Selective Relaying in Cooperative OFDM Systems: Two-Hop Random Network

Bo Gui, *Student Member, IEEE*, Lin Dai, *Member, IEEE*, and Leonard J. Cimini, Jr., *Fellow, IEEE*

Abstract

In this letter, we investigate two selective relaying schemes in cooperative OFDM systems. Selective OFDMA relaying, where the relay selection is performed in a per-subcarrier manner, and selective OFDM relaying, where one best relay among the L potential relays is selected to relay the entire OFDM block, are compared in a two-hop random network. The outage performance of Equal Bit Allocation (EBA), where each subchannel has the same number of bits, and Bit Loading (BL), where bits are adaptively allocated to each subchannel, are analyzed and compared for these two approaches. The outage analysis clearly shows that a significant performance gain can be achieved by selective OFDMA relaying, whether EBA or BL is employed, compared with selective OFDM relaying. With EBA, the performance gain increases with an increase in L, the number of relay nodes, and N, the number of independent subchannels. For BL, the performance gain further increases with an increase in R, the average number of bits per subchannel. Simulation results verify the outage analysis of these two schemes. Centralized and decentralized implementation issues are also considered.

I. INTRODUCTION

With the increasing needs for high speed wireless applications, future networks, no matter infrastructure-based or ad hoc, will be required to provide reliable high data-rate services in dynamic environments. The use of multiple antennas, which can significantly improve the power

---

This paper was presented in part at the IEEE WCNC 2007, Hong Kong, Mar. 2007.

B. Gui and L. J. Cimini are with Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716 USA. L. Dai is with Department of Electronic Engineering, City University of Hong Kong, Hong Kong.

This material is based on research sponsored by the Air Force Research Laboratory, under agreement number FA9550-06-1-0077. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon.

and spectral efficiency in single-link wireless communications, may be impractical in many instances due to limitations on the size and power of communications devices. Cooperative transmission, which utilizes the broadcast nature of the wireless medium and the numerous nodes in a network, is an efficient way to realize the benefits of multi-antenna transmission with only one antenna at each node (for example, see [1] and references therein).

Although there has been a significant effort on the study of cooperative systems, very little work has been done on the use of Orthogonal Frequency Division Multiplexing (OFDM) in these networks. In most of the work in this area, OFDM is simply the underlying transmission technology. How to use OFDM to facilitate relaying in a multihop network is still an open issue.

With L relays located between the source and the destination, the maximum diversity gain provided by the cooperation among these relays is L-fold. To achieve this full spatial diversity, selective relaying, i.e., only selecting the ”best” relay for forwarding the signal, is a good candidate which requires minimum cooperation among the relays and which can be performed in a distributed way [2]. In this paper, two selective relaying schemes, selective OFDMA relaying and selective OFDM relaying, are proposed to achieve the full cooperative diversity gain. Moreover, by using bit loading, where bits are adaptively allocated to each subchannel, frequency diversity can also be exploited.

In selective OFDMA relaying, relay selection is performed on a per-subchannel basis. In this way, different subchannels might traverse different paths. In selective OFDM relaying, however, only one “best” relay is selected to re-transmit the entire OFDM block. The end-to-end outage performance of these two relaying schemes are evaluated and compared in this paper. A two-hop *random* network including the effects of path loss is considered.

Two transmission schemes, Equal Bit Allocation (EBA), where each subchannel has the same number of bits to transmit, and Bit Loading (BL), where bits are adaptively allocated to each subchannel, are addressed. It is proved that, although selective OFDMA relaying and selective OFDM relaying achieve the same diversity gain, the power gain is different. The performance improvement of selective OFDMA relaying with EBA over selective OFDM relaying with EBA increases with an increase in L, the number of relay nodes, and N, the number of independent subchannels. For BL, an increase in R, the average number of transmission bits per subchannel, will also increase the performance improvement. We also show that the locations of the relay nodes do not affect the relative performance gap of these two approaches. Simulation results validate the analysis. Practical issues, such as centralized and decentralized implementations and complexity issues, are also addressed.

