# 1 Homework 1

1. In matlab, make a 3-D mesh plot of $w_1 \log\left(1 + \frac{H_{1,1}P_1}{H_{2,1}P_2+N}\right) + w_2 \log\left(1 + \frac{H_{2,2}P_2}{H_{1,2}P_1+N}\right)$ as a function of $P_1$ and $P_2$. Pick $H = \begin{pmatrix} 10^{-2} & 10^{-4} \\ 10^{-3} & 10^{-4} \end{pmatrix}$.

   (a) Open Matlab. This should open the command window and an editor. If no editor apears, type edit at the command prompt in the command editor. Enter the text below in the editor. Save the file to ShowUtility.m. Go to the command window and type ShowUtility.m. Or, after the text is entered into the editor and saved, press F5.

```
H = [1e-2, 1e-4 ; 1e-3, 1e-4]; % define H
P = [0 :  .01 :  1]; % Make vector of powers
NOISE = 1e-7;  % Set Constants
w = [1 1];  % set weighting
P1s = [ ]; % clear matrices
P2s = [ ];
U = [ ];
%Make matrix of throughputs
for i = 1 :  length(P)
    for j = 1 : length(P)
        P1s(i,j) = P(i);
        P2s(i,j) = P(j);
        U(i,j) = w(1) * log2(1 + H(1,1)*P(i) / (H(2,1)*P(j) + NOISE)) + w(2) * log2(1 +
H(2,2)*P(j) / (H(1,2)*P(i) + NOISE));
    end
end
%Make mesh plot
mesh(P1s,P2s,U)
xlabel('Power on link 1')
ylabel('Power on link 2')
zlabel('Utility');
```

   (b) Make a moive of the above as w(1) goes from 0 to 1. Modify the code above:

```
H = [1e-2, 1e-4 ; 1e-3, 1e-4]; % define H
P = [0 :  .01 :  1]; % Make vector of powers
NOISE = 1e-7;  % Set Constants
w = [1 1];  % set weighting
for w1 = 0:.01:1
    w(1) = w1;
    P1s = [ ]; % clear matrices
    P2s = [ ];
    U = [ ];
    %Make matrix of throughputs
    for i = 1 :  length(P)
        for j = 1 : length(P)
            P1s(i,j) = P(i);
            P2s(i,j) = P(j);
            U(i,j) = w(1) * log2(1 + H(1,1)*P(i) / (H(2,1)*P(j) + NOISE)) + w(2) * log2(1 +
H(2,2)*P(j) / (H(1,2)*P(i) + NOISE));
```

```
            end
        end
        %Make mesh plot
        mesh(P1s,P2s,U)
        xlabel('Power on link 1')
        ylabel('Power on link 2')
        zlabel('Utility');
        getframe;
    end
```

(c)
```
    H = [1e-2, 1e-4 ; 1e-3, 1e-4]; % define H
    P = [0 :  .01 :  1]; % Make vector of powers
    NOISE = 1e-7;  % Set Constants
    w = [1 1];  % set weighting
    for HI = -7:.1:-2
        H(1,2) = 10^HI;
        H(2,1) = H(1,2);
        P1s = [ ]; % clear matrices
        P2s = [ ];
        U = [ ];
        %Make matrix of throughputs
        for i = 1 :  length(P)
            for j = 1 : length(P)
                P1s(i,j) = P(i);
                P2s(i,j) = P(j);
                U(i,j) = w(1) * log2(1 + H(1,1)*P(i) / (H(2,1)*P(j) + NOISE)) + w(2) * log2(1 +
    H(2,2)*P(j) / (H(1,2)*P(i) + NOISE));
            end
        end
        %Make mesh plot
        mesh(P1s,P2s,U)
        xlabel('Power on link 1')
        ylabel('Power on link 2')
        zlabel('Utility');
        getframe;
    end
```

2.

$$\begin{aligned} &\max x_1 + x_2 + x_3 \\ &\text{subject to: } x_1 + 2x_2 - x_3 \geq 0 \\ &\qquad\qquad\quad |x_1| \geq 1 \\ &\qquad\qquad\quad x \geq 0 \end{aligned}$$

a linear programming problem?

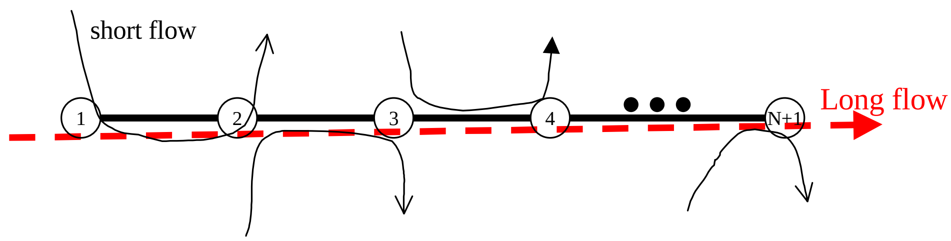


Figure 1:

3. Consider the goal of proportional fairness. And consider the topology show in Figure 1. We can guess that the long flow has rate $r$ and that all the other short flows have the same rate, which we denote $Mr$. We want to determine the value of $r$ and $M$. Suppose use the defintion of proportional fairness to evaluate the quality of the flows where the long flow has rate $r - \varepsilon$ and the short flows each have rate $Mr + \varepsilon$. Pick $M$ so that the test for whether a set of flow rates is proportional fair is always zero, and then determine the optimal flow rate.

