Observations and Models of Time-Varying Channel Gain in Crowded Areas

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***Abstract*— Testbeds are projected to be the next generation of mechanisms for protocol verification and performance validation of wireless networks. One important objective of these testbeds is to depict the propagation mechanism accurately. This paper presents measurements and models of propagation between stationary transmitters and receivers in a dynamic environment. These measurements indicate that in some environments, the signal strength displays wide variations, while in other environments, there is less variation in the signal strength. A diffusion-based stochastic model is presented that can be used to increase the accuracy of testbed emulations**

I. Introduction

One of the main challenges of mobile wireless networking is time-varying channels. This variability results in nodes being able to communicate at a high bit-rate and with high quality at one moment, and then, a moment later, not able to communicate at all or be able to only communicate at a low bit-rate. In order to understand the performance of mobile wireless networks in *realistic* environments, high-fidelity simulators or test-beds are necessary to capture the behaviors that are not amendable to analytic methods. In case of simulators and test-beds, it is important that the channels display the full range of realistic variations. In the case of test-beds, either the test-bed must include the physical mechanisms that result in realistic channel variation, or, if the physical means are not available, then some portion of the channel must be realized through artificial means. Thus, in order to verify the behavior of mobile wireless networks, the behavior of realistic channels variations must be well understood and perhaps modeled and simulated.

In the context of communication theory, variability of the channel gains is well known. In order to understand the performance of communication techniques, a number of models of the variability of channels have been developed. For example, a fast fading channel is assumed to be the result of the receiver and/or the transmitter moving. The resulting channel variations are often modeled with Jake's Model [1]. To account for the variability induced by the mobile node moving among large objects (e.g., buildings), a time-varying shadowing model can be used. One approach is to model the channel as a correlated Gaussian process [2], [3], [4].

It is important to note that the commonly used channel models used in communication theory assume that it is the mobility of the receiver or/and transmitter that causes the channels to vary. On the other hand, it is widely known that the mobility of objects in the environment can also result in time-varying channels. However, there has been little effort focused on quantifying the impact of mobility in these dynamic environments. Furthermore, to the best of our knowledge, there are no models for the channel variation in such environments. This paper reports on the characteristics of the time-varying channels that arise when the receiver and transmitter are stationary, but the environment contains mobile objects. We specifically focus on the impact of pedestrian mobility on the variability of the channel gain.

In the context of mesh networks, the infrastructure nodes are fixed. However, the environment could contain moving pedestrians. Furthermore, even when the receiver and/or transmitter are moving (e.g., a client node in a mesh network), the mobility of pedestrians may be responsible for a significant contribution to the channel variability. In this paper, it is found that the channel in such settings displays large variability, with variation in channel gain exceeding 10 dB. Recall that the dynamic range[1] of 802.11b at 11 Mbps is roughly 50 dB [5], so, in a sense, a 10 dB variation is 20% of the entire

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[1]Typically the transmission power of 802.11b is -10dBm (100mW). Considering the *first meter* loss to be $\equiv$ 40dB the pathloss has to be $\leq$ 50dB in-order for the received signal be above the receiver sensitivity.

dynamic range.

Considering the significance of the channel variation due to movement of objects in the environment, it is important that test-beds and simulators account for this type of channel variation. To support this need and to gain further understanding of channel variability, the paper develops a continuous-time diffusion model of the variability of the channel gain. It is important to note that the motivation for the diffusion-based model is that diffusion processes can provide a compact way to completely describe the observed process. In this paper, a one-dimensional, four-parameter diffusion process is found to provide a good approximation. Specifically, the stationary distribution of this process is the Gamma distribution, which closely matches with the observations (Section IV-A). And furthermore, we find that the transition probabilities of the diffusion process approximate the empirical transition probabilities (Section IV-B). While the four-parameter model is fairly compact, it appears that the number of parameters could be further reduced. As will be shown, the process's parameters are strongly dependent on the variance, and the variance is dependent on the pedestrian density. This points to the possibility of developing a single parameter model, where the parameter is a function of the pedestrian density. Future work will verify this possibility.