The paper is organized as follows. In Section II, we provide the system model and introduce selective OFDMA relaying. Sections III and IV present the outage analysis for selective OFDMA relaying and selective OFDM relaying with EBA, and with BL ,respectively. Finally, Section V summarizes and concludes the paper.

II. SYSTEM MODEL AND SELECTIVE RELAYING

We consider a single source(S)-destination(D) cooperative system with L relay nodes, as shown in Fig. 1. The relays are randomly located between the source and the destination. An OFDM transceiver with N subchannels is available at each node. We assume perfect time and frequency synchronization among nodes and the inclusion of a cyclic prefix that is long enough to accommodate the channel delay spread.

A two-stage transmission protocol is adopted. In the first stage, the source transmits and the relay nodes listen - the links in this stage are called the source-relay (SR) links. In the second stage, the relays retransmit the message to the destination - the links in this stage are called the relay-destination (RD) links. Here, we adopt a *selective* decode-and-forward relaying strategy. Compared to the other relaying strategies, selective relaying requires the least amount of signaling and can be performed in a distributed way [2]. With selective relaying, each source subchannel can only be relayed by one relay node. The selected relay node will fully decode the received information, re-encode it, and then forward it to the destination. In the RD links, a specific subchannel can only be used by a single node.

We assume that the total required data rate is R_{total} bits per OFDM symbol (block). On average, each subchannel will transmit $R = R_{total}/N$ bits. Further, denote the channel response of subchannel n from the source node to relay node i and from relay node i to the destination node as $H_{sr_i}(n)$ and $H_{r_id}(n)$, respectively. In general, these include path loss, shadowing, and Rayleigh fading. For convenience, let $G_{sr_i}(n)$ and $G_{r_id}(n)$ denote the channel power gains, $\|H_{sr_i}(n)\|^2$ and $\|H_{r_id}(n)\|^2$, respectively. Also, let $G_{sr_id}(n)$ refer to $\min\{G_{sr_i}(n), G_{r_id}(n)\}$.

Two transmission schemes are considered in this paper. In EBA transmission, the same number of bits is allocated to each subchannel regardless of the subchannel gains. In an OFDM system, however, the frequency diversity can be exploited by BL. In particular, more bits are placed in subchannels with larger channel gains, while subchannels which are faded carry less or even no bits. Hence, we also consider BL transmission to exploit the frequency diversity.

In previous work, OFDM is simply adopted as a physical-layer technique to overcome the frequency-selective fading in the network. As shown in Fig. 1(a), one relay is selected to forward the entire OFDM block so that all the subchannels traverse the same path. In particular, for EBA, the relay with the largest $\min_{n=1,\ldots,N} G_{sr_id}(n)$ is selected. For BL, the relay with the largest sum rate is selected. This type of relaying is referred to as *selective OFDM relaying*. Here, we propose *selective OFDMA relaying*; in this case, the relay selection is performed in a per-subcarrier manner. In particular, for subchannel n, the relay with the largest $\min_{i=1,\ldots,L} G_{sr_id}(n)$ is selected. Different relays might be selected to retransmit on different subchannels. As shown in Fig. 1(b), subchannels 1 and 2 are retransmitted by relay 3, while subchannel 3 is retransmitted by relay 2. At the destination, all the subchannels are collected. In this paper, we compare the performance of these two relaying strategies with EBA and BL.

The following assumptions are made:

A1) The destination node only uses the signal transmitted in the second stage.

A2) $G_{sr_i}(n)$ and $G_{r_id}(n)$ are independent exponential random variables with mean $1/\lambda_{sr_i}$ and $1/\lambda_{r_id}$, respectively. Also, let $\lambda_i = \lambda_{sr_i} + \lambda_{r_id}$.

A3) A high-SNR condition is assumed in deriving the diversity gain and power gain.

A4) Equal transmit power is allocated on each subchannel.

A1 allows us to concentrate on the comparisons of these two relaying schemes, and the results developed here can be easily extended to the case where the source-destination link is considered. Equal transmit power in each subchannel is a good choice for EBA because the channel gains are not available at the transmitter. For BL, at high SNR, varying the amount of transmit power as a function of the channel state yields minimal gains [3]; hence, A4 is a reasonable assumption.

