Submitted to IEEE/ACM Transactions on Networking

# Statistical Detection of Congestion in Routers

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***Abstract*—Detection of congestion plays a key role in numerous networking protocols, including those driving Active Queue Management (AQM) methods used in congestion control in Internet routers. This paper exploits the rich theory of statistical detection theory to develop simple detection mechanisms that can further enhance current AQM methods. The detection of congestion is performed using a Maximum Likelihood Ratio Test (MLRT) that is an asymptotically powerful unbiased test. The MLRT indicates that the likelihood of congestion grows super exponentially with the queue occupancy level. Performance evaluation of the likelihood detector shows it is robust to variations of the network parameters. The mathematical expression of the likelihood of congestion depends only on the current dropping rate, a desired queue occupancy level and the current queue occupancy. When incorporated into REM and PI, the MLRT-based detection improved the reaction time by at least $30\%$.**

***Index Terms*—Computer Networks, Computer Network Performance, TCP/IP, Congestion Control, Active Queue Management (AQM), QoS, Congestion Detection.**

## I. Introduction

RECENT evidence suggests, there is a need for Active Queue Management (AQM) in current networks [1]–[6]. The advances in AQM presented by Ryu $et.\ al.$ [7] conclude that currently proposed schemes do not always achieve the goal of pro-actively detecting incipient congestion. Rather, the main focus has been on the reaction to already present congestion. AQM requires more effective mechanisms to anticipate congestion, while taking into account the effects of the network delay.

To understand the need to detect congestion, consider AQM as a standard control problem [8] with a network flow as the controlled plant depicted in Fig. 1. As input, the network takes a packet drop or marking indication probability, $\delta$. After random delays, marks are received by the TCP sender and translated into a congestion window value, $W$, which impacts the queue occupancy. To make the system amendable to analysis, the actual TCP system is replaced with a TCP model that has an output $\overline{W}$, an approximated average window size. This average window size yields to an average queue occupancy, $\overline{q}$. The actual queue occupancy is modeled as $\overline{q} + N$, where $N$ is treated as noise expressed by $N = q - \overline{q}$.

The diagram in Fig. 1 is similar those described in [3], [4], [9], [10], where a linearized TCP model was proposed in order to describe the network box. However, in these efforts it is assumed that $q = \overline{q}$, the disturbance, $N$, is neglected, and thus there is no estimator. To understand the limitations of this assumption, linear control theory [11] can be applied, where the controller can be modeled as a linear system with Laplace transform $K(s)$, the network with $H(s)$, and the estimator is neglected, then

$$\bar{Q}(S) = \frac{K(s)H(s)}{1+K(s)H(s)}Q_d(s) + \frac{1}{1+K(s)H(s)}M(s) + \frac{K(s)H(s)}{1+K(s)H(s)}N(s).$$

It is possible to see that the impact of the linearization error, $M(s)$, is reduced by making the controller gain as large as possible, while maintaining stability. Conversely, the noise of approximating the mean queue occupancy with the actual queue occupancy, $N(s)$, is not impacted by large gain. From standard control theory it is known that disturbances in the output can only be reduced by using an estimator. This paper presents such an estimator.

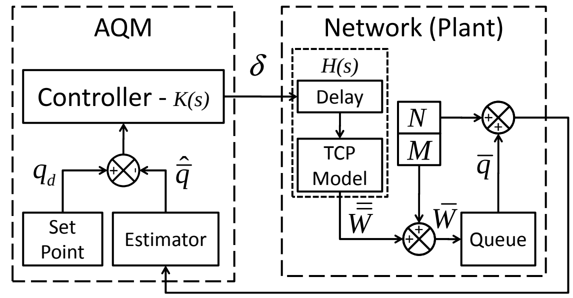


Fig. 1. Feedback loop analogy for a TCP network. The network (plant) introduces disturbances ($M$,$N$), modifying the linearized models ($\overline{\overline{W}}$) to produce a measurable variable $q$. AQM attempts to estimate the ergodic mean $\overline{q}$ and produce a dropping probability $\delta$ to control the level of congestion.

The paper is organized as follows: Section II provides some background on AQM methods and corresponding congestion detection schemes. Section III defines congestion from queue management perspective. Section IV characterizes the queue distribution based on traffic models and empirical results from several simulations. Section V details expressions for the likelihood of congestion based on different queue distributions as well as performance analysis of the detectors. Sections V and VI present simulations in support of the analytical results from Section V. Finally, in Section VII, discussion and conclusions are presented. In the next sections, this paper refers to packet dropping probabilities as packet marking probabilities since they are both congestion notification techniques, using the term drop when packet losses occur.

Manuscript received June XX, 2006. Prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon. This work was also supported in part by the National Science Foundation under the award ITR ANI-0312851.

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## II. Detection of Congestion in Previous AQM

The initial AQM work of Floyd $et.\ al.$ [2] recognized the need for some kind of congestion detection technique besides direct observations of the queue occupancy. Specifically, Random Early Detection (RED) measures congestion by smoothing the queue occupancy and relates this smoothed queue occupancy to marking probability via a piece-wise function to produce a marking probability. The smoother is an Exponentially Weighted Moving Average (EWMA) with equation

$$\bar{q}(k+1) = (1-w)\bar{q} + wq(k), \tag{1}$$

where $\bar{q}$ is the smoothed version of the queue occupancy $q$ and $w$ is the smoothing factor. The parameter $w$ controls a trade-off between smoothness and reactiveness. The design of $w$ is difficult as instabilities or even chaotic responses are observed if $w$ is not carefully chosen. Ranjan $et.\ al.$ modeled TCP-RED and presented simulations supporting the instabilities and erratic behavior of RED [12], while May $et.\ al$ recommended to avoid the deployment of RED until a better understanding of AQM was acquired [13]. Several variants of RED have since been proposed [14]–[16] , but the tuning of the algorithm is still complex.

Random Early Marking (REM) [4], a scheme based on optimization theory, proposed to control the arrival rate. In this case a simple smoother is used to estimate the arrival rate. Since the arrival rate only provides information on the variations in the queue, a term with the instantaneous value of the queue was introduced, aiming to maintain the queue occupancy level to a fixed value. REM detects congestion by using a weighted observation of the queue and an estimated arrival rate. Accurate estimates of arrival rates require a careful design of the smoother.

Using classical control theory, Hollot $et.\ al.$ proposed a proportional integral controller [3]. They used a linearized TCP model to describe the 'plant' and designed a controller based on that model. The congestion measure is performed using a second order filter, which periodically updates an estimated marking probability based on the current and previous values of queue occupancy. The update interval as well as the model parameters need to be carefully chosen, and to achieve optimal performance depend on network conditions.

