CISC640/440: Examples of Midpoint Algorithms

1 An example of Bresenham line

Draw a line with two endpoints (2,4) and (8,9). The line has a slope of $\frac{5}{6} < 1$, with

$$\Delta x = 6, \qquad \Delta y = 5.$$

The initial decision parameter has the value:

$$p_0 = 2\Delta y - \Delta x = 4,$$

and the increments for calculating successive decision paramters are

$$2\Delta y = 10, \qquad 2\Delta y - 2\Delta x = -2.$$

Successive pixels of the Bresenham line are listed in the following table:

k	p_k	(x_{k+1}, y_{k+1})
0	4	(3,5)
1	2	(4,6)
2	0	(5,7)
3	-2	(6,7)
4	8	(7,8)
5	6	(8,9)

Table 1: Successive pixels in a Bresenham line.

2 An example of Midpoint circle

Draw a circle with radius $r = 8$, centering at (0,0). Initial point is

$$(x_0, y_0) = (0, 8)$$

Initial decision parameter is

$$p_0 = 1 - r = 7.$$

Successive pixels of the Midpoint circel are listed in the following table:

k	p_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0	-7	(1,8)	2	16
1	-4	(2,8)	4	16
2	1	(3,7)	6	14
3	-6	(4,7)	8	14
4	3	(5,6)	10	12
5	2	(6,5)	12	10

Table 2: Successive pixels in a Midpoint circle.

3 An example of Midpoint ellipse

Draw an ellipse with r_x =10 and r_y = 8, centering at (0,0).

For region 1, the initial point for the ellipse centered on the origin is $(x_0, y_0) = (0, 8)$, and the initial decision parameter value is

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 = -711.$$

Successive midpoint decision-parameter values and the pixel positions along the ellipse are listed in the following table:

k	$p1_k$	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-711	(1,8)	128	1600
1	-519	(2,8)	256	1600
2	-199	(3,8)	384	1600
3	249	(4,7)	512	1400
4	-575	(5,7)	640	1400
5	129	(6,6)	768	1200
6	-239	(7,6)	896	1200
7	721	(8,5)	1024	1000

Table 3: Successive pixels in the region 1 of a Midpoint ellipse.

We now move out of region 1, since $2r_y^2 x > 2r_x^2 y$.

For region 2, the initial point is $(x_0, y_0) = (8, 5)$ and the initial decision parameter is

$$p2_0 = f_{\text{ellipse}}(8 + \frac{1}{2}, 4) = -704.$$

The remaining positions along the ellipse path in the first quadrant are then calculated as:

k	$p2_k$	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-704	(9,4)	1152	800
1	-252	(10,3)	1280	600
2	528	(10,2)	1280	400
3	228	(10,1)	1280	200
4	128	(10,0)	-	-

Table 4: Successive pixels in the region 2 of a Midpoint ellipse.