III. OUTAGE ANALYSIS FOR EBA

In this section, we will evaluate the end-to-end outage performance, using EBA, of selective OFDM relaying and selective OFDMA relaying. In this case, each subchannel has the same number of bits, R. An outage occurs when at least one subchannel cannot successfully support the end-to-end transmission of the R bits.

Since relay selection is performed independently for each subchannel and channel gains are independent (A2), the end-to-end outage of selective OFDMA relaying with EBA is given by

$$P_{out,EBA}^{OFDMA} = 1 - \prod_{n=1}^{N} \left(1 - P_{out,EBA}^{OFDMA}(n)\right) \approx \sum_{n=1}^{N} P_{out,EBA}^{OFDMA}(n) \qquad (1)$$

The final approximation comes from the fact that $P_{out,EBA}^{OFDMA}(n)P_{out,EBA}^{OFDMA}(m)$, $n \neq m$, are small compared to $P_{out,EBA}^{OFDMA}(n)$, and, to first-order, we ignore them. $P_{out,EBA}^{OFDMA}(n)$ is the outage probability of subchannel n, and is given as

$$P_{out,EBA}^{OFDMA}(n) = \Pr\left[\frac{1}{2}\log(1 + \max_{i=1,\ldots,L} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < R\right] \qquad (2)$$

where the logarithms are base-2 unless otherwise noted, Γ is the SNR gap [4] determined by coding techniques, and γ is the SNR in each subchannel without fading. With perfect coding, $\Gamma = 1$, and $\Gamma = 8.8$ dB without any coding [4].

Theorem 1. For high SNR, the end-to-end outage of selective OFDMA relaying with EBA transmission is given by

$$P_{out,EBA}^{OFDMA} \approx \prod_{i=1}^{L} \lambda_i N \left(2^{2R} - 1\right)^{L} \left(\frac{\Gamma}{\gamma}\right)^{L} \qquad (3)$$

Proof: Rewrite (2) as

$$P_{out,EBA}^{OFDMA}(n) = \Pr\left[\max_{i=1,\ldots,L} G_{sr_id}(n) < \frac{(2^{2R}-1)\Gamma}{\gamma}\right] = \prod_{i=1}^{L} \Pr\left[G_{sr_id}(n) < \frac{(2^{2R}-1)\Gamma}{\gamma}\right] \qquad (4)$$

The last step results from A2. We also know that $G_{sr_id}(n)$ is an exponential random variable with parameter $\lambda_i = \lambda_{sr_i} + \lambda_{r_id}$ [5]. Then

$$\Pr\left[G_{sr_id}(n) < \frac{(2^{2R}-1)\Gamma}{\gamma}\right] = 1 - \exp\left(-\lambda_i \frac{(2^{2R}-1)\Gamma}{\gamma}\right) \approx \lambda_i \frac{(2^{2R}-1)\Gamma}{\gamma} \qquad (5)$$

The last step is a high-SNR approximation. Substituting (4) and (5) into (1), we obtain (3). ■

From Theorem 1, we can see that selective OFDMA relaying with EBA can achieve the full spatial diversity gain, i.e., L-fold. Also, λ_i varies with different relay locations, and, so, the outage probability will vary with the locations of the relays. It can be easily proven that the outage probability is minimized when the L relay nodes are located in the middle of the source-to-destination path. In particular, each relay is equidistance from the source and the destination. The L relay nodes form a relay cluster, and we assume that the distance between the source or the destination and the relay nodes is much larger than the distance between any two relay nodes. Also, the distance between any two relay nodes is sufficiently large so that the channel gains of different relay nodes are independent. Therefore, $G_{sr_i}(n)$ and $G_{r_id}(n)$ are i.i.d. random variables.