Kunniyur and Srikant proposed AQM based on an Adaptive Virtual Queue (AVQ) [5]. This approach uses the arrival rate as a measure of congestion attempting to keep the value smaller than the total capacity. AVQ detects congestion in the same reactive way drop tail does, once the virtual queue overflows it assumes the link is congested and marks the arriving packets.

Although, all these schemes have achieved good performance improvements in steady state, Le $et.\ al.$ [17] noted no significant differences in reaction speed between well known AQM schemes and the traditional drop tail. Results of these studies suggest that better understanding of the queue statistics is necessary for designing AQM algorithms. The statistical analysis presented in this paper provides better understanding of the queuing process and contributes with mechanisms for congestion detection.

## III. Congestion from Queue Management Point of View

There are many observables that could be used for detecting congestion. For example, congestion can be defined as when the average packet arrival rate exceeds the outgoing link speed, therefore it may be useful to use the packet arrival rate to detect congestion. However, statistical detection research indicates that the queue occupancy, not the arrival rate, is best suited to derive the level of congestion [18].

To illustrate this concept, consider the goal of detecting the event of the average arrival rate exceeding its nominal mean. In this case, it is well-known that sequential change-point detection will provide the fastest detection for a given false alarm rate [19]. Under the assumption that the number of arrivals during a sample interval is Gaussian (a reasonable assumption if there are many flows), optimal change-point detection declares that the mean arrival rate has exceeded its nominal rate when the cumulative sum $Q(k)$ exceeds a threshold, where $Q(k)$ is defined as

$$Q(k+1) = \max\left(0, Q(k) + A(k) - C\right), \tag{2}$$

where $A(k)$ is the number of arrivals in the $k$-th sample interval and $C$ is the nominal departure rate. Since $Q(k)$ is simply the queue occupancy, it is optimal to detect congestion. However, AQM does not require a binary indication of congestion but rather a continuous variable to indicate the level of congestion. A real-valued measure of congestion can be obtained by using hypothesis testing. For the purpose of this paper, the hypothesis of a link being congested is said to be true if the ergodic mean queue occupancy, $\overline{q}$, is greater than a desired queue occupancy, $q_d$. Similarly, if $\overline{q} \leq q_d$, the link is said to be underutilized. However, it is not possible to directly measure the ergodic average of the queue occupancy, $\overline{q}$. Therefore, the observed value $q_o$ is used to attempt the detection of $\overline{q}$, turning the problem into speed and accuracy of the estimated $\overline{q}$.

Observables that might impact a determination of the level of congestion include: the number of flows, the round trip delay for each flow, and the past marking probability applied to each flow. While round trip delays and number of flows are difficult to measure, it is possible to take the marking probability applied by a router to be the marking probability experienced by the flows. Thus the smoothed marking probability is taken as an observable.

## IV. Statistical Characteristics of TCP flows

Several probabilistic models have been studied for the statistical characterization of queue occupancy in network routers. Appenzeller $et.\ al.$ [20] used a Gaussian approximation of the sum of TCP congestion windows based on the equation

$$Q_i\left(k\right) = W_i\left(k\right) - \overline{RTT_i} \times C_i - \epsilon_i, \tag{3}$$

where $Q_i$ is the number of packets of the $i^th$ flow in the queue at time $k$, $W_i$ is the congestion window, $\overline{RTT_i} \times C_i$ is the number of packets currently in the links and $\epsilon_i$ is the number of packets dropped. Summing (3) over all the flows and using the relationship $\overline{RTT_i}C_i = c/\sqrt{\delta}$, where $c$

is a proportionality constant that relates the throughput and delay to the marking probability, $\delta$ [21], the router's queue occupancy can be expressed as

$$Q\left(t\right)=\sum_{i}Q_i\left(t\right)=\sum_{i}W_i\left(t\right)-c\sum_{i}\frac{1}{\sqrt{\delta_i}}-\sum_{i}\epsilon_i. \quad (4)$$

Note from (3) and (4) that the distribution of the router's queue occupancy, $Q=\sum Q_i$, depends not only on the distributions of the congestion windows, but also the round trip delays, $\overline{RTT}_i$, per flow throughput, $C_i$ and even the method used to drop packets, $\epsilon_i$. It is possible to assume that each flow gets a similar marking probability $\delta_i$ [22]. Thus, from (4), the queue occupancy distribution is, by approximation, the distribution of the sum of the congestion windows. Using a Gaussian approximation of the queue occupancy distribution may at first seem reasonably simpler [20]. However, the non-negative values of the observed queue and the goal of keeping the queue occupancy low indicate that a skewed distribution provides a better representation.

Bohacek and Shah modeled the distribution of the congestion window of a TCP flow $i$ as a negative binomial random variable with parameters $N_i$ and $r_i$ [23]. The distribution described by

$$P\{W_i=w\}=\frac{\Gamma\left(N_i+w-1\right)}{\Gamma\left(N_i\right)\Gamma\left(w\right)}\left(1-r_i\right)^{N_i}{r_i}^{w-1}, \quad (5)$$

with mean value $N_i r_i/(1-r_i)+1$ and variance $N_i r_i/(1-r_i)^2$, for $w>0$. Their simple approximation relates the parameters of the negative binomial distribution with the packet marking probability $\delta_i$ of the particular flow,

$$r_i=1-\frac{\delta_i}{\gamma}\left(\frac{c_1}{\sqrt{\delta_i}}+1\right), \quad (6)$$

$$N_i=\frac{1-r_i}{r_i}\left(\frac{c_1}{\sqrt{\delta_i}}-1\right), \quad (7)$$

where $c_1$ is the TCP constant $\sqrt{3/2}$, and $\gamma=0.31$. With identical marking probabilities, $\delta_i$, the aggregate throughput is also negative binomial distributed with parameters $r=r_i=r_j$ and $N=\sum N_i$.

On the other hand, Bhatnagar [24] and Kim $et.\ al.$ [25] assumed the packet interarrival time process follows a Gamma distribution. The Gamma distribution is a continuous time approximation to a negative binomial distribution, but the Gamma distribution is arithmetically simpler [26]. Considering a constant departure rate, this distribution translates to the queue occupancy.

For these reasons, the Gamma distribution is used for the model in this paper. The probability density function of the Gamma distribution can be expressed as

$$P\left\{Q=q;\overline{q},\theta\right\}=\frac{q^{\frac{\overline{q}}{\theta}-1}e^{-\frac{q}{\theta}}}{\theta^{\frac{\overline{q}}{\theta}}\Gamma\left(\frac{\overline{q}}{\theta}\right)}, \quad (8)$$

for $q>0$, where $\overline{q}$ is the mean of the Gamma-distributed random variable $Q$, $\theta$ is the scale parameter and $\Gamma$ is the Gamma function.Using this notation, $E\left[Q\right]=\overline{q}$ and $Var\left[Q\right]=\overline{q}\theta$.