The remainder of this paper is as follows. In the next section a brief description of the measurement setup and the measured scenarios is provided. Then the class of diffusion models used is briefly described in Section III. An overview of how the model parameters are estimated is provided in Section III-A. In the data analysis section several feature are examined. Section IV-A focuses on the stationary distributions, while Section IV-B focuses on the transition probabilities. And, concluding remarks are provided in Section V.

II. Measurement setup

The central goal of this effort was to measure the *channel gain* when the channel might be subjected to blockage due to mobile pedestrians. Furthermore, the impact of the antenna height was investigated. Two Linksys BEFW11S4 routers were used. The parameters of the BEFW11S4s were set such that the data rates were fixed at 1Mbps. They were also outfitted with a 2 dBi antenna.

To determine the received signal strength, the Yellowjacket [6] receiver made by Berkeley Varitronics Systems was used. The yellowjacket 1.40 firmware was used with

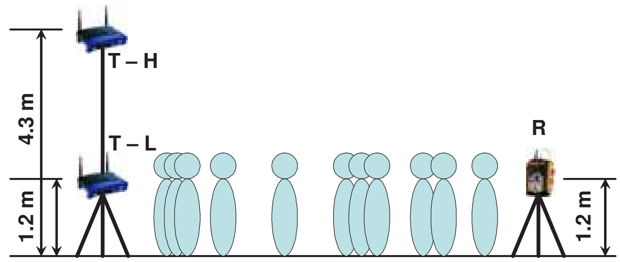

Fig. 1. Measurement Configuration. T-L and T-H indicate Transmitter at lower and elevated positions respectively. R indicates the reciver position.

the system. The base stations and the receiver where configured to support 20 packets per second.

The configuration of the equipment is illustrated in Figure 1. Note that two transmitters are used at the same time. We refer to the higher antenna as the *elevated antenna*, and refer to the other antenna as the *lower antenna*.

Three scenarios were investigated. The *high pedestrian node density scenario* was measured between 4 PM and 5 PM on Walnut Street between 15th and 16th Streets in Philadelphia. The other two scenarios are from Trabant University Center at the University of Delaware. The *low pedestrian density* measurements where made from 4 to 5 PM, while the *moderate pedestrian density* measurements were made from 12 to 1 PM, when the building is fairly crowded due to lunchtime.

III. Diffusion process

Diffusions processes have been used to model a large number of continuous time (and continuous space) processes [7]. While diffusion processes have many properties that make them especially amendable to various calculations, our purpose for employing diffusion is limited. Specifically, a diffusion process allows for a few parameters to specify the invariant distribution as well as the transition probabilities. Furthermore, the transition probability is specified for all times. That is, let $p(x, t|x_0)$ denote the probability density of the process taking the value x at time t, given that the process took the value x_0 at time 0. A diffusion model provides the function p for all the values taken by the variables x, t, and x_0. Finally, it is straightforward to simulate the process. This last property makes it convenient and important for the hi-fidelity simulations and test-bed experiments of wireless networks. The challenge in using diffusion models is to find a model that has minimum number of parameters. We will utilize a process with four parameters.

In general, a diffusion process can be described through a stochastic differential equation, i.e.,

$$dX_t = \mu(X_t)\,dt + s(X_t)\,dW_t, \tag{1}$$

where X_t is the value of the process at time t, μ and s are one-dimensional functions and W_t is Brownian motion. Note that if $s \equiv 0$, then (1) is an ordinary differential equation. Here we are interested in the case where

$$\begin{aligned} \mu_{\lambda,\phi,\gamma,\sigma}(x) &= \frac{\sigma^2}{2}\left((\gamma+\lambda-1)\,x^{\gamma-1} - \phi x^{\gamma}\right) \\ s^2_{\gamma,\sigma}(x) &= \sigma^2 x^{\gamma}. \end{aligned} \tag{2}$$

In this case, it can be shown that $X_t \in [0,\infty)$ and that the probability density of the invariant (or stationary) distribution of X_t is given by $h_{\lambda,\phi}(x) = \frac{\phi^{\lambda}}{\Gamma(\lambda)} x^{\lambda-1}\exp(-\phi x)$ [8]. That is, X_t is Gamma distributed with parameters λ and ϕ. Note that if $\gamma = 1$, then (1) is the widely used CIR model [9].