For selective OFDM relaying with EBA, one relay is selected to retransmit the entire OFDM symbol. Under A2, the end-to-end outage for this case is given by

$$P_{out,EBA}^{OFDM} = \prod_{i=1}^{L} P_{out,EBA,r_i}^{OFDM} \tag{6}$$

where P_{out,EBA,r_i}^{OFDM} is the end-to-end outage probability through relay i, and is given as

$$P_{out,EBA,r_i}^{OFDM} = 1 - \prod_{n=1}^{N} \left(1 - \Pr\left[\frac{1}{2}\log(1 + G_{sr_id}(n)\frac{\gamma}{\Gamma}) < R\right]\right) \tag{7}$$

Theorem 2. For high SNR, the end-to-end outage of selective OFDM relaying with EBA transmission is given by

$$P_{out,EBA}^{OFDM} \approx \prod_{i=1}^{L} \lambda_i N^L \left(2^{2R} - 1\right)^L \left(\frac{\Gamma}{\gamma}\right)^L \tag{8}$$

Proof: Rewrite (7) as

$$P_{out,EBA,r_i}^{OFDM} = \Pr\left[\frac{1}{2}\log(1 + \min_{n=1,..,N} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < R\right] \tag{9}$$

From A2, we know that $\min_{n=1,..,N} G_{sr_id}(n)$ is exponentially distributed with parameter $N\lambda_i$. Hence, for high SNR,

$$P_{out,EBA,r_i}^{OFDM} \approx N\lambda_i \frac{(2^{2R} - 1)\Gamma}{\gamma} \tag{10}$$

Eq. (8) follows easily by substituting (10) into (6). ■

As for selective OFDMA relaying with EBA, the same L-fold diversity gain can be achieved. The power gain, however, is different. In [6], we showed that no diversity gain can be obtained for selective OFDM relaying with EBA if the relay node with the highest combined SNR is chosen. Here, the relay with the largest minimum SNR among the N subchannels is chosen, and L-fold diversity gain can be achieved. Comparing (3) and (8), we see that the relative performance improvement of OFDMA relaying over OFDM relaying is

$$\Delta\gamma_{EBA} = 10\frac{L-1}{L}\log_{10} N \tag{11}$$

Thus, using EBA, selective OFDMA relaying is always preferred. This improvement is not a function of the location of the relays or the data rate R. In addition, it increases with an increase in L, the number of relays, and N, the number of subchannels.

Fig. 2 illustrates the performance improvement of OFDMA relaying over OFDM relaying. In the simulation, we assume the channels between the source and each relay and between each relay and the destination are independent. A perfect match can be observed between the theoretical and simulation results. As expected, the performance gap increases with an increase in L, the number of relays, and N, the number of subchannels. An almost 6-dB performance improvement can be achieved by using OFDMA relaying with EBA when there are $L = 4$ relays and $N = 6$ subchannels.

Using EBA, both OFDMA relaying and OFDM relaying can be implemented in a decentralized way [2]. Each relay only needs to know channel gains of its own $2N$ subchannels. For selective OFDM relaying with EBA, each relay first finds the $\min\limits_{n=1,\ldots,N} G_{sr_id}(n)$, which requires $2N-1$ comparisons. Then, the relay with the largest $\min\limits_{n=1,\ldots,N} G_{sr_id}(n)$ will be self-elected using the timer scheme described in [2]. In the case of selective OFDMA relaying with EBA, however, each subchannel at each relay has a timer according to $G_{sr_id}(n)$, which would increase the selection delay N-fold. In addition, the comparisons among relays are implemented by the timers. Therefore, with a decentralized EBA, selective OFDMA relaying achieves better performance, but at the expense of a larger selection delay and less computational complexity. In Table 1, we summarize the implementation complexities.

IV. OUTAGE ANALYSIS FOR BL

We have shown above that, with EBA, selective OFDMA relaying and selective OFDM relaying both achieve L-fold diversity gain; however, only space diversity is exploited. In this section, we analyze and compare the outage of OFDMA relaying and OFDM relaying with bit loading (BL), where frequency diversity gain can also be achieved.

With BL, an outage occurs when the sum rate of all subchannels is less than NR bits. The end-to-end outage of selective OFDMA relaying with BL is

$$P_{out,BL}^{OFDMA} = \Pr\left[\sum_{n=1}^{N} \frac{1}{2}\log(1 + \max_{i=1,\ldots,L} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < NR\right] \tag{12}$$

The exact outage performance can be given in terms of the Meijer G-function [7]. However, we can not obtain any insight from resulting complicated close-form expression. Here, we focus on the high-SNR approximation to obtain some insight.