An approximation of the negative binomial by a Gamma distribution is achieved by equating the first two moments [26]. The resulting distribution of the queue occupancy is modeled as Gamma distributed with parameters $\overline{q}$ and

$$\theta=\frac{\gamma}{c_1\sqrt{\delta}}\left(\frac{c_1-\sqrt{\delta}}{c_1+\sqrt{\delta}}\right). \quad (9)$$

In order to confirm (8), more than 70,000 simulations were run using the Network Simulator [27] based on a dumbbell topology, varying the parameters of the network as shown in Table I of Section VI. The queue occupancy histograms obtained from the simulations were used to fit mean, variance and skewness to a Gamma distribution. The $L_1$ norm, $\int|f(q)-\hat{f}(q)|dq$, is used to evaluate the quality of fit [28]. When calculating the $L_1$ distance between the empirical distributions and the Gamma distribution, an average error of $0.02$ was obtained, which indicates that the probabilities of the data sets are off by at most $0.01$ [28]. These values decreased as the marking probability increased and also when the mean queue occupancy decreased. Smaller error for lower average queue occupancy points that the Gamma distribution is useful to represent AQM controlled buffers, where desired mean queue occupancies are low. Smaller error when the marking probability increases indicates this type of distribution is also appropriate for describing heavy congestion. Examples of the simulated pdf and fitted pdf are shown in Figs. 2 and 3.

Figure 4 shows the fitted values of $\theta$ for several simulations. As can be observed, $\theta(\delta)$ from (9) provides a good approximation when characteristics between flows are similar. However, the differences in network conditions for all the flows in real case scenarios increase the variability of the queue occupancy. Therefore, this paper uses a scaled version of $\theta$ to provide an upper bound for several network conditions and improve the noise rejection of the measured queue occupancy. The upper bound is required for the next section, where the maximized probabilities are used to calculate the likelihood of congestion.

## V. Likelihood Ratio Test and Detection of Congestion

### A. Maximum Likelihood Ratio Test

A fundamental objective of this paper is to determine the likelihood that the link is congested. As discussed in section III, the observables are the current queue occupancy, the smoothed marking probability and the desired queue occupancy, $q_d$. Define $q_o$ as an observation of the router's queue occupancy level and $\overline{q}$ the ergodic average of the queue. The probability that $q=q_o$, for a given set of parameters $\overline{q}$ and $\theta$, is given by $P\left\{Q=q_o;\overline{q},\theta\right\}$.

To detect congestion, we must decide between two hypotheses, being congested, which is referred to as $H_1$ and not being congested ($H_0$). Define $P\{H_1\}$ as the probability of being congested and $P\{H_0\}$ as the probability of not being congested. One of the hypotheses must be true, therefore $P\{H_0\}+P\{H_1\}=1$.

The Likelihood Ratio Test (LRT) [29] proposes

$$\frac{P\{Q=q_o;\overline{q},\theta|H_1\}}{P\{Q=q_o;\overline{q},\theta|H_0\}}\ \underset{\text{accept } H_0}{\overset{\text{accept } H_1}{\gtrless}}\ T, \quad (10)$$

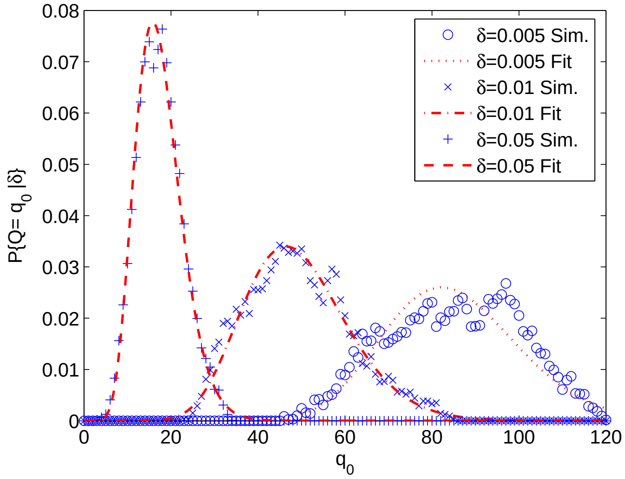


Fig. 2. Queue occupancy distribution fitted to a Gamma distribution for 5 TCP flows passing traffic through a 1 Mbps bottleneck link for different marking probabilities $\delta$.

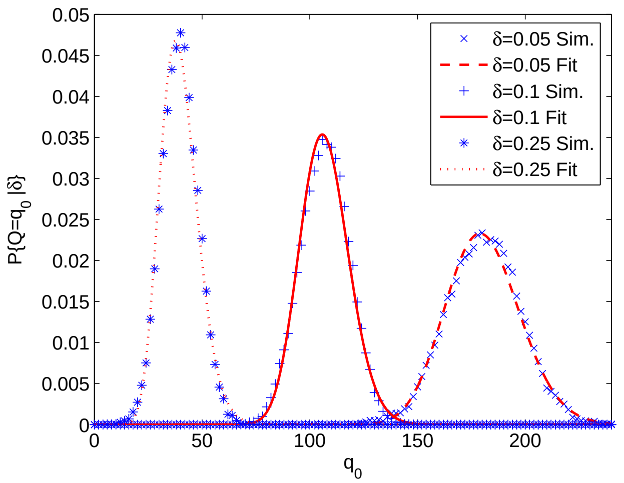


Fig. 3. Queue occupancy distribution fitted to a Gamma distribution for 50 TCP flows passing traffic through a 10 Mbps bottleneck link for different marking probabilities $\delta$.

where $T$ is a threshold. According to (10), hypothesis $H_1$ can be safely accepted if the ratio is greater than the threshold $T$. The higher the value of $T$ the smaller the false alarm probability, thus the ratio is a measure of the probability that hypothesis $H_1$ is true. The optimal threshold $T$, for equally probable events, is when $T = 1$, i.e. the decision rule is determined by the higher probable event.