For stochastic calculus [7], it can be shown that the transition probability of X_t obeys

$$\begin{aligned} &\frac{\partial p_{\lambda,\phi,\gamma,\sigma}(x,t|x_0)}{\partial t} = \\ &-\frac{\partial}{\partial x}\left(\mu_{\lambda,\phi,\gamma,\sigma}(x)\,p_{\lambda,\phi,\gamma,\sigma}(x,t|x_0)\right) \\ &+\frac{1}{2}\frac{\partial^2}{\partial x^2}\left(s^2_{\gamma,\sigma}(x)\,p_{\lambda,\phi,\gamma,\sigma}(x,t|x_0)\right), \end{aligned}$$

where $p_{\lambda,\phi,\gamma,\sigma}(x,t|x_0)$ is the probability density of X_t given that $X_0 = x_0$ and given the parameters λ, ϕ, γ and σ. The above partial differential equation can be solved numerically, or Monte Carlo simulations of 1 can be used to determine the transition probability.

A. Parameter estimation methodology

The parameters, γ, λ, ϕ, and σ, are estimated in a two-stage process. First, all the data samples are used to estimate the stationary distribution, from which λ and ϕ, the parameters of the Gamma distribution, are found. Once λ and ϕ are found, σ and γ can be estimated. While it is possible to estimate the model parameters via maximum likelihood estimation, here we elected to select the parameters that minimize the L^1 norm between the observed probability density function (i.e., the histogram) and the modeled probability distribution. The motivation for using the L^1 norm is that it can be shown that if f and g are probability densities, then $\int |f(x) - g(x)|\,dx = 2\sup_A \left|\int_A f(x)\,dx - \int_A g(x)\,dx\right|$. In other words, the error from using f to estimate the probability of event A as oppose to using g, is bounded by one-half of the L^1 difference between f and g [10].

The parameters are estimated as follows. Let $u_\Delta(x)$ be the fraction of observations between $x-\Delta$ and $x+\Delta$. Similarly, let $v_\Delta(x,t|x_0)$ be the fraction the observations such that $x_0 - \Delta < X_\tau \le x_0 + \Delta$ and $x - \Delta < X_{\tau+t} \le x + \Delta$. Then, the λ and ϕ are selected such that

$$\min_{\lambda,\phi} \sum_{k=1}^{\infty} \left| u_\Delta(k2\Delta) - \int_{k2\Delta-\Delta}^{k2\Delta+\Delta} h_{\lambda,\phi}(y)\,dy \right|. \tag{3}$$

Once λ and ϕ are determined, γ and σ can be found in a similar fashion, by solving

$$\begin{aligned} \min_{\gamma,\sigma} \sum_{k=1}^{\infty} h_{\lambda,\phi}(k2\Delta) \sum_{j=1}^{\infty} &\left| v_\Delta(2j\Delta, T|\,2\Delta k) \right. \\ &\left. - \int_{j2\Delta-\Delta}^{j2\Delta+\Delta}\int_{k2\Delta-\Delta}^{k2\Delta+\Delta} p_{\lambda,\phi,\gamma,\sigma}(x,T|\,y)\,dydx \right|. \end{aligned} \tag{4}$$

Note that the above is the weighted L^1 norm of the transition probabilities where the weighting is given by the invariant probability distribution. The rational behind this is to force the parameters to provide the best fit for the most likely initial conditions x_0.

In the computations below, $\Delta = 1/2$ and $T = 1$ sec.

IV. Data analysis

The data collected represents the channel gain in dBm. We transform the data as follows. Denote the collected data at time t as Y_t dBm. Then, define $X_t := -Y_t + \max_t(Y_t)$. Thus, $X_t \ge 0$. Note that the smaller X_t, the louder the received power. It is important to note that the received signal power has been normalized by $\max_t(Y_t)$. Hence statistics such as the mean of X_t cannot be used to directly determine the probability of transmission error. However, the offset $\max t\,(Y_t)$ does not impact the variability, which is of primary focus here.