Theorem 3. For high SNR, the end-to-end outage of selective OFDMA relaying with BL transmission can be approximated as

$$P_{out,BL}^{OFDMA} \approx \prod_{i=1}^{L} \lambda_i^N 2^{2NRL} \left(L \ln 2\right)^{N-1} \times \left(\prod_{j=1}^{N-1} \frac{1}{j}(2NR)^{N-1}\right) \left(\frac{\Gamma}{\gamma}\right)^{-NL} \tag{13}$$

Proof: See Appendix I.

From Theorem 3, we can see that selective OFDMA relaying with BL can achieve full spatial and frequency diversity gain, i.e., NL-fold. This means that all of the possible diversity gains can be exploited. Also, as for EBA, the outage probability is minimized when the L relay nodes are located in the middle of the path from the source to the destination.

For selective OFDM relying with BL, one *best* relay is selected for retransmission. Under A2, the end-to-end outage of selective OFDM relaying with BL is

$$P_{out,BL}^{OFDM} = \prod_{i=1}^{L} \Pr\left[\sum_{n=1}^{N} \frac{1}{2} \log(1 + G_{sr_id}(n)\frac{\gamma}{\Gamma}) < NR\right] \tag{14}$$

Theorem 4. For high SNR, the end-to-end outage of selective OFDM relaying with BL transmission can be approximated as

$$P_{out,BL}^{OFDM} \approx \prod_{i=1}^{L} \lambda_i^N 2^{2NRL} \left(\ln 2\right)^{L(N-1)} \times \left(\prod_{j=1}^{N-1} \frac{1}{j}(2NR)^{N-1}\right)^{L} \left(\frac{\Gamma}{\gamma}\right)^{-NL} \tag{15}$$

Proof: : Letting $L = 1$ in Theorem 3, we get

$$\Pr\left[\sum_{n=1}^{N} \frac{1}{2} \log(1 + G_{sr_id}(n)\frac{\gamma}{\Gamma}) < NR\right] \approx \lambda_i^N 2^{2NR} \left(\ln 2\right)^{N-1} \left(\prod_{j=1}^{N-1} \frac{1}{j}(2NR)^{N-1}\right) \left(\frac{\Gamma}{\gamma}\right)^{N} \tag{16}$$

Substituting (16) into (14), gives (15). ■

Comparing (13) and (15), we see that the two selective relaying schemes both achieve the same NL-fold diversity gain, but with different power gains. The performance improvement of OFDMA relaying over OFDM relaying is

$$\Delta\gamma_{BL} = \frac{10(N-1)}{N} \log_{10}\left((CR)^{(L-1)/L} L^{-1/L}\right) \tag{17}$$

where $C = 2N\left(\ln 2\right)\left(\prod_{j=1}^{N-1} \frac{1}{j}\right)^{1/(N-1)}$

In Fig. 3, this performance improvement of OFDMA relaying over OFDM relaying is illustrated. The same simulation environment as in Section III is adopted. We see that the analysis is quite accurate, especially for a high data rate. For BL, as for EBA, the performance gain increases with an increase in L, the number of relay nodes. In addition, it also increases with an increase in R, the average number of bits per subchannel.

In the previous simulations, we assume that the channel gains of neighboring subchannnels are independent; this assumption, however, is not valid for realistic OFDM systems. Next, we relax this assumption. In particular, we consider a channel with an exponential power delay profile with a root-mean-square (RMS) delay spread $\tau_{rms} = \eta T$, where T is the time duration of one OFDM symbol (block), $T = NT_s$ and $0 < \eta \leq 0.1$. In the simulations, $N = 64$. The smaller RMS delay spread results correlations among neighboring subchannels. In the previous example, τ_{rms} was much larger than this (even though we assume no ISI). In the simulation, we use a discrete-time model with an impulse response limited to 16 samples spaced by T_s; this is sufficient to encompass all of the paths with significant energy for these values of rms delay spread. The total bit rate is 256 bits per OFDM symbol (block), i.e., $R = 4$ bits. BL, which can exploit the frequency diversity and which is interesting for practical systems, is adopted. The same network setup as the previous simulation is assumed. Four relay nodes are located in the middle of the source-to-destination path as discussed in Section III. From Fig. 4, we can see that the performance improvement of OFDMA relaying over OFDM relaying increases with an increase in the delay spread. This is because the number of independent subchannels also increases with the delay spread.