Since the a priori probabilities $P\{H_0\}$ and $P\{H_0\}$ are not known, it is necessary to obtain these expressions. Replacing the hypotheses by using the definition of congestion from section III results in,

$$\frac{P\{Q = q_o; \overline{q} > q_d, \theta\}}{P\{Q = q_o; \overline{q} \leq q_d, \theta\}} \underset{\text{accept } \overline{q} \leq q_d}{\overset{\text{accept } \overline{q} > q_d}{\gtrless}} T. \quad (11)$$

The generalized form of the likelihood ratio test (GLRT) is obtained when the probabilities are maximum for each of the

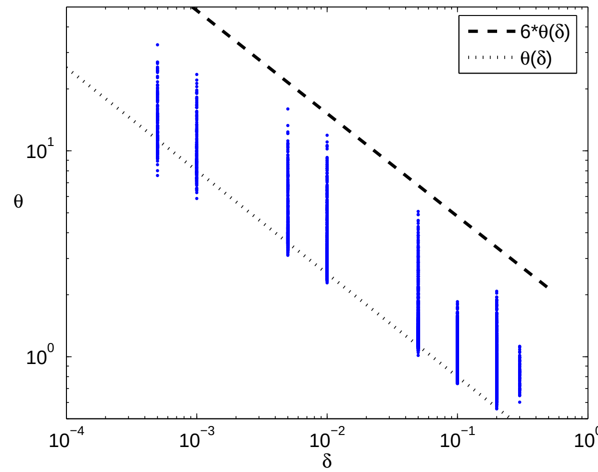


Fig. 4. Fitted values of $\theta$ vs. marking probability and theoretical value $\theta(\delta)$ and a scaled version $6\theta(\delta)$.

hypotheses. The expression for the GLRT is

$$\Lambda(q_o) = \frac{\max_{\overline{q} > q_d} P\{Q = q_o; \overline{q}, \theta\}}{\max_{\overline{q} \leq q_d} P\{Q = q_o; \overline{q}, \theta\}}, \quad (12)$$

where $\Lambda(q_o)$ is defined as the likelihood of congestion given an observation $q_o$.

The function $\Lambda$ takes values in $(0, \infty)$, where large numbers indicate certainty of congestion, values close to 0 indicate certainty of link under-utilization, and values around 1 indicate uncertainty about the degree of congestion.

### *B. Likelihood of a Gamma-Distributed Queue*

To compute the likelihood using the proposed Gamma distribution from Section IV, it is required to maximize (8) with respect to $\overline{q}$ subject to the conditions of the hypotheses, and replace it into the likelihood ratio test function (12).

*Theorem 1: Given an observation, $q_o$, an average marking probability, $\delta$, and a desired queue occupancy, $q_d$. The likelihood of congestion of a router's outbound link with a Gamma-distributed queue occupancy is,*

$$\Lambda_\Gamma(q_o) = \left( \frac{\Gamma\left(\frac{q_d}{\theta}\right)}{\Gamma\left(\frac{q_o}{\theta} + \frac{1}{2}\right)} \right)^{\operatorname{sgn}\left(\frac{q_o - q_d}{\theta} + \frac{1}{2}\right)} \left(\frac{q_o}{\theta}\right)^{\left|\frac{q_o - q_d}{\theta} + \frac{1}{2}\right|}, \quad (13)$$

*where*

$$\theta = \frac{\kappa\gamma}{c_1\sqrt{\delta}} \left( \frac{c_1 - \sqrt{\delta}}{c_1 + \sqrt{\delta}} \right).$$

*The parameter $\gamma$ is 0.31, $c_1$ is the TCP constant, $\sqrt{1.5}$, and $\kappa = 6$, which counteracts the effects of differences between flows.*

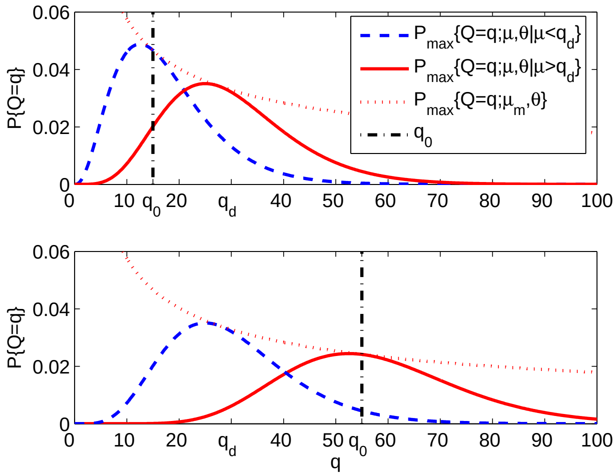


Fig. 5. Maximized probability for a queue occupancy observation, $q_o$, given that $q_o < q_d$ (top) and $q_o > q_d$ (bottom), with $q_d = 30$ and $\theta = 5$.

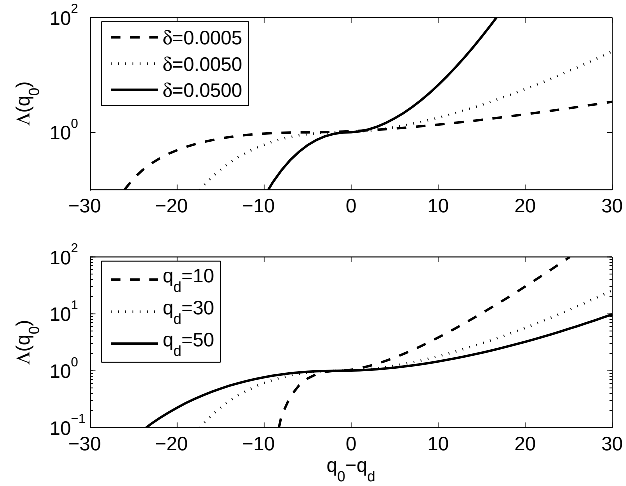


Fig. 6. Likelihood of congestion with respect to the observed queue occupancy Top: For different marking probabilities, given a desired queue occupancy $q_d = 30$. Bottom: For different values of desired queue occupancy, given a current marking probability $\delta = 0.005$.

*Proof:* Taking the derivative of (8) with respect to $\overline{q}$,

$$\frac{dP\{Q=q_o;\overline{q},\theta\}}{d\overline{q}} = \frac{e^{-\frac{q_o}{\theta}} q_o^{\frac{\overline{q}}{\theta}-1}}{\theta^{\frac{\overline{q}}{\theta}+1}\Gamma\left(\frac{\overline{q}}{\theta}\right)} \left(\ln\left(\frac{q_o}{\theta}\right) - \psi\left(\frac{\overline{q}}{\theta}\right)\right), \tag{14}$$

where $\psi$ corresponds to the Digamma function,

$$\psi(z) = \frac{1}{\Gamma(z)}\frac{d}{dz}\Gamma(z).$$

Letting $\overline{q}^*$ be the value of $\overline{q}$ that maximizes (8), note that $\overline{q}^*$ satisfies,

$$\ln\left(\frac{q_o}{\theta}\right) = \psi\left(\frac{\overline{q}^*}{\theta}\right). \tag{15}$$

A close form expression for $\overline{q}^*$ cannot be obtained, but using the approximation of Muqattash and Yahdi [30],

$$\psi(z) \approx \ln(z+a) - \frac{1}{z},$$

where $a\epsilon[0,1]$, (15) can be rewritten as,

$$q_o = (\overline{q}^* + a\theta)\, e^{-\frac{\theta}{\overline{q}^*}}. \tag{16}$$

For $\theta/\overline{q}^*$ small,

$$\overline{q}^* \approx q_o + \frac{1}{2}\theta. \tag{17}$$

The maximized probability for an observation $q_o$ given that the hypotheses are true is obtained by replacing (17) into (8),

$$P\{Q=q_o;\overline{q}^*,\theta\} = \frac{q_o^{\frac{q_o}{\theta}-(1-a)} e^{-\frac{q_o}{\theta}}}{\theta^{\frac{q_o}{\theta}+a}\Gamma\left(\frac{q_o}{\theta}-(1-a)\right)}. \tag{18}$$

Note the small error introduced for large values of $\theta/q_o$ is strongly attenuated by the distribution function.

