- World-coordinate system represents a coordinate system in given units, that represents a given application program of the world.

  Graphics packages must be told how to map world coordinates onto screen coordinates.

  Mapping can be done by,

  i) application programmer provides the graphics package with a transformation matrix,

  ii) application programmer specifies a rectangular region in world coordinates called the world-coordinate window and a corresponding rectangular region in screen coordinates called viewport. The transformation that maps the window into the viewport is applied to all of the output primitives in world coordinates.

- The modifier "world-coordinate" is used with window to emphasize that we are not talking about the window-manager window.

2D viewing:

i. Specify a window on the 2D world
ii. Viewport on the 2D view surface
iii. Objects in the world are clipped against the window
iv. Transformation of the remaining objects from window to viewport.

3D viewing

Slightly more complex due to the extra dimension, since the display devices are only 2D.

- *Projections* will transform 3D objects onto 2D projection plane. Thus the sequence of operations for 3D viewing is,

  i. *view volume* is specified in the world. Objects in 3D are clipped against this view volume.
  ii. Contents of the view volume are projected onto a given projection plane thus forming the *window*.
  iii. Window is then transformed into the viewport for display.

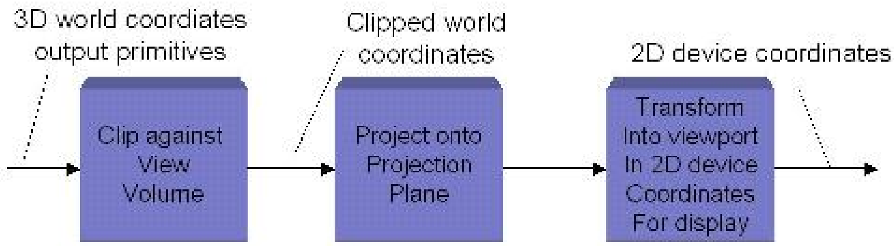

Figure 1: Conceptual model of the 3D Viewing Process

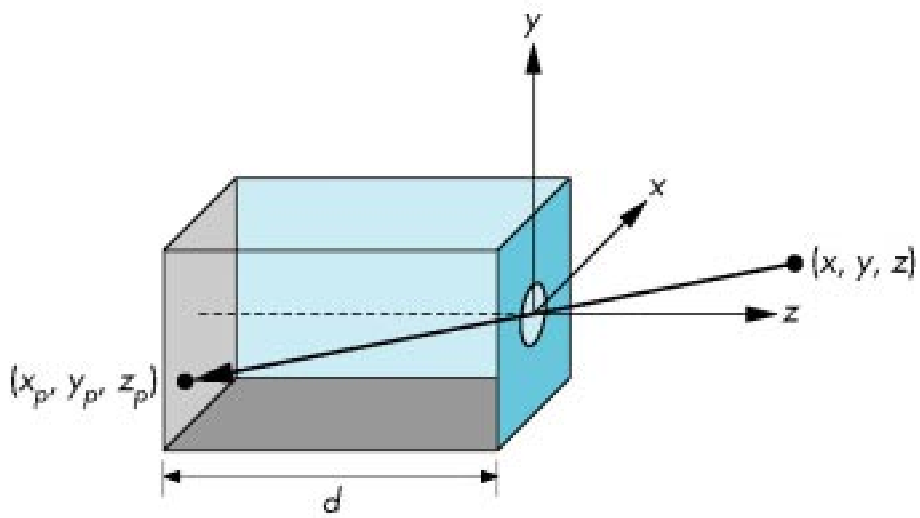

Figure 2: Pihole Camera

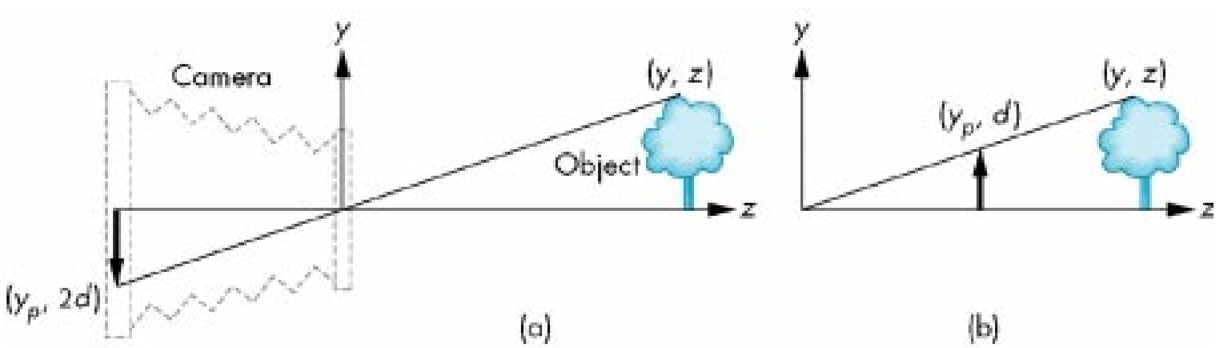

Figure 3: a: image formed on the back of camera, b) image plane moved to front of camera

Projections

- Projections in-general, transform points in a coordinate system of dimension n into points in a coordinate system of dimension less than n. We concentrate on 3D to 2D.
- The projection of a 3D object is defined by straight projection rays (*projectors*) emanating from a *center of projection*, passing through each point of the object, and intersecting a *projection plane* to form the projection.
- Planar Geometric Projection: i) Projectors are straight lines, ii) Projection surface is a plane (picture plane, projection plane, view plane).