For selective OFDMA relaying with BL, a significant amount of communications overhead is required for the center controller to collect the channel gains of all $2NL$ SR and RD links. After collecting the required channel information, the center controller will perform bit loading and broadcast the decisions. For selective OFDM relaying with BL, however, each relay calculates the maximum rate based on its own $2N$ channel gains. That is, L times the complexity is required for selective OFDM relaying with BL. The relay with the largest rate is selected to relay all the subchannels. We see that, for BL, selective OFDMA relaying achieves a better performance at the expense of more communications overhead and less computational complexity. In Table 2, we summarize the implementation complexities.

V. CONCLUSIONS

In this paper, we analyzed and compared the outage performance of selective OFDMA relaying and selective OFDM relaying in a two-hop random network. We showed that the same diversity gain can be achieved by both relaying schemes; the power gains, however, are different. Simulation results validated the analysis and showed that superior performance can always be achieved by selective OFDMA relaying, and the performance improvement is not a function of the locations of the relays. This approach is preferred for centralized systems because of its good performance. For decentralized systems, selective OFDMA relaying works well with EBA; if BL is employed, selective OFDM relaying is a good choice because of its simpler implementation.

APPENDIX I

PROOF OF THEOREM 3

The proof of Theorem 3 requires the results of Theorem 1 in [8], which is rewritten here: *Theorem 1 in [8]*: Let u_s and v_s be two independent random variables with the property that

$$\lim_{s\to\infty} s\cdot\Pr[u_s<t]=f(t)$$

$$\lim_{s\to\infty} s^d\cdot\Pr[v_s<t]=g(t),$$

where $f(t)$ and $g(t)$ are monotonically increasing and integrable, and $f'(t)$ is integrable. Then

$$\lim_{s\to\infty} s^{d+1}\cdot\Pr[u_s+v_s<t]=\int_0^t g(t-x)f'(x)dx. \tag{18}$$

The end-to-end outage of OFDMA-relaying with BL is

$$P_{out,BL}^{OFDMA}=\Pr\left[\sum_{n=1}^{N}\frac{1}{2}\log(1+\max_{i=1,\dots,L}G_{sr_id}(n)\frac{\gamma}{\Gamma})<NR\right]\approx\Pr\left[\sum_{n=1}^{N}\frac{1}{2}\log(\max_{i=1,\dots,L}G_{sr_id}(n)\frac{\gamma}{\Gamma})<NR\right] \tag{19}$$

The last step comes from the fact that $\log_2(1+x)\approx\log_2 x$, when $x\gg 1$. Let $u_n=\log\left(\max_{i=1,\dots,L}G_{sr_id}(n)\frac{\gamma}{\Gamma}\right)$, then

$$\Pr[u_n<t]=\Pr\left[(\max_{i=1,\dots,L}G_{sr_id}(n)\frac{\gamma}{\Gamma})<2^t\right]=\prod_{i=1}^{L}\Pr\left[G_{sr_id}(n)<\frac{2^t\Gamma}{\gamma}\right] \tag{20}$$

We know that $G_{sr_id}(n)$ is an exponential random variable with parameter $\lambda_i=\lambda_{sr_i}+\lambda_{r_id}$ [5]. Then

$$\Pr[u_n<t]=\prod_{i=1}^{L}\left(1-\exp\left(-\lambda_i\frac{2^t\Gamma}{\gamma}\right)\right) \tag{21}$$

Therefore

$$\lim_{\gamma\to\infty}\gamma^L\cdot\Pr[u_n<t]=\left(2^t\Gamma\right)^L\prod_{i=1}^{L}\lambda_i \tag{22}$$