Figure 5 shows two examples for the observations $q_o < q_d$ and $q_o > q_d$ and their corresponding estimated distributions. The maximized probability for an observation $q_o$ given that the hypotheses are false is simply $P\{Q=q;q_d,\theta\}$. Thus, the likelihood of congestion as a function of the instantaneous queue occupancy $q_o$ can be written as

$$\Lambda(q_o) = \begin{cases} \frac{P\{Q=q_o;\overline{q}^*,\theta\}}{P\{Q=q_o;q_d,\theta\}} & \text{if } q_o > q_d - \theta/2 \\ \frac{P\{Q=q_o;q_d,\theta\}}{P\{Q=q_o;\overline{q}^*,\theta\}} & \text{otherwise.} \end{cases} \tag{19}$$

Replacing both (8) using $\overline{q} = q_d$ and (18) into (19), a simplified expression for the likelihood of congestion is obtained.

As mentioned, the likelihood given by (13) represents how likely the link is congested based on a single a sample $q_o$. The likelihood depends on the parameters $\theta$ and $q_d$, and the observed variable $q_o$. However, note that $\theta$ is function of $\delta$, which corresponds to the current average marking probability and can be approximated by the router. The value of $q_d$ is set by the network administrator. Figure 6 shows the behavior of $\Lambda(q_o)$ for different values of the parameters $\theta(\delta)$ and $q_d$.

In Fig. 6 is possible to see that as the queue empties the likelihood goes to 0, indicating a certainty of not being congested. When the observed queue occupancy is around the desired value the likelihood goes to 1, indicating uncertainty about being congested. The likelihood goes to infinity to indicate certainty that congestion is occurring. For high values of the marking rate, the congestion certainty grows faster as the queue occupancy increases, compensating for the variance of the queue occupancy.

## C. Likelihood Based On Other Queue Distributions

It is useful to analyze the results obtained if other, less realistic, queue distributions are assumed. For example, the simplest distribution that could be used is the uniform distribution. Thus, assuming a uniform-distributed queue with mean $\mu$ and variance $\sigma^2$. The random variable $q$ is uniformly distributed in $(q_d - \sqrt{3}\sigma, q_d + \sqrt{3}\sigma)$, where $\sigma^2 = q_d\theta$. The evaluation of the likelihood is straight-forward, since the ratio

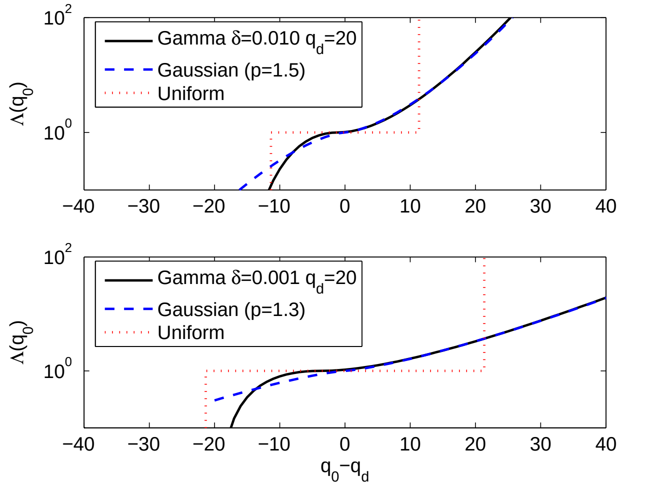


Fig. 7. Comparison between likelihood functions for Gamma (solid), Gaussian (dashed) and Uniform (dotted) distributed queues, given a desired queue occupancy of 20 packets.

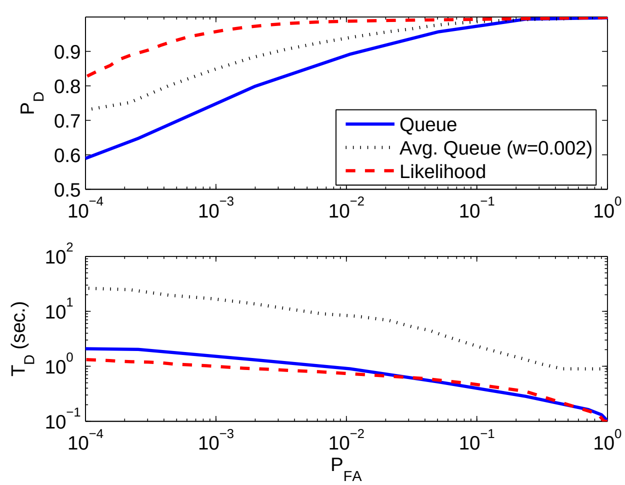


Fig. 8. Top: Probability of detecting congestion, $P_D$, with respect to the probability of false alarm, $P_{FA}$. Bottom: Average time to detect congestion, $T_D$ versus $P_{FA}$.

between the distributions takes only three values $\{0, 1, \infty\}$, namely,

$$\Lambda_U(q_o) = \begin{cases} 0 & \text{if } q_o < q_d - \sqrt{3\sigma^2} \\ 1 & \text{if } q_d - \sqrt{3\sigma^2} < q_o < q_d + \sqrt{3\sigma^2} \\ \infty & \text{otherwise.} \end{cases} \quad (20)$$

Thus, an AQM scheme that assumes a uniformly distributed queue occupancy would mark all the arriving packets when $q_o > q_d + \sqrt{3q_d\theta}$, where $q_d$ is the desired queue occupancy and $\theta$ is given by (9), which depends on $\delta$. Thus assuming uniform distributed queue occupancy results in a virtual queuing approach, where the size of the virtual queue depends on the past values of marking probability.

It is also of interest to analyze a Gaussian-distributed queue. Therefore, using a generalized Gaussian distribution with mean $\overline{q}$ and variance $\sigma^2$

$$P\{Q = q_o; \overline{q}, \sigma^2\} = \frac{p}{2\sqrt{2\sigma^2}\Gamma\left(\frac{1}{p}\right)} e^{-\left(\frac{|q_o - \overline{q}|}{\sqrt{2\sigma^2}}\right)^p}, \quad (21)$$

where the $\overline{q}$ that maximizes the probability function is $q_o$. The likelihood simplifies to,

$$\Lambda_G(q_o) = e^{\operatorname{sgn}(q_o - q_d)\left(\frac{|q_o - q_d|}{\sqrt{2\sigma^2}}\right)^p}. \quad (22)$$

Figure 7 shows a comparison between the likelihood functions for the Gamma, Gaussian and uniform distributions for different parameter values. The generalized Gaussian adds another parameter, $p$, which also depends on network parameters. Through fitting distributions found from simulations, $p$ was found to be in the interval $[1, 2]$, where tails of Gamma and Gaussian are similar. Figure. 7 shows two cases for different marking probability, $\delta$, and the approximated values of the parameter $p$ are shown in the legend.