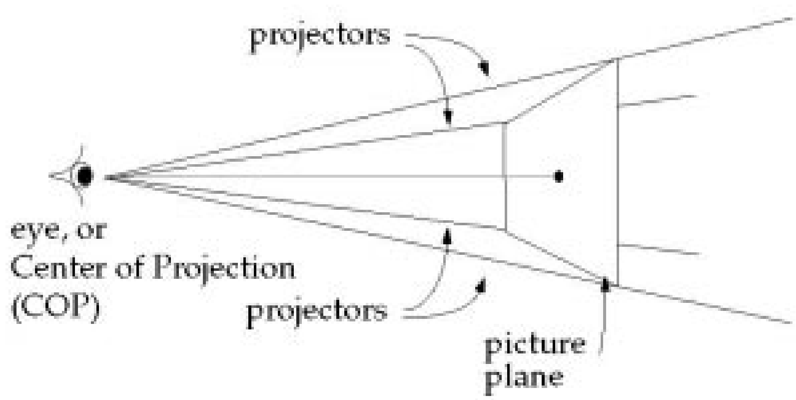

Figure 4: A perspective projection (Drawing itself is in perspective projection).

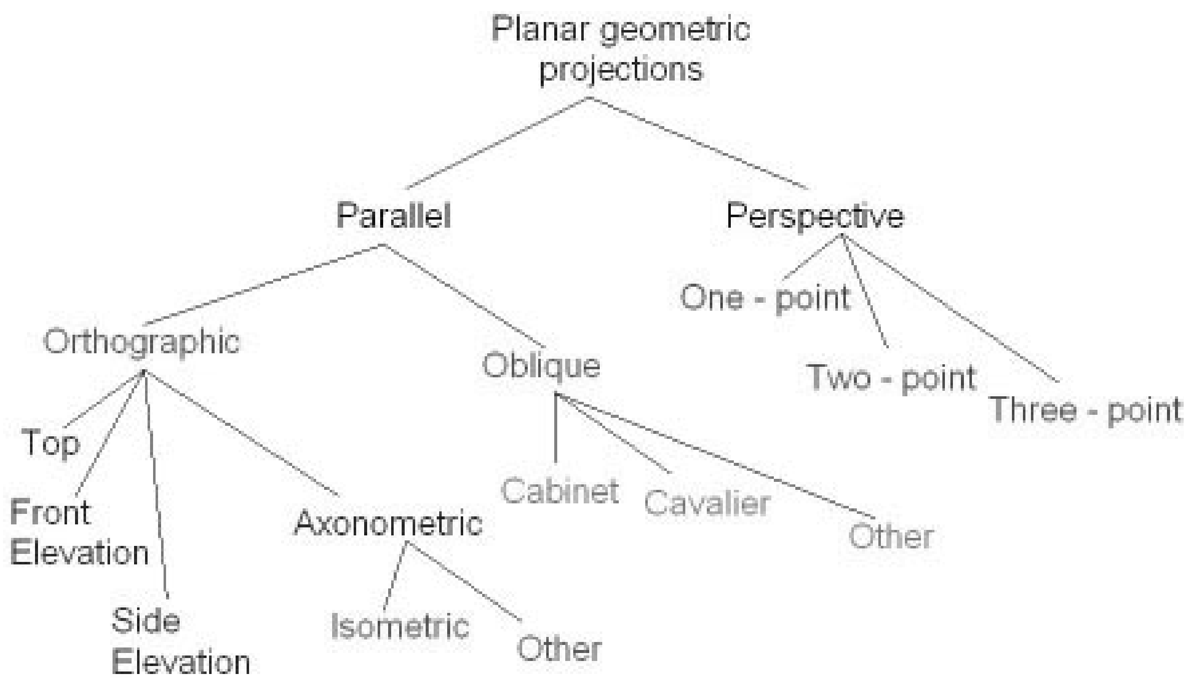

Figure 5: Types of Projections

Main Classes of Planar Geometric Projections:

Perspective: If the distance between COP and projection plane is finite.

Parallel: If this distance is infinite.

- Perspective projection can be specified by COP (Center of Projection); Parallel projection is specified by DOP (Direction of Projection).

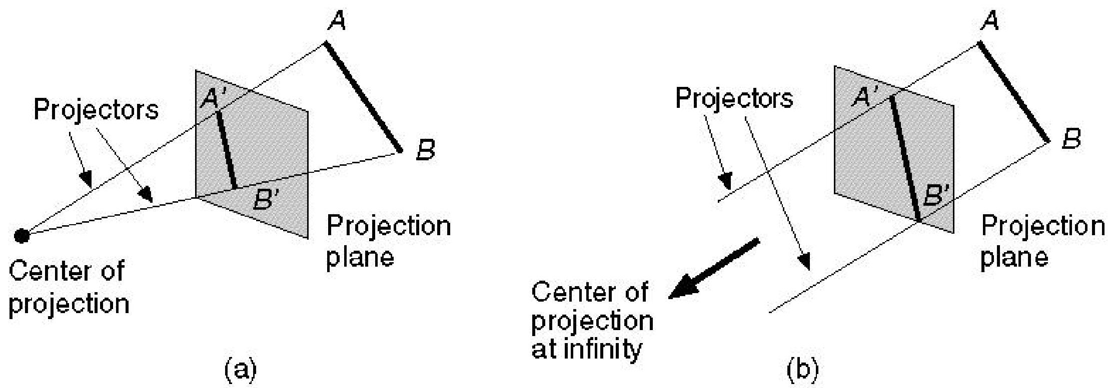

Figure 6:

- Parallel projections are categorized into two classes, depending on the relation between the direction of projection and the normal to the projection plane:

  i. Orthographic parallel projections: directions are the same (or the reverse of each other).

  ii. Oblique parallel projections: directions are not the same.

