

University of Delaware -- Computer and Information Science

CIS621 -- Homework 5

Handed out: November 8, 2011

Due date: November 29, 2011

13. [Individual Problem]

The Music Album Problem: Before the days of iTunes (and after the days of “45s”), it was not possible to purchase music one song at a time. If you wanted a song you had to buy an album that included that song. Consider then that:

- U is a set of songs
- $L = (S_1, S_2, \dots, S_n)$ is a list of the songs that you want to own
- Albums A_1, A_2, \dots, A_m are available for purchase where album A_i consists of a subset of the songs in U
- All albums cost the same amount
- You have enough money in your budget to purchase B albums

Formally the *Instance* consists of U , L , the A_i 's and B .

Question: Are there B albums that together include all of the songs in your list L ?

Show that the Music Album Problem is NP-complete.

14. [Group Problem]

Exercise 34.5-2 from CLRS (0-1 integer programming)

15. [Group Problem]

A **Carpenter's Ruler** consists of line segments of integral lengths, hinged together consecutively. The line segments (called **links**) can rotate freely about the hinges. Such a ruler models a robot arm. In moving a robot arm it is found convenient to fold the arm as compactly as possible. In such a folding, we allow the links to cross each other, but the angle at a hinge is restricted to be either 0 or 180 degrees.

Prove that given a carpenter's ruler in which the successive links have lengths $\omega_1, \omega_2, \dots, \omega_n$, and an arbitrary bound D , it is NP-complete to determine whether the ruler can be folded into a length of at most D .

16. [Group Problem]

This problem also deals with the Carpenter's Ruler.

a) Suppose m is the largest of the ω_i 's. Prove that the Carpenter's Ruler can always be folded into a length of no more than $2m$. Do this by giving an algorithm for folding the ruler and basing the result on the folding that the algorithm produces.

b) Prove that the bound in part a is the best possible. That is, for any given $\epsilon > 0$, explain how to construct a ruler for which the optimal folded length is at least $2m - \epsilon$.