Let $g_1(t) = f(t) = \left(2^t\Gamma\right)^L \prod_{i=1}^{L} \lambda_i$ and $f'(t) = 2^{tL} L \Gamma^L \prod_{i=1}^{L} \lambda_i \ln 2$. Applying Th. 1 in [8],

$$\lim_{\gamma\to\infty} \gamma^{2L} \cdot \Pr[u_1 + u_2 < t] = \int_0^t g_1(t-x) f'(x) dx = \prod_{i=1}^{L} \lambda_i^2 2^{tL} \Gamma^{2L} \left(L \ln 2\right) t \tag{23}$$

Repeating the application of Th. 1 in [8], that is, let $g_2(t) = \prod\limits_{i=1}^{L} \lambda_i^2 2^{tL} \Gamma^{2L} \left(L \ln 2\right) t$ and $f'(t) = 2^{tL} L \Gamma^L \prod_{i=1}^{L} \lambda_i \ln 2$, then

$$\lim_{\gamma\to\infty} \gamma^{3L} \cdot \Pr\left[\sum_{n=1}^{3} u_n < t\right] = \prod_{i=1}^{L} \lambda_i^3 2^{tL} \Gamma^{3L} \left(L \ln 2\right)^2 \left(\frac{1}{2} t^2\right) = g_3(t) \tag{24}$$

In this way, we can easily prove the following by induction,

$$g_N(t) = \prod_{i=1}^{L} \lambda_i^N 2^{tL} \Gamma^{NL} \left(L \ln 2\right)^{N-1} \left(\prod_{j=1}^{N-1} \frac{1}{j} t^{N-1} \right). \tag{25}$$

Thus, for sufficiently high SNR,

$$\Pr\left[\sum_{n=1}^{N} u_n < t\right] = \prod_{i=1}^{L} \lambda_i^N 2^{tL} \left(L \ln 2\right)^{N-1} \left(\prod_{j=1}^{N-1} \frac{1}{j} t^{N-1} \right) \left(\frac{\Gamma}{\gamma}\right)^{NL} \tag{26}$$

With $t = 2NR$ and substituting (26) into (19), (13) is obtained.

REFERENCES

[1] F. H. P. Fitzek and M. D. Katz, *Cooperation in Wireless Networks: Principles and Applications*, Springer, 2006.

[2] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659-672, Mar. 2006.

[3] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, Cambridge University Press, 2005.

[4] Y. Li and G. L. Stuber, *Orthogonal Frequency Division Multiplexing for Wireless Communications*, Springer, 2006.

[5] S. M. Ross. *Introduction to Probability Models*, 9th edition, Academic Press, 2006.

[6] L. Dai, B. Gui, and L. J. Cimini, Jr., "Selective relaying in OFDM multihop cooperative networks," in *Proc. of IEEE WCNC 2007*, Hong Kong, Mar. 2007.

[7] J. Salo, H. M. E1-Sallabi, and P. Vainikainen, "The distribution of the product of independent Rayleigh random variables," *IEEE Tran. on Antennas and Propagation*, vol. 54, no. 2, pp. 639-643, Feb. 2006.

[8] J. N. Laneman, "Limiting analysis of outage probabilities for diversity schemes in fading channels," in *Proc. of IEEE Globecom*, vol. 3, pp. 1242-1246, Dec. 2003.

TABLE I

IMPLEMENTATION COMPLEXITY FOR EBA

Relaying Schemes	Number of Channel Gains	Number of Comparisons	Number of Relay Selection	Implementation Method
OFDM	2N subchannels	2LN-L	1	Decentralized
OFDMA	2N subchannels	LN	N	Decentralized

TABLE II

IMPLEMENTATION COMPLEXITY FOR BL

Relaying Schemes	Number of Channel Gains	Number of Performing Bit Loading	Implementation Method
OFDM	2N subchannels	L	Decentralized
OFDMA	2NL subchannels	1	Centralized

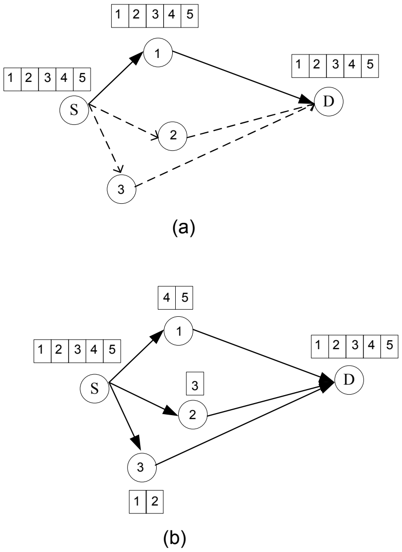

Fig. 1. Two-hop random network (a) Selective OFDM relaying. (b) Selective OFDMA relaying.