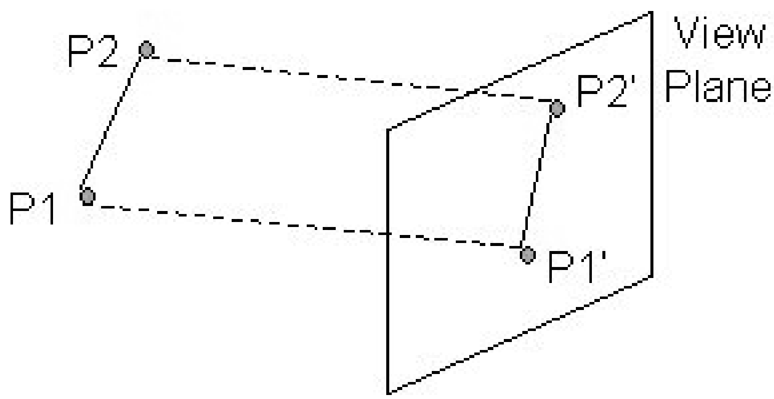

Figure 7: Parallel projection of an object to the view plane.

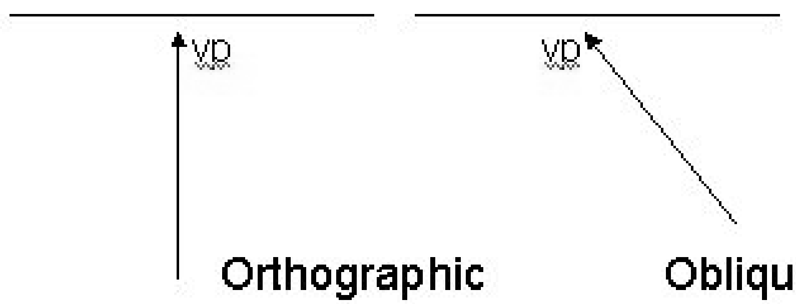

Figure 8: Orientation of the projection vector V_p to produce an orthographic projection (a) and an oblique projection (b).

- Most common types of orthographic projections: front-elevation, top-elevation and side-elevation. In all of these, the projection plane is perpendicular to the principal axis.

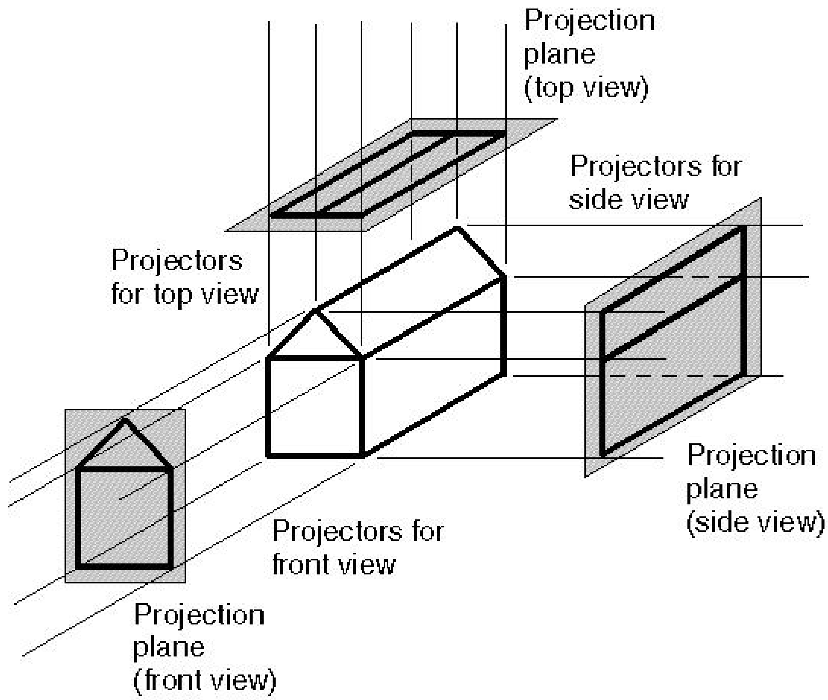

Figure 9:

- Used for:
  - engineering drawings of machines, machine parts
  - Working architectural drawings
- Pros:
  - accurate measurement possible
  - All the views are at same scale
- Cons:
  - Does not provide "realistic" view or sense of 3D form
  - Usually need multiple views to get a 3D feeling for object

- Axonometric orthographic projections use projection planes that are not normal to a principal axis, thus showing several faces of an object at once.
- Foreshortening is uniform (but not proportional like in perspective, where foreshortening is proportional to z.)
- Parallel lines remain parallel but angles are not preserved.
- Distances can be measured along each principal axis (with different scale factors).
- Isometric projection is a commonly used axonometric projection. Here, the projection plane normal (direction of projection) makes equal angles with each principal axis.
- Isometric projection can be generated by aligning the projection plane so that it intersects each coordinate axis in which the object is defined (principal axes) at the same distance from the origin.
- There are eight possible isometric projection directions.