4. Gradient

Let $f(x,y) = x^3y^2$.

(a) Find the gradient at 1,1.

(b) Use the gradient to estimate $f(1+h,1)$. (i.e., $f(1+h,1) \approx f(1,1) + \nabla f(1,1) \cdot [1,0] \times h$

(c) Use the gradient to estimate $f(1,1+h)$.

(d) In Matlab, plot $f(1+h,1)$ and the estimate of $f(1+h,1)$ using the gradient.

Answer:

```
GRAD = [??  ??]; % you must enter a a vector here (two numbers)
F=[ ];
L = [ ];
H = [ -1 :  .01 :  1];
for h = H
    F = [F; (1+h)^3*1^2];
    L = [L; 1 + GRAD * [1, 0]'*h];
end
plot(H,F)
hold on
plot(H,L,'r')
hold off
legend('Function', 'Linear Approximation')
xlabel('h')
```

(e) Do the same as above, but in the direction [0,1]. and Then in the direction [1 1]

(f) By direct calculation, take the derivative of $f(1+h,1)$, the derivative of $f(1,1+h)$, and $f(1+h,1+h)$ with respect to $h$. Now use the gradient to compute these derivatives.

(g) Plot the linear approximation of $f(x,y) = x^4 + y^4$ at $(1,1)$.

i. 
```
Y=[];
X=[];
F = [];
L = [];
H = [-1:.1:1];
for i = 1 : length(H)
    for j = 1 : length(H)
        Y(i,j) = 1 + H(j);
        X(i,j) = 1 + H(i);
        F(i,j) = X(i,j)^4 + Y(i,j)^4;
        L(i,j) = 2 + GRAD * [H(i) H(j)]';
    end
end
mesh(X,Y,F)
hold on
surf(X,Y,L)
hold off
```

Rotate the view so that the function and linear approximation are viewable. Is this function comvex at $(1,1)$

(h) Do the same thing as above for the function $f(x,y) = x^3y^2$. If this function convex?

5. Nondifferentialable functions and the gradient

Define $f(x,y) = \left(x^{1/3} + y^{1/3}\right)^3$. Show that $\frac{\partial f}{\partial x}(0,0) = 1$ and $\frac{\partial f}{\partial x}(0,0) = 1$. Now find the derivative in the [1,1] direction. To do this, set $x = t$ and $y = t$. So $f(x,y)$ become $f(t) = \left(t^{1/3} + t^{1/3}\right)^3$. Take the derivative of this function with respect to $t$. What is wrong?

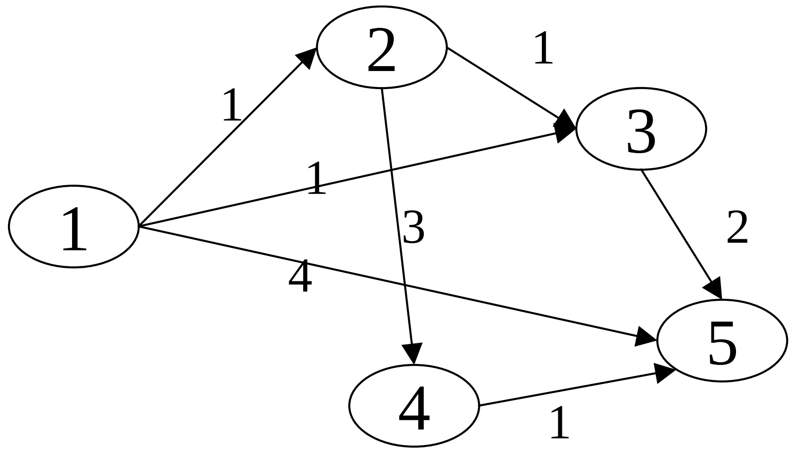


Figure 2:

6. Max-Flow

   Consider the graph in Figure 2. The souce is 1 and destination is 5. The link capacities are as shown. Make the incidence matrix (i.e, the A matrix), and the $E$ matrix so that the conservation conditions are $\begin{bmatrix} A & E \end{bmatrix} \begin{bmatrix} X \\ f \end{bmatrix} = 0$. Now use the Matlab's x = linprog(f,A,b,Aeq,beq,lb,ub); to find the link rates and the total flow

7. Min-Cut

   Show that the min-cut problem given in class can be rewritten as

$$\begin{aligned}
&\min K^T w \\
&\begin{bmatrix} A^T & -I \end{bmatrix} \begin{bmatrix} z \\ w \end{bmatrix} \leq 0 \\
&w \geq 0 \\
&\text{no lower bound on } z
\end{aligned}$$

   where $I = eye\,(size(A,1))$ ; and $K$ are the vector of capacities.

8. Redefine the max-flow problem with the capacity from node 1 to 5 being $4 + h$ instead of 4. Solve the max-flow problem for a set of $h$ with $h$ going from $-1$ to 1 in steps of 0.1. Make a plot of the value of the solution versus $h$. What is the slope of this curve. How does this compare to the an element of the optimal $w$?

9. Considering problem 6 and 7, check that the complementasry sensitivty condtions hold for the max-flow and min-cut.