## VI. Evaluation of Performance

### A. Detector Evaluation

In this section, the performance of the proposed detector is compared to the commonly used detectors, specifically, the smoothed queue occupancy (as in RED, REM, PI, etc.) and the direct observation of the queue. Normally, detectors use thresholds to determine whether the hypothesis tested is valid. Poorly set thresholds can either detect congestion when it is not occurring (i.e. false alarm probability) or fail to detect congestion when it is occurring (i.e. probability of detection). To evaluate the performance of the detector, the classical performance metrics for detectors were used [31]. Specifically, the false alarm probability, $P_{FA}$, the probability of correctly detecting congestion, $P_D$, and the time to detect, $T_D$. $P_{FA}$ is the proportion of time that the detector input was above the threshold during non-congestion. $P_D$ corresponds to the proportion of time the input of the detector was above the threshold during congestion conditions. Note that the metric $P_D$ does not reveal the detection speed but only certainty. Therefore, it is necessary to evaluate $T_D$, which is the time it takes to first cross the threshold after congestion begins (i.e. when new flows start).

To obtain these measurements, another set of simulations were run. These simulations used a fixed marking probability to maintain the average queue occupancy produced by a fixed number of flows. The simulation parameters included bottleneck bitrates ranging from 1Mbps up to 100Mbps, a number of initial flows varying from 5 to 150 with uniformly-distributed delays between 1ms and 40ms, marking probabilities from $3 \times 10^{-4}$ to 0.3, and the smoothing parameter $w = 0.002$, originally proposed in [2]. After time 2000 seconds, the number of flows is doubled, creating a congestion condition. Several threshold values were used to obtain different values of $P_{FA}$, $P_D$ and $T_D$.

Figure 8 summarizes the performance using the metrics explained above. On the top plot, $P_D$ versus $P_{FA}$, the likeli-

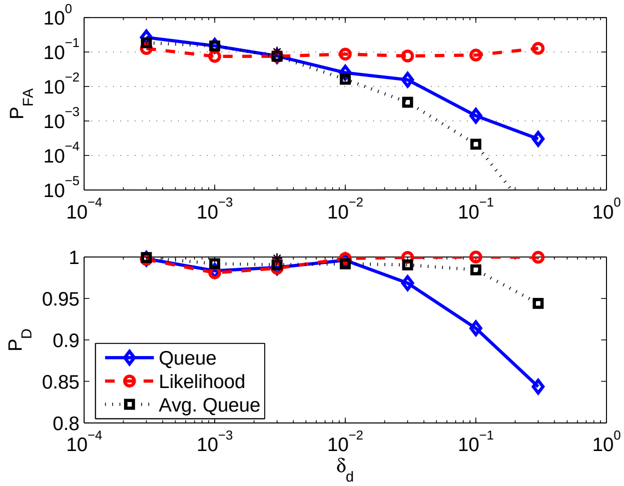


Fig. 9. False alarm probability, $P_{FA}$, and probability of detection, $P_D$ versus initial packet marking probability, $\delta_d$. Threshold adjusted to achieve $P_{FA} = 0.1$ in a randomly selected baseline scenario.

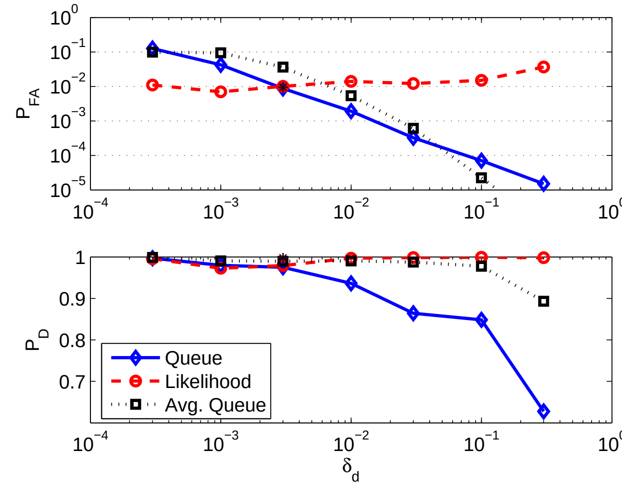


Fig. 10. False alarm probability, $P_{FA}$, and probability of detection, $P_D$ versus initial packet marking probability, $\delta_d$. Threshold adjusted to achieve $P_{FA} = 0.01$ in a randomly selected baseline scenario.

hood detector shows larger probability of detection and lower false alarm probability than the instantaneous and smoothed queue. The performance of the smoothed queue improves as the parameter $w$ becomes smaller. In fact, the Network Simulator [27] implements values of $w$ that decrease as the link throughput grows. However, the average detection times at the bottom of Fig. 8 reveal the trade-off between $T_D$ and $P_D$ for this approach. Therefore, the parameter $w$ cannot be arbitrarily small given its direct impact on the detection time. On the other hand, likelihood maintains small detection times, similar to those of the instantaneous queue occupancy.

In order to evaluate the detector robustness to changes in network parameters, a baseline scenario was randomly selected and thresholds were set to obtain false alarm probabilities on each detector of $10^{-1}$, $10^{-2}$ and $10^{-3}$ fdepending on the experiment. All the scenarios were then evaluated using these thresholds. Results in Figs. 9, 10 and 11 show that the likelihood detector maintains the desired $P_{FA}$ and keeps $P_D$ high as the initial marking probability, $\delta_d$ changes. These figures also show that both the instantaneous and the smoothed queue reduce the $P_{FA}$ when $\delta_d$ increases, with the penalty of decreased $P_D$. Note also that the behavior of the smoothed queue was not considerably impacted by changes on the baseline thresholds (i.e the curves corresponding to the smoothed queue in the figures are nearly the same). Thus, in the case of the smoothed queue detector thresholds set for a single scenario might only work for that particular scenario. On the other hand, the queue detector reduces the $P_{FA}$ with the penalty of small $P_D$.

The independence of $P_{FA}$ with respect to different scenarios is a powerful result of the likelihood detector. It reveals an immediate relationship between the likelihood and the reaction to congestion, which increases as the certainty of congestion increases. Therefore, the likelihood provides real-valued degree of congestion, instead of a binary decision. This is particularly useful for the development of AQM schemes.