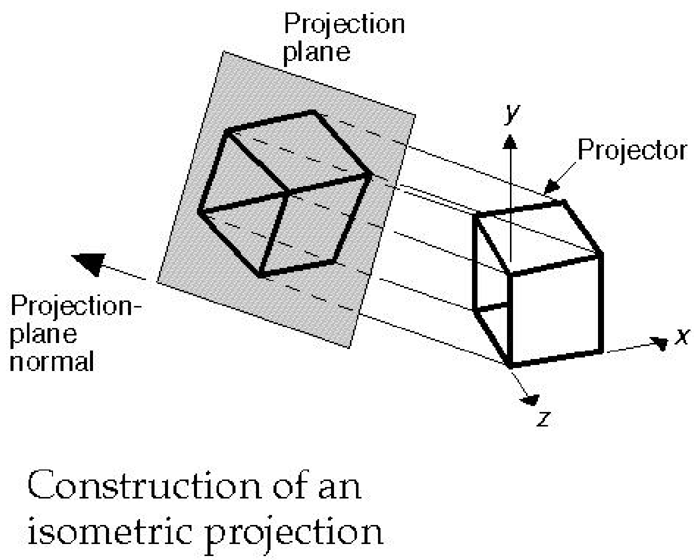

Figure 10: Construction of an isometric projection

- Used for:
  - Catalogue illustrations
  - Patent office records
  - Furniture design
  - Structural design
- Pros:
  - Don't need multiple views
  - Illustrates 3D nature of object
  - Measurements can be made to scale along principal axes
- Cons:
  - Lack of foreshortening creates a distorted appearance.
  - More useful for rectangular than curved shapes.

- Oblique Projections: The projection-plane normal and the direction of the projection differ.
- Projectors are at an oblique angle to the projection plane.
- It combines the properties of front, top, and side orthographic projections with those of axonometric projection.
- Projection of the face of the object parallel to projection plane allows measurement of distances and angles. In case of the rest of the faces, distances along the principal axes can be measured but not angles.

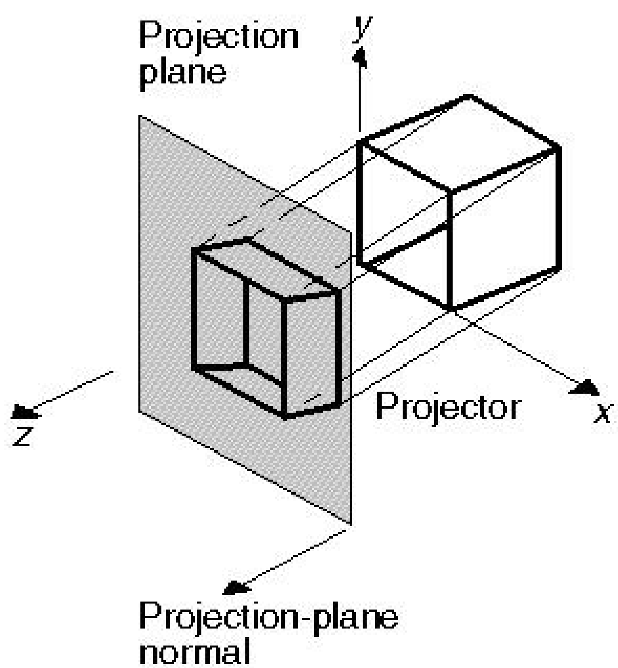

Figure 11:

- Pros:

  - Can represent the exact shape of one face of an object (can take accurate mea surements): better for elliptical shapes than axonometric projections, better for "mechanical" viewing.

  - lack of perspective foreshortening makes comparison of sizes easier

  - displays some of the object's 3D appearance

- Cons:

  - objects can look distorted if careful choice not made about position of PP (ex., circles become ellipses)

  - lack of foreshortening (not realistic looking)

Example: Oblique View

- Rules for placing projection plane for oblique views: Projection plane should be chosen according to one or several of the followings

  - It is parallel to the most irregular of the principal faces, or to the one which contains circular or curved surfaces

  - It is parallel to the longest principal face of the object

  - It is parallel to the face of interest

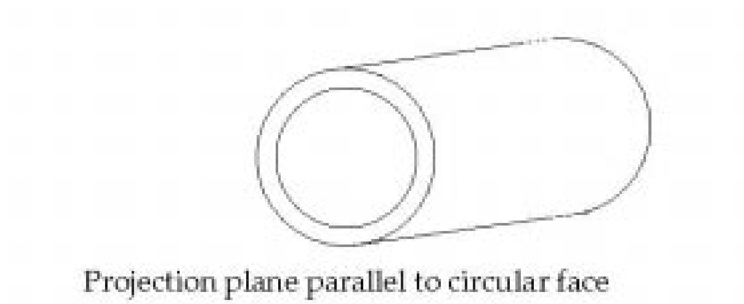

Figure 12: Projection plane parallel to circular face

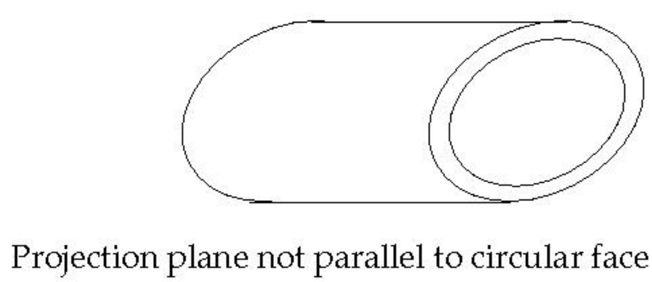

Figure 13: Projection plane not parallel to circular face

- Main Types of Oblique Projections:

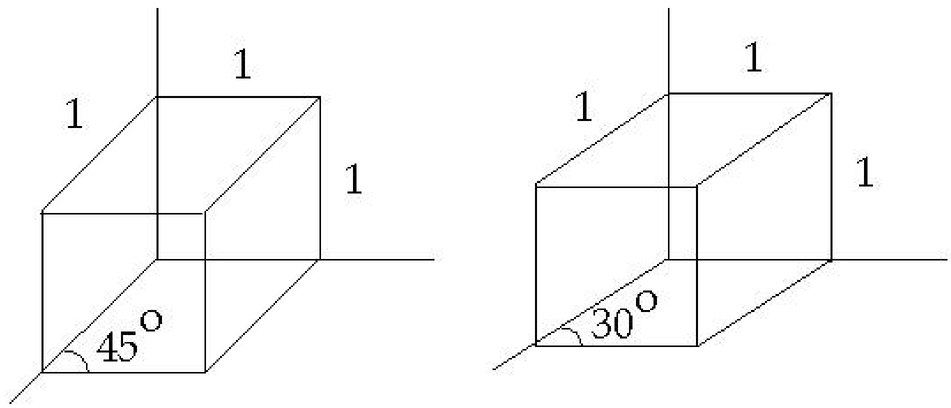

Figure 14: cavalier projection of unit cube.

- *Cavalier* Angle between projectors and projection plane is 45 degrees. Perpendicular faces are projected at full scale.

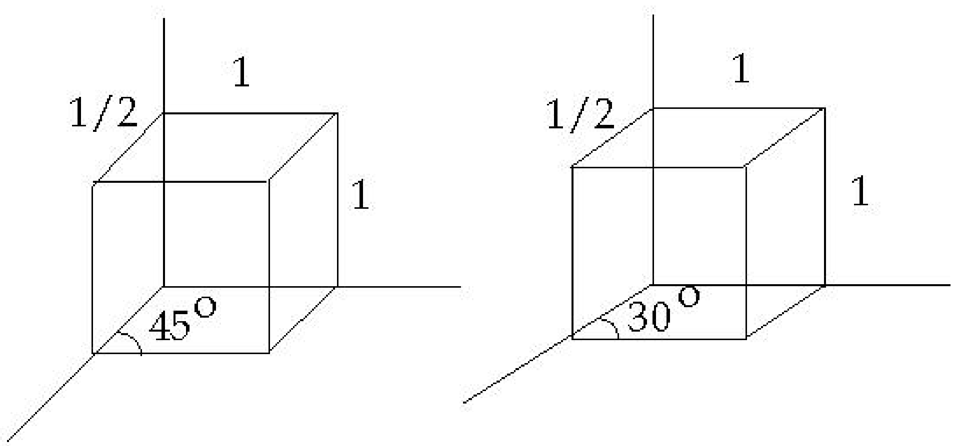

Figure 15: cabinet projection of unit cube

- *Cabinet* Angle between projectors and projection plane is arctan(2) = 63.4 degrees. Perpendicular faces are projected at 50% scale.

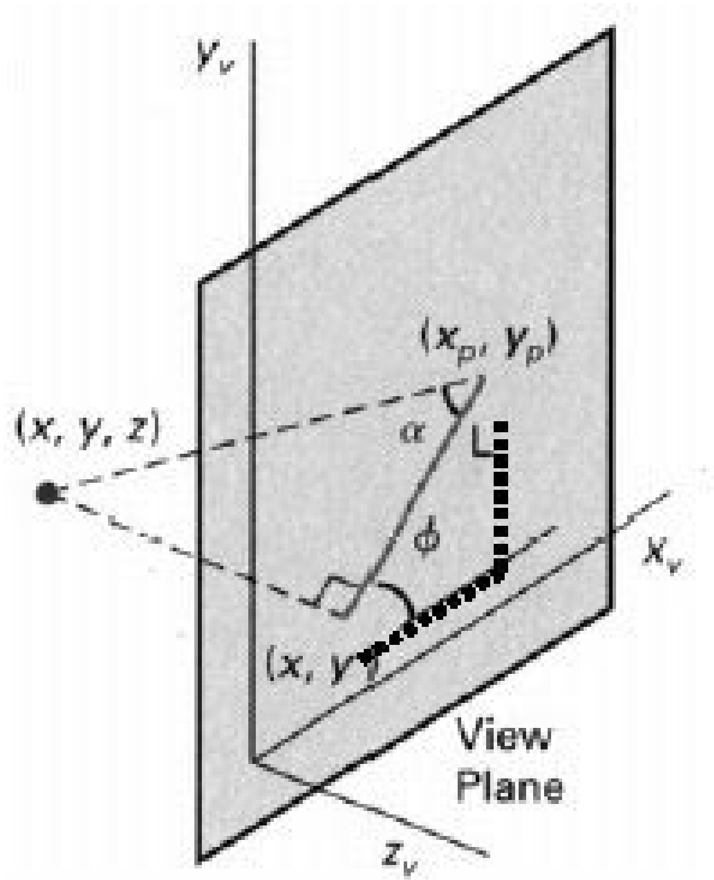

Figure 16: Oblique projection of coordinate position (x, y, z) to (x_p, y_p)

$$x_p = x + Lcos\phi$$
$$y_p = y + Lsin\phi$$

$$L = \frac{z}{tan\alpha} = zL_1$$

where L_1 is the inverse of $tan\alpha$ which is also the value of L when $z = 1$.

