- Ellipse is an elongated circle.
- Sum of distances from any point on the ellipse to two fixed points (called foci points) is constant.
- $$\sqrt{(x - x_1)^2 + (y - y_1)^2} + \sqrt{(x - x_2)^2 + (y - y_2)^2} = const.$$
- By squaring above equation twice and manipulating, we get equation of the forms

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$$

- Simpler form obtained when the major and minor axis are oriented parallel to the x and y axes.

$$\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 = 1$$

- We can also define polar form as below

$$x = x_c + r_x cos\theta \quad y = y_c + r_y sin\theta$$

- To Draw the Ellipse: We can determine (x,y) for ellipse centered at (0,0) and then shift all points by (x_c, y_c). And rotate the ellipse if it has to be in non-standard position. Also use the symmetry of each quadrant (not octant as in circle).

- The midpoint ellipse method is applied in two parts of the first quadrant. We process the first quadrant by taking unit steps in the x direction till absolute value of slope is less than 1.0. Afterwards we switch the roles of x and y.
- i.e, we take steps in the y direction where the slope has magnitude greater then 1.0.

- Define,

$$f_{ell}(x, y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$$

f_{ell} is less than 0 if inside, equal to 0 if on the ellipse, and greater than 0 if outside.

- Starting at $(0, r_y)$, take steps in the x direction till we touch region2. Ellipse slope:

$$\frac{dy}{dx} = -\frac{2r_y^2 x}{2r_x^2 y}$$

At the boundary, slope is -1, so, $2r_y^2 x = 2r_x^2 y$. Hence, we move into region2 when $2r_y^2 x$ is greater than or equal to $2r_x^2 y$.

- At the midpoint of candidate pixels at sampling position $x_k + 1$ in the region1:

  $p1_k = f_{ell}(x_k{+}1, y_k{-}1/2) = r_y^2(x_k{+}1)^2{+}r_x^2(y_k{-}1/2)^2{-}r_x^2 r_y^2$

  if $p1_k$ is less than 0, choose pixel on scanline y_k, else choose $y_k - 1$.

- At the next sampling position $(x_{k+1}{+}1 = x_k{+}2)$, decision parameter is:

$$p1_{k+1} = f_{ell}(x_{k+1} + 1, y_{k+1} - 1/2)$$
$$= r_y^2[(x_k + 1) + 1]^2 + r_x^2(y_{k+1} - 1/2)^2 - r_x^2 r_y^2$$

$= p1_k + 2r_y^2(x_k + 1) + r_y^2 + r_x^2[(y_{k+1} - 1/2)^2 - (y_k - 1/2)^2]$

where y_{k+1} is either y_k or $y_k - 1$ depending on sign of decision parameter. So,
increment of decision parameter $= 2r_y^2 x_{k+1} + r_y^2$ if $p1_k < 0$, else

- Initial value for $p1_0$ can be obtained by evaluating the ellipse function at $(0, r_y)$ as $r_y^2 - r_x^2 r_y + 1/4 r_x^2$.

- Region 2: sample at unit intervals in the negative y-direction. Decision parameter is:

$$p2_k = f_{ell}(x_k + 1/2, y_k - 1)$$
$$= r_y^2(x_k + 1/2)^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2$$

  If $p2_k > 0$, midpoint is outside the ellipse boundary, select pixel at x_k. If $p2_k <= 0$, the midpoint is inside or on the ellipse boundary, and we select x_{k+1}.

- Evaluate ellipse function at $y_{k+1} - 1 = y_k - 2$, we get

$$p2_{k+1} = f_{ell}(x_{k+1} + 1/2, y_{k+1} - 1)$$
$$= r_y^2[(x_k + 1) + 1/2]^2 + r_x^2[(y_k - 1) - 1]^2 - r_x^2 r_y^2$$

$$= p2_k - 2r_x^2(y_k - 1) + r_x^2 + r_y^2[(x_{k+1} + 1/2)^2 - (x_k + 1/2)^2]$$

with x_{k+1} set either to x_k or to x_{k+1}, depending on the sign of $p2_k$.
Initial position of region2 is taken as last position of region1. Initial decision parameter of region2 is:

$$p2_0 = f_{ell}(x_0 + 1/2, y_0 - 1) = r_x^2(x_0 + 1/2)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$$