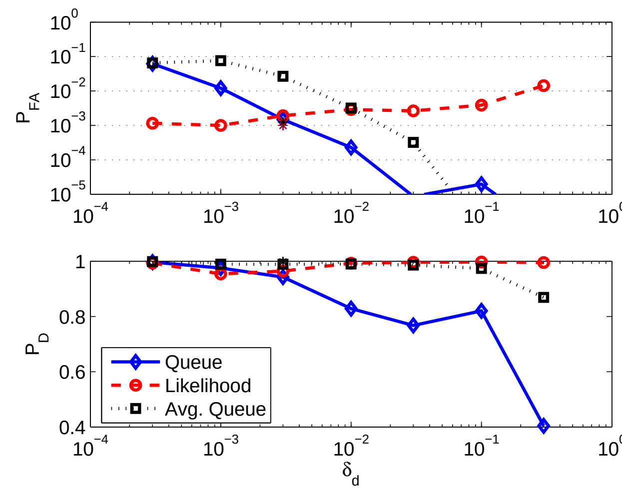


Fig. 11. False alarm probability, $P_{FA}$, and probability of detection, $P_D$ versus initial packet marking probability, $\delta_d$. Threshold adjusted to achieve $P_{FA} = 0.001$ in a randomly selected baseline scenario.

### *B. Likelihood of Congestion from Simulations*

Using the Network Simulator (NS2) [27], more than 70,000 simulations were run based on a dumbbell topology, different values of marking probability, bottleneck link capacity, transmission delays and number of flows. The values of the parameters are listed in the Table I. The buffer size of the router was set to 1000 packets to reduce interference from the buffer size in the measurement. Packet marking was accomplished with Explicit Congestion Notification (ECN) [32].

Based on the results of these simulations, the likelihood of congestion was calculated for different marking rates and desired queue occupancies. Figs. 12 and 13 show the empirical likelihood found from simulations and the likelihood based on the assumption in (13) that the queue occupancy is Gamma-

TABLE I
PARAMETERS USED IN THE SIMULATIONS

| | |
|---|---|
| Marking Probability | 0.0001 - 0.6 |
| Bottleneck Capacity | 0.2Mbps - 100Mbps |
| TCP Flows | 1 - 30 |
| Bottleneck Link Delay | 0.1ms - 5ms |
| Per Flow Delays | 1ms to 40ms |

distributed. Values of $q_d$ of 20 and 30 packets respectively were used. The figures show that the simulated results closely match the analytical results from section V-B.

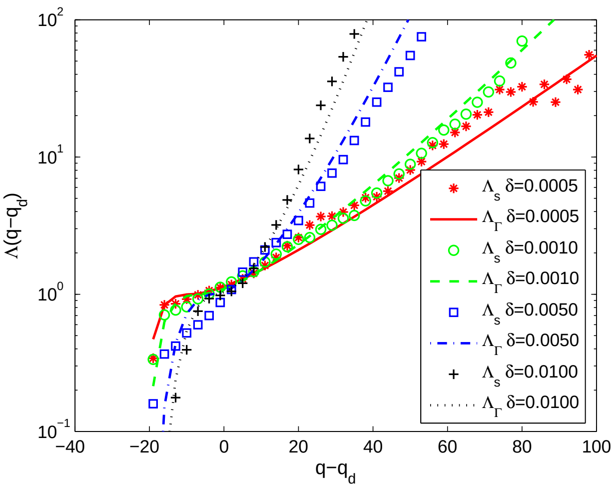


Fig. 12. Simulation based likelihood ($\Lambda_s$) compared to Gamma based likelihood of congestion ($\Lambda_\Gamma$) for different values of marking probability when $q_d = 20$.

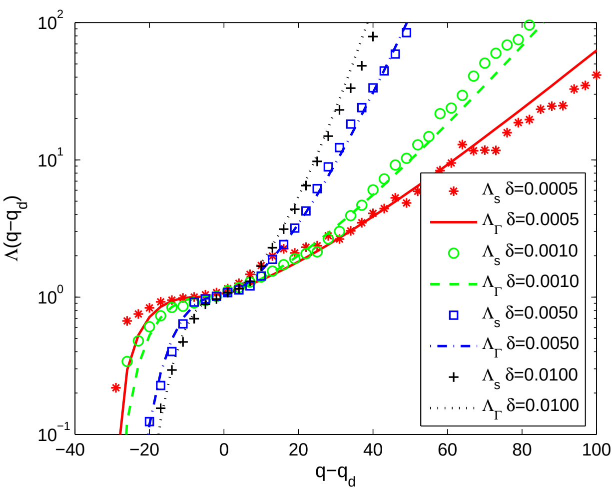


Fig. 13. Simulation based likelihood ($\Lambda_s$) compared to Gamma based likelihood of congestion ($\Lambda_\Gamma$) for different values of marking probability when $q_d = 30$.

## C. The Impact of Congestion Likelihood in AQM Performance

The likelihood-based detector provides a level of certainty about congestion based on network parameters and it is reasonable to include it in the estimator of Fig. 1. However,

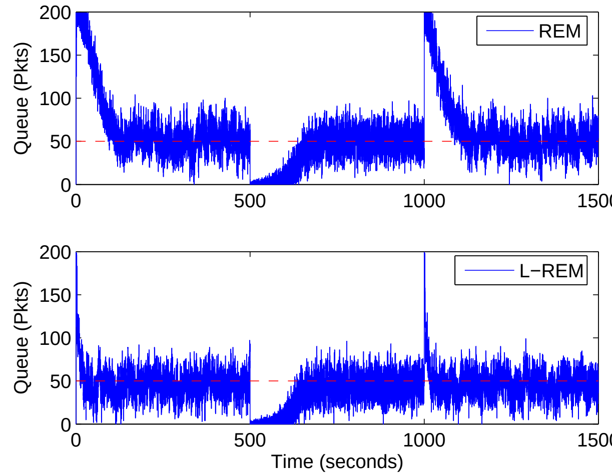


Fig. 14. Reaction speed comparison between REM and Likelihood-REM, for $q_d = 50$

the design of AQM schemes is out of the scope of this paper. Therefore, for the purpose of a quick evaluation of the detector characteristics in AQM schemes, the measuring element block in Fig. 1 was replaced by $\hat{\hat{q}} = q_d\Lambda(q_o)$. Design of an AQM using the likelihood-based detector is presented in [33].

The reaction speed of REM and PI was evaluated for a dumbbell topology. Access links of 100Mbps with uniformly-distributed delays are connected to a gateway router. The gateway router has a 10Mbps outbound link, with a buffer for 200 packets.

## D. Experiment 1

The simulation used in this experiment starts at time 0 with 140 ftp flows, after 500 seconds, 105 flows are terminated, leaving only 35 flows crossing traffic through the gateway router. At time 1000 seconds, 105 flows start, resulting in a total of 140 flows again. The parameters of the AQM were set to the preferred values except for the desired queue occupancy, which was set to 50 packets aiming to observe a non-empty queue when in steady state.