Figure 2 shows several time series of the channel gains collected in the different scenarios described in Section II. This figure also shows a sample time series plot generated by the diffusion process 1 and 2 with model parameters derived from the observations and using the techniques discussed in Section III-A. As can readily be seen, the observed time series and the generated data are qualitatively the same. Hence, the diffusion model can be used to generate channel gains that are, in some ways, realistic. In this section, the observed data and quality of the diffusion process fit is analyzed in terms the stationary probability function (Section IV-A) and in terms of the transition probability functions (Section IV-B).

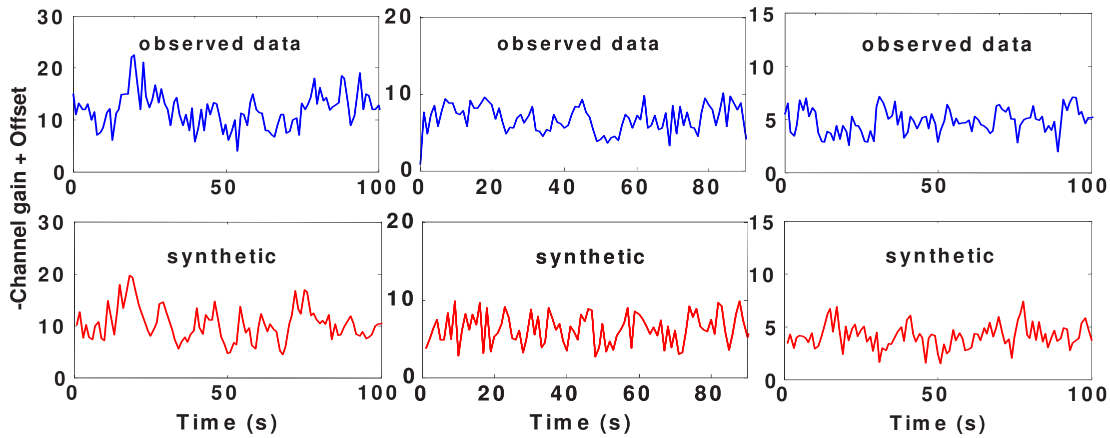

Fig. 2. Observed Time Series and Synthetic Time Series of Channel Gains in Different Scenarios. The synthetic time series were generated with a diffusion model. Note that difference in scales.

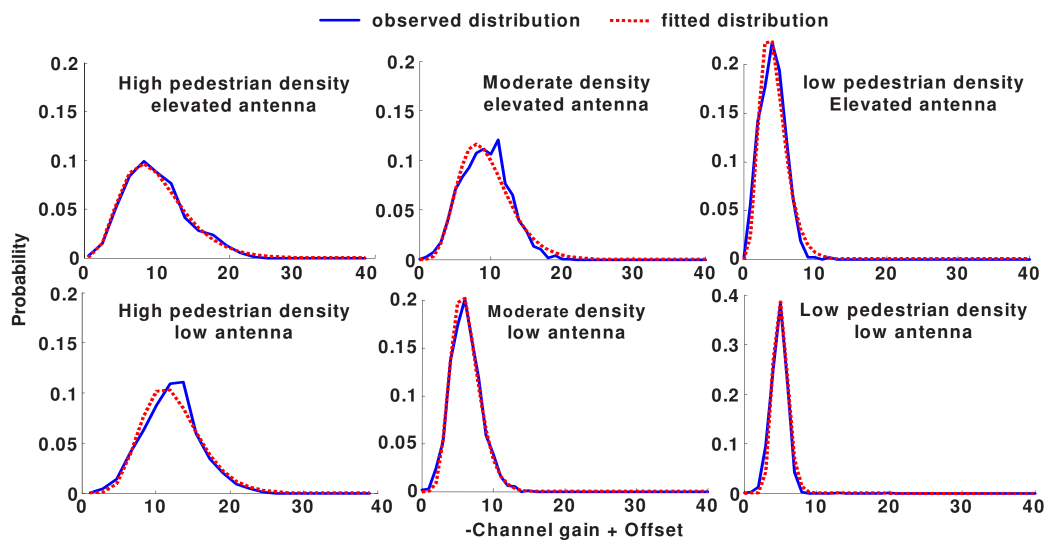

Fig. 3. Observed and Fitted Stationary Distribution for Three Different Pedestrian Densities and Two Different Antenna Heights