$$x_p = x + Lcos\phi$$
$$y_p = y + Lsin\phi$$

$$L = \frac{z}{tan\alpha} = zL_1$$

$$x_p = x + z(L_1 cos\phi)$$
$$y_p = y + z(L_1 sin\phi)$$

$$M_{parallel} = \begin{bmatrix} 1 & 0 & L_1 \cos\phi & 0 \\ 0 & 1 & L_1 \sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Commonly used: i) $tan\alpha = 1$ (cavalier), and ii) $tan\alpha = 2$ (cabinet).

Summary of parallel projections

- Assume object face of interest lies in principal plane, i.e., parallel to xy, yz or zx planes. (DOP = Direction of Projection, VPN = View Plane Normal)

1. Multiview Orthographic: i) VPN is parallel to a principal coordinate axis, ii) DOP is parallel to VPN, iii) shows single face, exact measurements
2. Axonometric: i) VPN is not parallel to a principal coordinate axis, ii) DOP is parallel to VPN, iii) adjacent faces, none exact, uniformly foreshortened (as a function of angle between face normal and DOP)
3. Oblique: i) VPN is parallel to a principal coordinate axis, ii) DOP is not parallel to VPN, iii) adjacent faces, one exact, others uniformly foreshortened

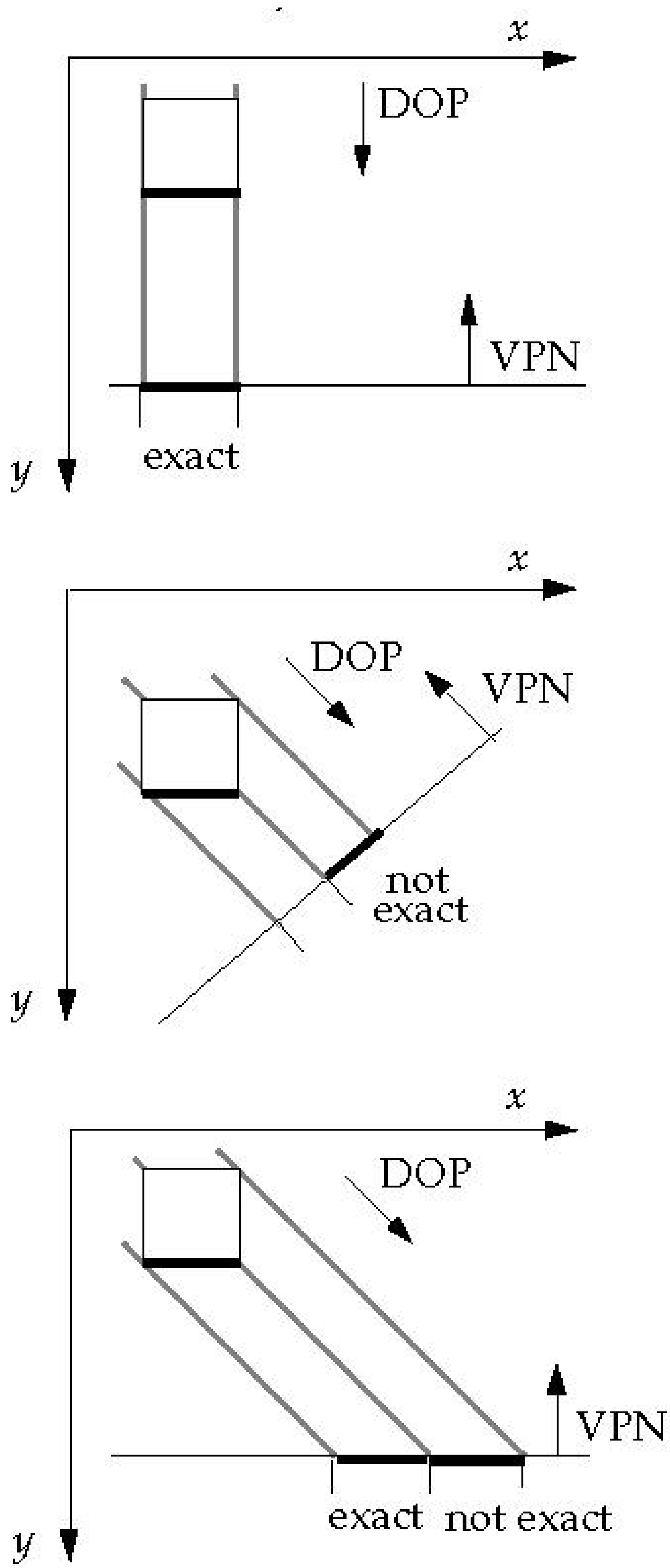
x
DOP
VPN
exact
y
x
DOP
VPN
not
exact
y
x
DOP
VPN
exact
not exact
y

Figure 17:

- Perspective Projections: Characterized by *diminution* of size. When objects are moved farther from the viewer, their images become smaller, giving a natural appearance.
- However, because the amount by which a line is foreshortened depends on how far the line is from the viewer, one cannot make measurements from a perspective view.
- The perspective projections of any set of parallel lines that are not parallel to the projection plane converge to a *vanishing point*.
- Parallel lines in 3D meet at infinity; vanishing point can be thought of as the projection of a point at infinity.
- If the set of lines is parallel to one of the three principal axes, its called axis vanishing point.
- Perspective projections are categorized by their number of principal vanishing points (utmost 3) and therefore by the number of axes the projection plane cuts.

