**A1. Computer Graphics (25 points)**

Texture Mapping.

1. [5pts] Build the Summed Area Table for the following texture.

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 3 | 5 | 2 |
| 6 | 2 | 4 | 1 |
| 7 | 5 | 2 | 3 |
| 6 | 2 | 3 | 1 |

1. [5pts] Build the MIPMAP hierarchy for the same texture. (Feel free to round the floating point values to integer values at each level if needed).
2. [3pts] If the original 2D texture is of size M (bytes), what is the size of its corresponding MIPMAP?

1. [12pts] Environment Mapping.

a) [5pts] Given an environment map (spherical map) surrounding a scene, assume the incident ray has normalized direction *i*, the vertex normal is *n*, what is the reflected ray direction *r*?

b) [7pts] Derive the texture coordinate of the reflected ray in the environment map in terms of r (assume r = [rx, ry, rz])?

**A2. Computer Graphics (25 points)**

Transformation.

1. [10pts] Recall that a viewing transformation from one homogenous coordinate system to another can be written as the product of a rotation matrix *R* and a translation matrix *T*. Given a camera viewpoint at [0, 0, 0], a viewing direction of [1, 1, 1], and a “view-up” or sky vector of [0, 1, 0], write down the equation for computing the image right vector *r*, the image up vector *u*, and the rotation matrix *R*. (If you use dot or cross product, you need to write out the explicit result rather than just a x b).

2. Normal Transformation Matrix. If we transform the vertices of a 3D model by a transformation matrix M, derive the corresponding transformation matrix N (in terms of M) for the surface normal by answering the following two parts.

a) [5pts] Given two 3D points **x**1 and **x**2 on a plane that has normal **n**, what constraint (equation) should **n**, **x**1 and **x**2 satisfy?

b) [10pts] Now that we transform the points **x**1 and **x**2 with matrix **M**, we need to transform the normal **n** accordingly so that the points and the normal still satisfy the constraint above after transformation. Please derive this normal transformation matrix **N** in terms of **M**. Show all your work.

**A3. Computer Graphics (25 points)**

Visibility.

1. [8 pts] Explain the Cohen-Sutherland line clipping algorithm, including the role of outcodes. Where does it typically occur in the pipeline? How do the 2-D and 3-D cases differ?
2. [8 pts] What are the Z-buffer algorithm and the painter's algorithm for visibility testing? Briefly explain how they work and give at least one positive and one negative feature of each.
3. [5 pts] Consider the set of 2-D line segments in the figure below. Suppose we pick the partitioning line for a given level of a binary space partitioning (BSP) tree as the first in a list ordered using the lines' numerical labels. The ordering rule is “smallest-to-largest even numbers, then largest-to-smallest odd numbers."

Give the steps to construct a BSP tree from the figure's line segments using this rule, indicating any splits that are necessary. For clarity, the only splits possible are: line 2 splits 3 and 4, line 1 splits 3, and line 3 splits 5.



4. [4 pts] Now suppose the viewer is positioned at the dot in the figure. In what order would the BSP tree you constructed be traversed for the painter's algorithm?

**A4. Computer Graphics (25 points)**

Shading.

1. [4 pts] Assume that a certain full-color (24 bit per pixel) RGB raster system has a 2048 by 2048 frame buffer. If 360 megabytes/second is the allowable transfer rate, what is the maximum possible frame rate (frames per second)?
2. [3 pts] If objects that a programmer creates would not appear in the final scene, do they still participate in shading? Explain your answer with proper justification.
3. a) [10 pts] Give and explain each term of both the diffuse and specular terms of the illumination equation for a single light source. ii) What does the ambient term in the illumination equation model? iii) What effect does ‘shinyness’ term have in the specular term.
	* + 1. [4 pts] Modify the above illumination equation to include multiple light sources and light source distance attenuation.
			2. [4 pts] Explain how shading of transparent objects is handled.