Experimental Scenario	Mean	variance	λ	Φ	Stationary distribution L^1 error	S	γ	Probability transition L^1 error
Philadelphia (Peak hour) Elevated – high density	13.9401	17.4602	4.9440	0.4920	0.1288	0.8400	1.2700	0.0584
Philadelphia (Peak hour) Low – high density	12.1627	13.8924	9.6395	0.7817	0.2302	0.8600	1.3000	0.0604
Trabant (Lunch time) Elevated – mod density	9.2259	10.7039	6.5718	0.6989	0.1467	0.8900	1.3500	0.0899
Trabant (Lunch time) Low – mod density	6.1820	4.3546	9.4812	1.5267	0.0777	0.9200	1.3800	0.1532
Trabant–(late afternoon) Elevated – low density	4.1192	2.8668	5.2893	1.2332	0.1633	0.9300	1.5300	0.1922
Trabant–(late afternoon) Low – low density	4.8699	1.2866	23.176	4.6211	0.1269	0.9800	1.9300	0.4132

Fig. 4. The above table shows the description of different experimental scenarios and the parameters and errors for stationary distribution and probability transition functions

A. Stationary distribution

Utilizing (3), the parameters of the Gamma distribution were found that minimize the L^1 norm between the histogram of the experimental data and the modeled distribution. Figure 3 shows the resulting Gamma distribution along with the observed histogram for various measurement scenarios. As can be seen, the Gamma distribution provides a good fit to the observed behavior for all pedestrian densities observed.

As can be observed in Figure 3 and in Figure 4, as the pedestrian density increases, the variance of the channel gain increases. This conclusion is reasonable. In a low pedestrian density setting, the received signal is a multipath signal that has bounced off of several stationary objects (e.g., the ground, walls, etc.). However, when the pedestrian density is high, there are a larger number of objects for the signal to be bounced off. The result of the increase in the number of reflectors results in a higher variability of the received signal power.

Comparing the distribution of the channel gain for elevated antennas to lower antennas, there is a relatively small difference. It can be seen that the variance is larger when the antenna is elevated. One possible explanation for this behavior is that the wireless signal transmitted from the elevated antenna is sometimes able reach the receiver via a strong line-of-sight path. When a line-of-sight path is not available, then the propagation environment facing the elevated antenna is similar to the environment facing the lower antenna, and hence, has similar channel gain variation. As a result, the received signal strength ranges from a line-of-sight strength to a rather weak signal that has experienced many reflections

Figure 5 shows the relationship between the mean and variance for different pedestrian densities. As expected, as the pedestrian density increases, the channel gain shows more variability. We also see that the mean increases with the pedestrian density. Furthermore, the mean approximately obeys an affine relationship with the variance, with slope of 0.68 and y-intercept 2.67. Hence, while the Gamma distribution requires two parameters, if the variance is specified, then the mean is also approximately specified. Future work will examine the relationship between the pedestrian flow rate and the variance.

B. Transition probabilities

Now we turn to the transition probabilities. While the transition probabilities are important for determining channel behaviors such as the outage duration, we

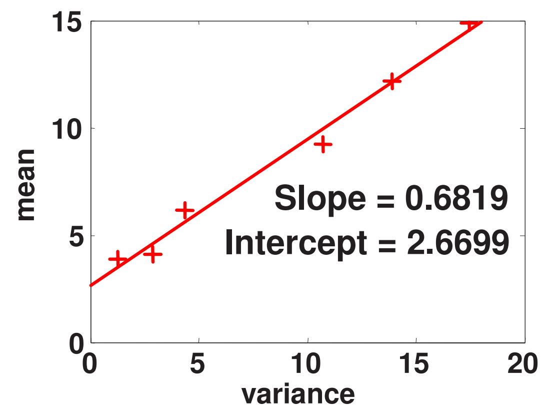

Fig. 5. The plot shows the least squares fit for relationship between the variance and the mean channel gain

