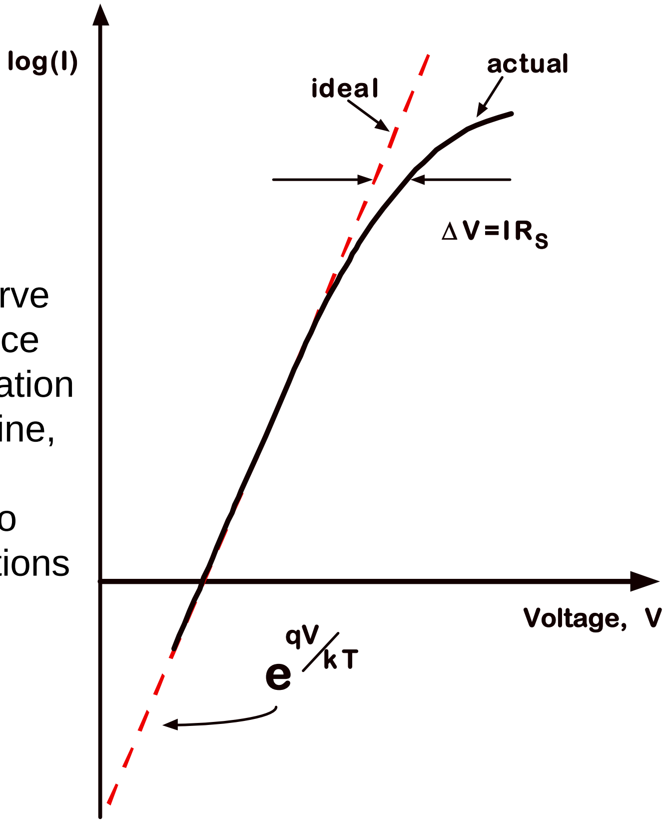
Non-idealities

- Series Resistance
  - Majority carrier currents flow in device due to Electric Field, but usually the voltage drop associated with this voltage drop is negligible compared to the junction.
  - As current densities increase, the voltage at the contacts becomes less than the voltage at the junction itself This decreases the current compared to the case without series resistance.
  - Diode equation becomes:

$$I = I_0\left[\exp\left(\frac{q\left(V - IR_S\right)}{kT}\right) - 1\right]$$



Often plot IV curve on log plots, since ideal diode equation will be straight line, and hence non-idealities easy to detect as deviations from line.



High Injection

- High injection in a “non-ideality” in that high injection effects are not included in the ideal diode equation
- High injection refers to the condition when the minority carrier concentration begins to be equal to the majority carrier concentration (ie doping)
- Under forward bias in a pn junction, the number of Due to the exponential increase of injected minority carrier concentrations with the forward bias voltage, they will eventually approach the doping..
- High injection will occur first on the more lightly doped side.
- Since the derivation of the ideal diode equation assumes low injection, high injection represents a breakdown of the previous theory.
- High injection occurs when injected carriers = doping:

$$p_{nb} = \frac{n_i^2}{N_D} \exp\left(\frac{qV_{hi}}{kT}\right) = N_D$$

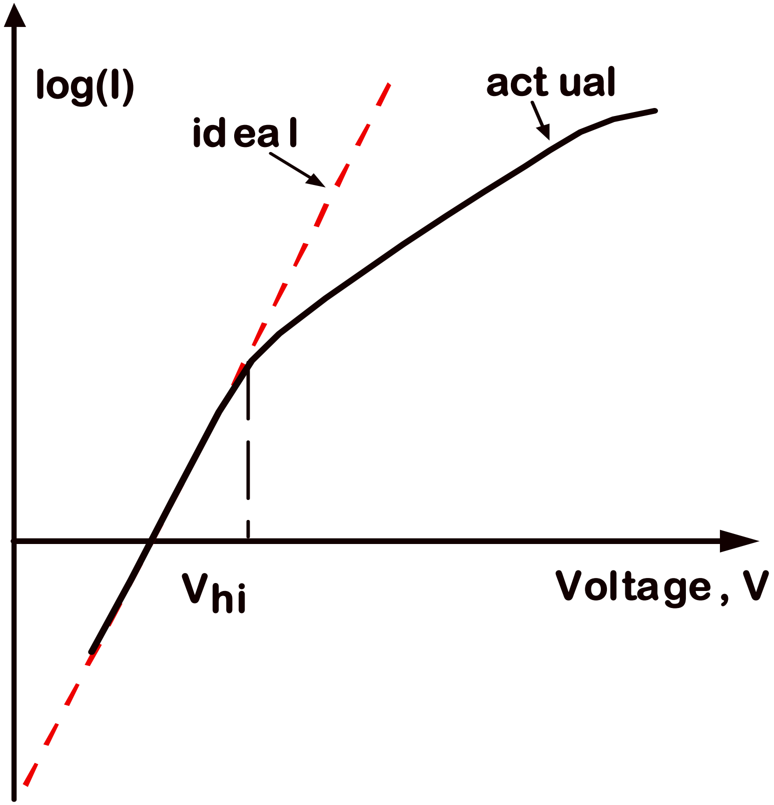
where p_{nb} carrier concentration on the n-type side of the depletion region, and V_{hi} is the voltage for the onset of high injection