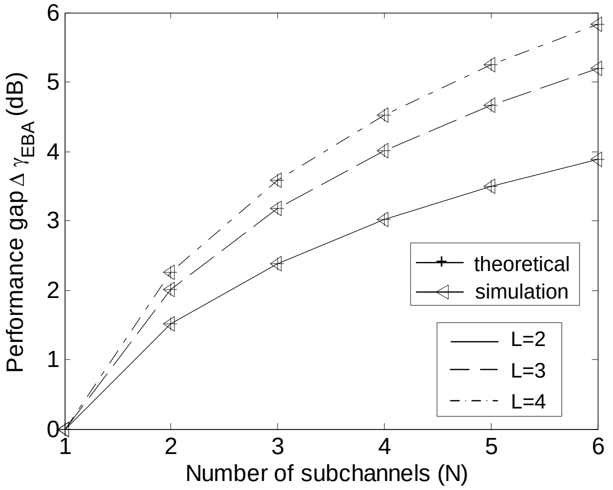

Fig. 2. Performance improvement of selective OFDMA relaying over selective OFDM relaying with EBA ($L = 2$, 3, and 4).

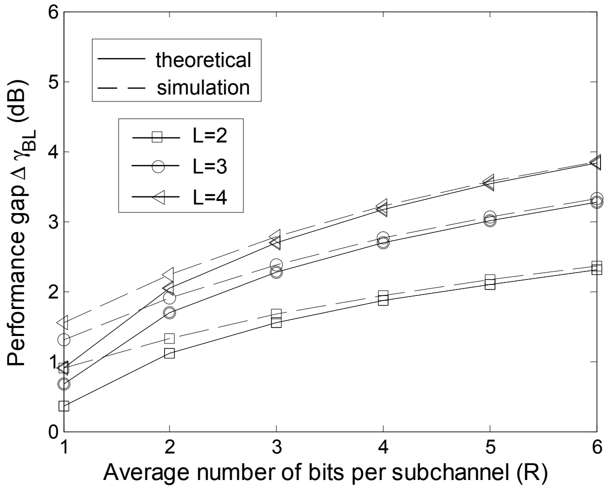

Fig. 3. Performance improvement of selective OFDMA relaying over selective OFDM relaying with BL ($L = 2$, 3, 4 and $N = 2$).

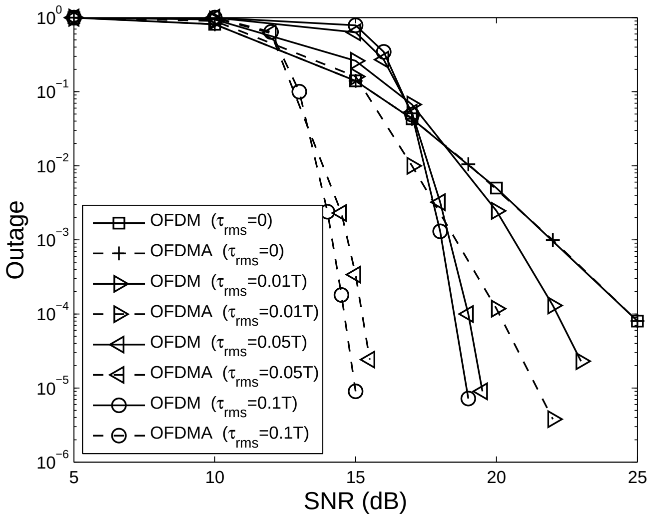

Fig. 4. Performance improvement of selective OFDMA relaying over selective OFDM relaying with BL for different delay spreads.