Figure 14 shows a significant performance improvement in the reaction speed of the Likelihood REM (L-REM) over the traditional REM scheme. This figure shows a convergence to steady state in less than 40 seconds when congestion is detected. This time is considerably small compared to the 140 seconds of the traditional REM. Note that queue length variability is not negatively impacted when the system achieves steady state and the large buffer size limitation was improved.

The likelihood block was also implemented in PI, obtaining performance improvements in detection of congestion. Figure 15 compares the reaction speed of Likelihood-based PI and REM, with respect to AVQ, which is a fast AQM scheme. The improvements in REM reduced the amount of packet drops due to overflow during the last 500 seconds from more than 900 packets to 150 packets, and, in PI, from 690 packets to 61 packets in identical conditions. Both REM and PI maintained

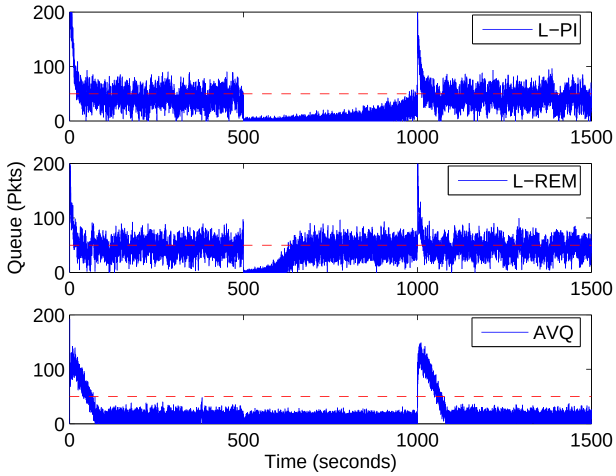


Fig. 15. Experiment 1: Reaction speed comparison between L-PI, L-REM and AVQ for $q_d = 50$, when 105 flows are terminated at time 500 and restarted at time 1000.

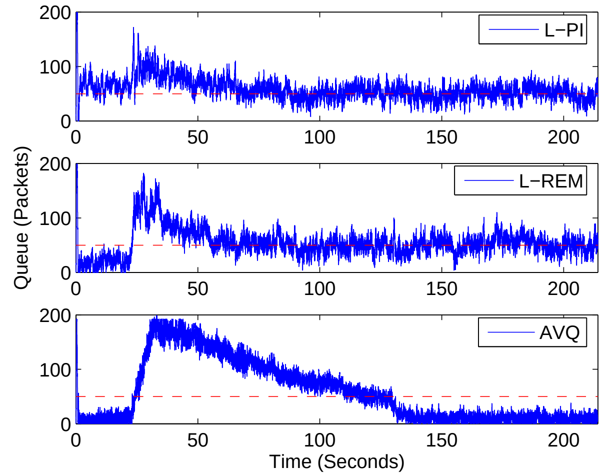


Fig. 16. Experiment 2: Reaction speed comparison between L-PI, L-REM and AVQ, when 140 flows are introduced at a rate of 10 flows per second.

full link utilization during that period of time. Note that even though the congestion detection was significantly improved, the reaction to underutilization was only slightly impacted. The reason for this is that the algorithms are not designed for the likelihood-based detector. This detector outputs values in $(0, \infty)$. When the likelihood goes to $\infty$, indicating that the link is heavily congested, REM and PI take extreme action as they should. However, when the likelihood goes to $0$, indicating that congestion is unlikely, REM and PI do not take appropriate action. In fact, they keep marking packets with a probability that slowly decreases. As previously explained, the likelihood-based detector requires of an AQM that takes full advantage of its characteristics and the purpose of this paper is to introduce these characteristics.

### *E. Experiment 2*

To illustrate an effect of slow reaction speeds in AQM routers, the previous experiment was slightly changed. Thirty flows are introduced at time $0$, then, at time $20$, $110$ flows are introduced in groups of $10$ every second, generating an arrival rate of $10$ flows per second. The results shown in figs. 16 and 17 correspond to the queue occupancy and link utilization respectively. AVQ struggles to achieve the goal of maintaining the utilization at $98\%$, even losing packets due to buffer overflow. However, Likelihood-REM and Likelihood-PI converge to the desired queue size of 50 packets faster with no packet losses. Original implementations of REM and PI would take longer to converge, dropping hundreds or even thousands of packets under similar conditions.

## VII. Conclusions

This paper explored a statistical technique applied to AQM, namely, maximum likelihood estimation of congestion. The investigation of maximum likelihood estimation of congestion revealed a relationship between the observed marking rate, the observed queue occupancy and the likelihood of congestion.

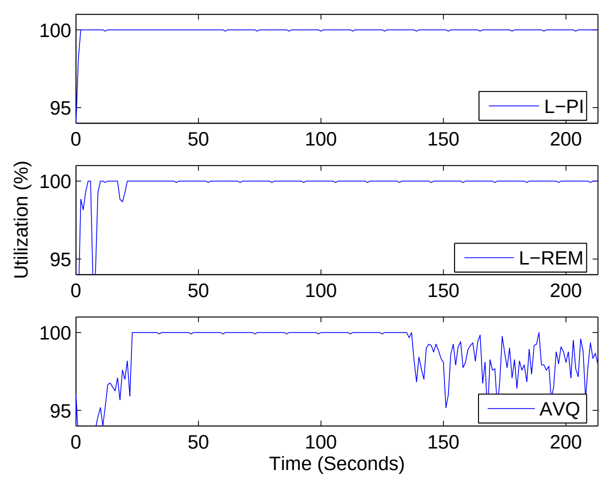


Fig. 17. Experiment 2: Reaction speed comparison between L-PI, L-REM and AVQ, when 140 flows are introduced at a rate of 10 flows per second.

This technique represents significant improvement to many current AQM algorithms since current schemes use either binary point of detection or reactive schemes that wait for the router's queue to be congested before reacting.

This new approach provides additional information of the level of congestion of the router's outbound link based on the current known parameters. One of the most important results is that the likelihood of congestion grows super-exponentially with the queue occupancy.

An AQM based on this detection scheme can boost its reaction speed, outperforming other schemes. The mathematical expression of the likelihood of congestion was implemented in well known AQM algorithms. Simulations showed that these algorithms can achieve a quick detection of congestion when used the congestion measurement. An AQM designed to work with the likelihood of congestion will take full advantage of this statistical analysis, resulting in better performance.

AQM schemes based on arrival rate estimation can also take advantage of the likelihood of congestion by extending the statistical analysis to packet inter-arrival times.

## ACKNOWLEDGMENT

The authors would like to thank Khushboo Shah for assistance on early versions of this work and support on initial simulations for the MLRT approach to congestion detection.

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