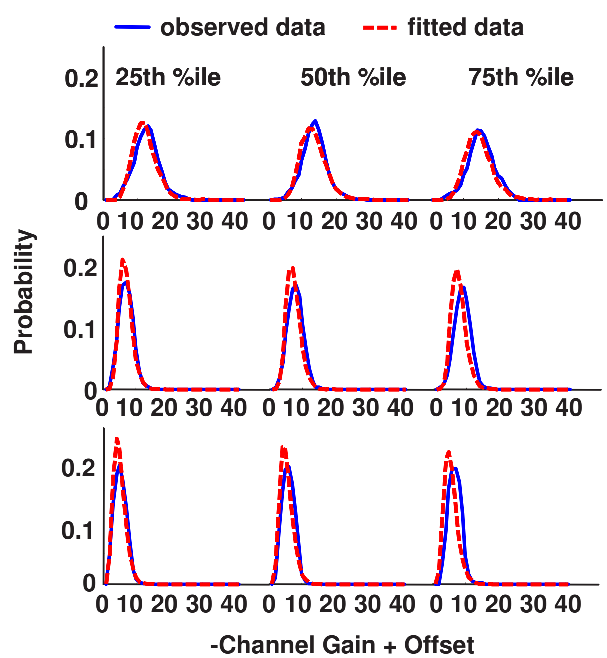

Fig. 6. Transition Probabilities. The upper plots show the transition probabilities from the high density pedestrian scenario with a low antenna height, while lower set of plots show the transitions probabilities from the lower pedestrian density with the elevated antenna. In each case three transition probabilities are shown corresponding to three different initial conditions (x_0), namely, x_0 is the 25th percentile, the 50th percentile, and the 75th percentile.

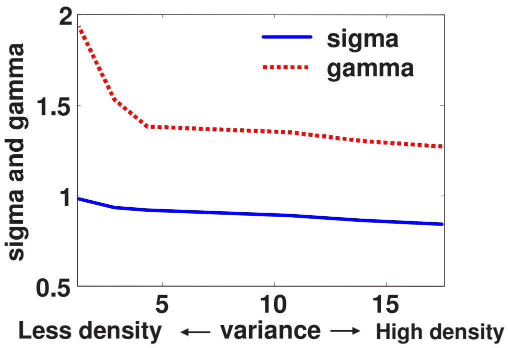

Fig. 7. Relationship between variance and the model parameters γ and σ.

investigate these probabilities for the simple reason that the combination of the stationary probability distribution and transition probability completely characterizes a stochastic process. Hence, if the transition probabilities observed match those derived from the diffusion process given by (1) and (2), then the diffusion model can be used to generate synthetic channel gains that realistically mimic the channel gains that arise in crowded areas.

Following the approach discussed in Section III-A, the parameters γ and σ were found. These parameters were used to numerically determine the transition probabilities. Figure 6 shows several observed and fitted transition probabilities for the high pedestrian density and low antenna scenario and the low pedestrian density and high antenna scenario. The other scenarios 4 resulted in similar quality of fit.

In Section IV-A it was shown that the mean and variance approximately obey an affine function. Here we determine if any simple relationship exists for the parameters $\gamma$and σ. To this end, Figure 7 shows the relationship between the variance of the channel gain and these parameters. Besides the low pedestrian density scenario, both γ and σ follow an affine relationship with the channel gain variance. While more work is required to understand the low pedestrian density case, it appears that if the variance is known, then γ and σ can be found.

V. Conclusions

While the variability of the channel is often attributed to the mobility of the receiver or transmitter, it is also possible that objects in the environment are partly responsible for the channel variability. This paper verifies that hypothesis. Furthermore, a four-parameter, diffusion process is found to approximate the observed variability of the channels. Finally, it was found that the model parameters are closely related to one another and appear to follow a deterministic relationship. Thus, the only independent variable is the variance of the channel gain, which seems to be highly correlated to the pedestrian density. However, further work is required to determine the relationship between pedestrian density and the variance.

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