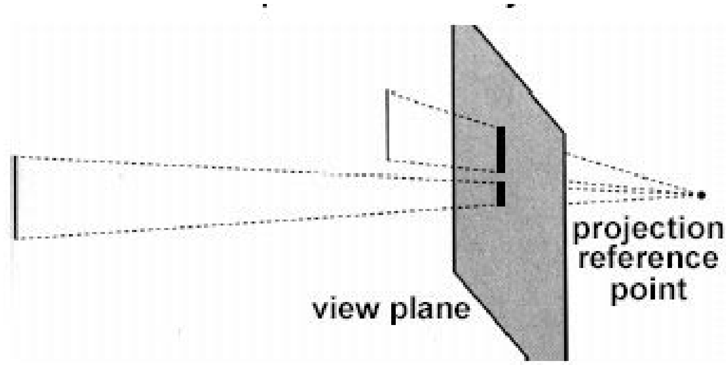

Figure 18:

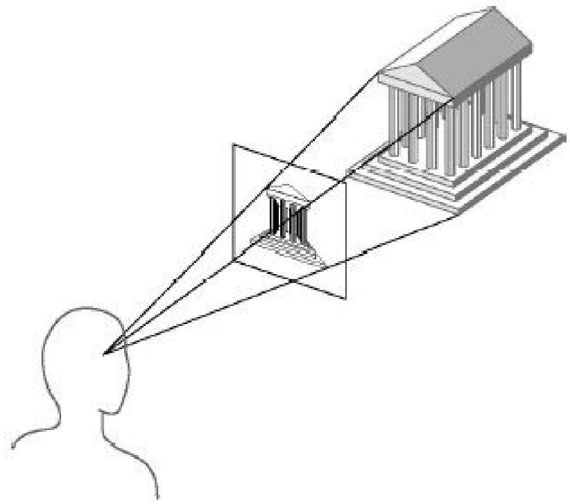

Figure 19:

Figure 20:

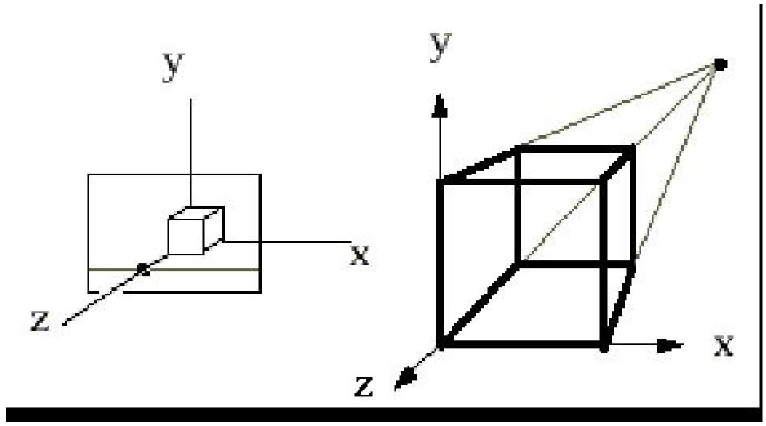

Figure 21: one-point projection: z-axis vanishing point

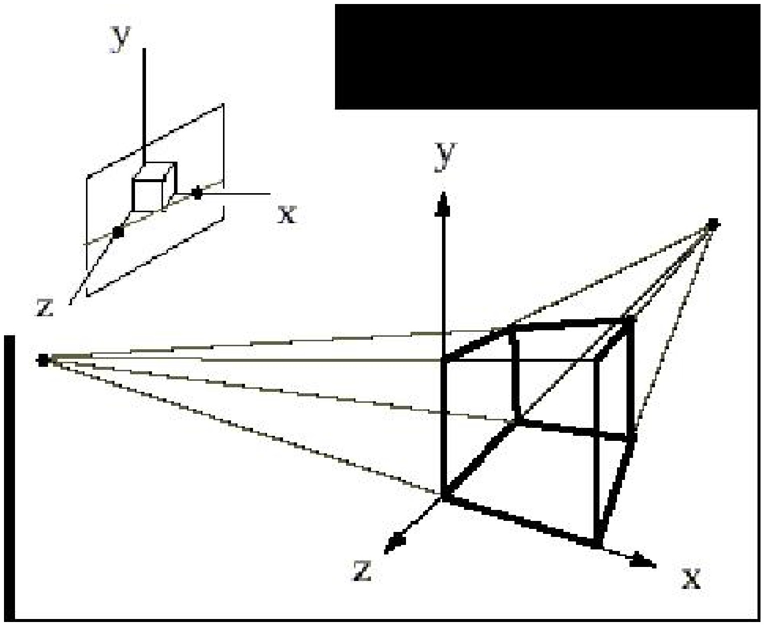

Figure 22: two-point projection

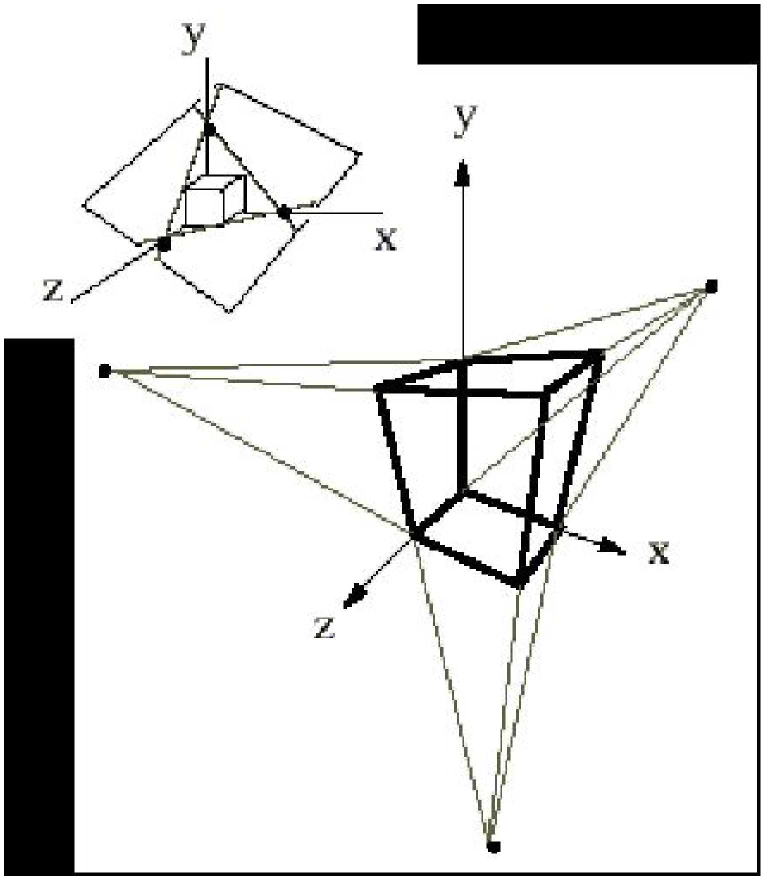

Figure 23: three-point projection

- Perspective image is intersection of a plane with light rays from object to eye (COP)
- Perspective image is result of foreshortening due to convergence of some parallel lines toward vanishing point

  Combining these two views:

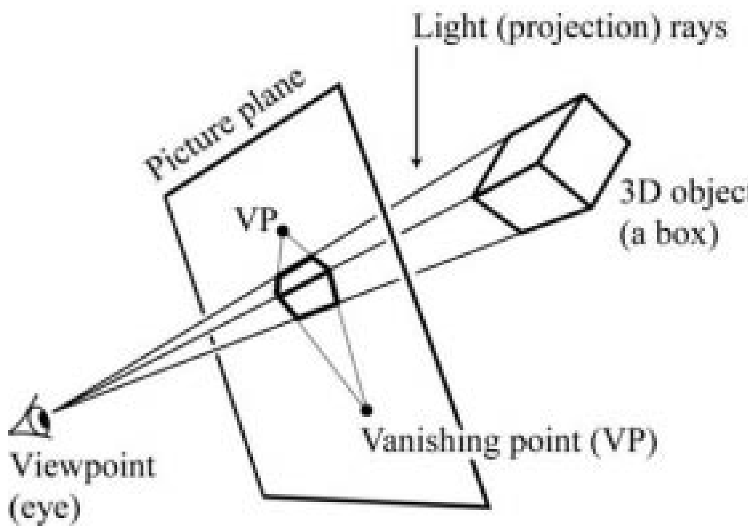

Figure 24:

Perspective Projections

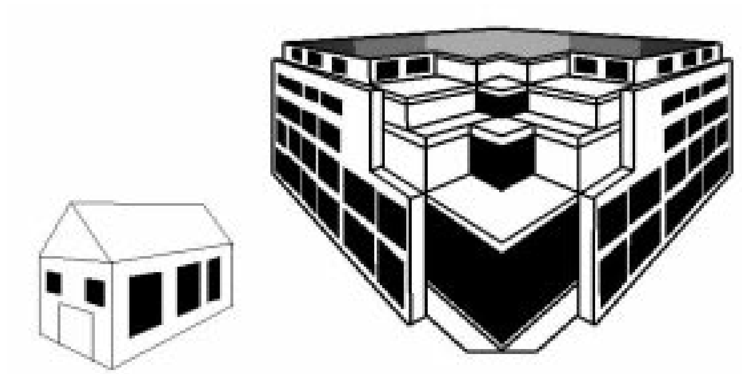

Figure 25:

- Used for:

  - advertising

  - presentation drawings for architecture, industrial design, engineering

  - fine art

- Pros

  - gives a realistic view and feeling for 3D

- Cons

  - does not preserve shape of object or scale (except where object intersects PP)

- Different from parallel projection because: i) parallel lines not parallel to the PP converge, ii) size of the object diminished with distance, iii) foreshortening is not uniform.