$$V_{hi} = \frac{2kT}{q} \ln\left(\frac{N_D}{n_i}\right)$$

High Injection

- **High injection effects modify the current flow in a pn junction and change the IV curve.**
- **The recombination mechanisms no longer are controlled solely by the minority carrier concentration expressed by ($\Delta n/\tau$)**
- **The current changes to a form:**

$$I = I_S \exp\left({qV}/{2kT} \right)$$



* **In devices with other recombination mechanisms, a general form of the IV curve is:**

$$I = I_0 \left[\exp\left({qV}/{nkT} \right) - 1 \right]$$

* **The term n is the diode ideality factor and is related to the number of carriers that control the recombination**

* **For high injection, when both minority and majority carriers are significant, its value is 2.**

Depletion Region Recombination

- Previously recombination in the space charge region has been neglected. However, this type of recombination actually dominates at low currents.
- The general expression for recombination is:

$$U = \frac{\left(np - n_i^2\right)}{\left(\tau_p\left(n + n_1\right) + \tau_n\left(p + p_1\right)\right)}$$

- In the quasi-neutral regions and assuming n = ND and ni2=n0p0, the above equation reduces to

$$U = \frac{\left(N_D p - n_i^2\right)}{\left(\tau_p N_D + smaller\ temrs\right)} = \frac{\left(p - p_{n0}\right)}{\tau_p}$$

- In depletion region, can’t assume that

$np = n_i^2$ but instead $np = n_i^2 \exp\left({}^{qV}\!/\!{}_{kT} \right)$

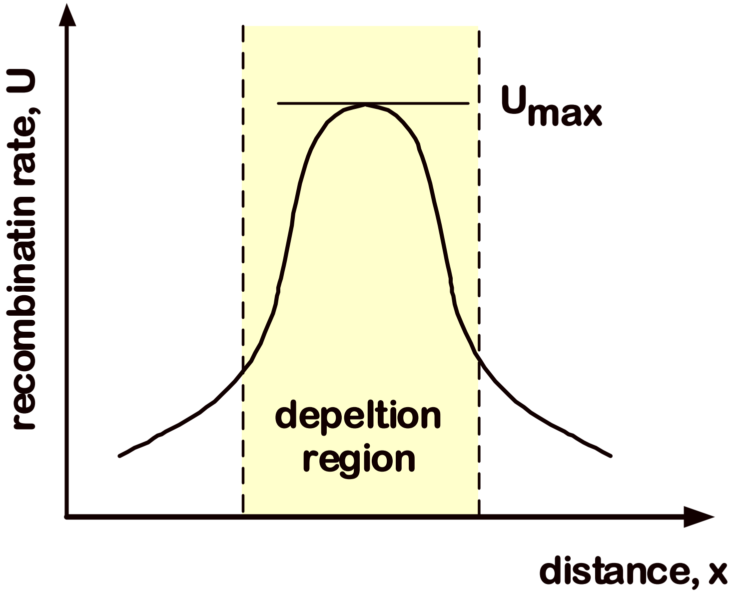
Depletion Region Recombination

- Recombination rate then becomes:

$$U = \frac{n_i^2\left(\exp\left(\frac{qV}{kT}\right) - 1\right)}{\left(\tau_p n + \tau_n p + \tau_p n_1 + \tau_n p_1\right)}$$

where the terms with n_1 and p_1 are small and can be ignored.

- Plotting the recombination rate (for a given voltage) across the depletion region (ie., n and p vary) shows that the recombination peaks near the center of the depletion region.
- U_{max} occurs when the denominator is a minimum. The denominator is a minimum when:



$$\frac{d\left(\tau_p n + \tau_n p\right)}{dn} = \tau_{p0} - \frac{n_i^2}{n^2}\tau_{n0}\exp\left(\frac{qV}{kT}\right)$$

Depletion Region Recombination

- **The current then becomes:**

$$I = I_{rec}\, exp\left({}^{qV}\!\big/_{2kT} \right) \qquad I_{rec} \approx \frac{qAn_i^2 W}{2\tau_e}$$

where

$$\tau_e t = 2\sqrt{\tau_{n0}\tau_{p0}}$$

in forward bias and

$$\tau_e t = \tfrac{1}{2}\left(\tau_{n0} + \tau_{p0}\right)$$

in reverse bias