

Image Enhancement in the Spatial Domain

Image Processing with Biomedical Applications
ELEG-475/675
Prof. Barner

Spatial Domain Processing

- Utilize neighborhood operations

$$g(x,y) = T[f(x,y)]$$

- Simple case: point operations

$$s = T(r)$$

- Contrast stretching
- Thresholding

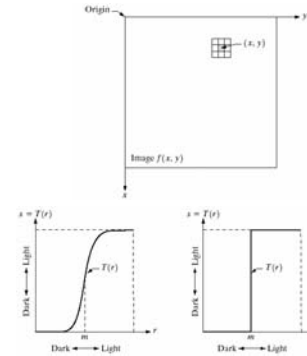


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Image Enhancement in the Spatial Domain

- Algorithms for improving the visual appearance of images
 - Gamma correction
 - Contrast improvements
 - Histogram equalization
 - Noise reduction
 - Image sharpening
- Optimality is (often) in the eye of the observer
 - Ad hoc
- Reading assignments: papers on WebCT

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Basic Gray Level Transformations

- Image negatives
 - Easier visualization of detail embedded in dark regions

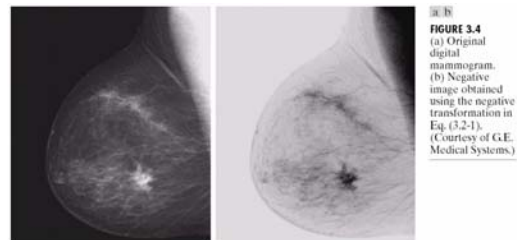


FIGURE 3.4
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)

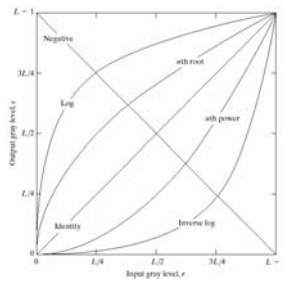
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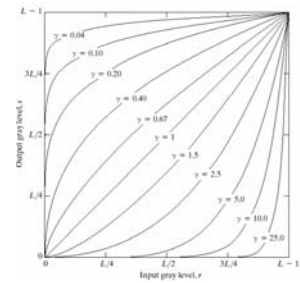
Gray Level Transformation Curves

- More general transformations
 - log
 - Inverse log
 - n 'th power
 - n 'th root
- Used to map narrow dark (log/root) or bright (inverse log/power) range to a greater dynamic range



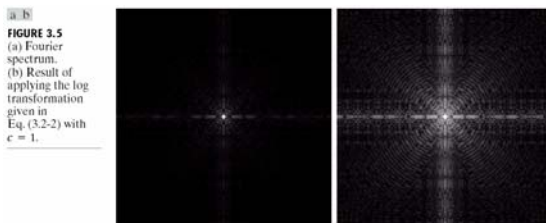
Power-Law (Gamma) Transformations

- Power-law transformation:
 $s = cr^\gamma$
- Many devices require gamma correction
 - CRTs have power function intensity-to-voltage responses
 - Monitor specific
 - Applied in color planes
 - Embedded in some image representations

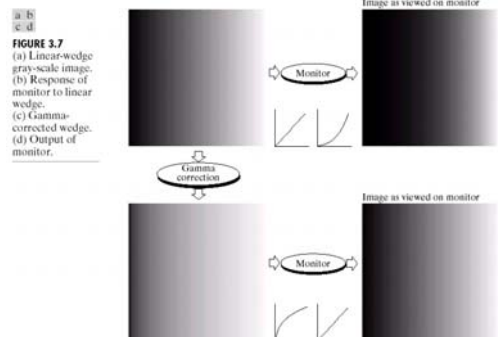


Fourier Spectrum Example

- The spectrum has a large dynamic range
 - Example below: 0 to 10^6
 - Compress range to view visually (dB's)



Gamma Correction Example – Monitor



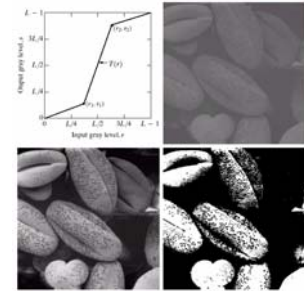
Gamma Correction Example – MR



FIGURE 3.8
(a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2.3) with $c = 1$ and $\gamma = 0.4, 0.4$, and 5.0 , respectively. (Original image for this example courtesy of Dr. David R. Pelkon, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Piecewise-Linear Transformations

- Contrast stretching
 - Poor illumination
 - Sensor dynamic range
 - Lens aperture settings
- Feature extraction
 - Medical imaging
 - Bone
 - Soft tissue
- Gray-level slicing
 - Only show range of interest



Gamma Correction Example – Aerial

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

(m)

(n)

(o)

(p)

(q)

(r)

(s)

(t)

(u)

(v)

(w)

(x)

(y)

(z)

(aa)

(ab)

(ac)

(ad)

(ae)

(af)

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Histogram Equalization

- We focus on the (normalized) scalar mapping

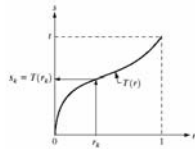
$$s = T(r) \quad 0 \leq r \leq 1$$

where the following are satisfied:

- $T(r)$ is single-valued and monotonically increasing in $[0,1]$
- $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$
- The single-valued condition allows the inverse transformation to be defined

$$r = T^{-1}(s) \quad 0 \leq s \leq 1$$

- The monotonically increasing condition prevents inversion



CDF Distribution

- Set $T(r) = P_r(r)$

$$\begin{aligned} \frac{ds}{dr} &= \frac{dT(r)}{dr} \\ &= \frac{d}{dr} \left[\int_0^r p_r(w) dw \right] \\ &= p_r(r) \end{aligned}$$

- Thus the PDF of s is

$$\begin{aligned} p_s(s) &= p_r(r) \left| \frac{dr}{ds} \right| \\ &= p_r(r) \left| \frac{1}{p_r(r)} \right| \\ &= 1 \end{aligned}$$

- The CDF is uniformly distributed
- Independent of the PDF

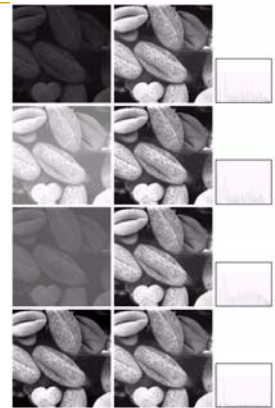
Probability Density Function

- Let the PDF of r be $p_r(r)$
- The CDF is: $P_r(r) = \int_0^r p_r(w) dw$
 - Note CDFs are monotonically increasing and have range $[0,1]$
- Defined the RV $s = T(r)$
 - The PDF of a RV function is

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

Histogram Equalization

- CDF mapping of Gray values
 - Yields uniform histogram
 - Simple, parameter-free
- Discrete case
 - Results not strictly uniform
 - Implementation issues



Histogram EQ Mappings

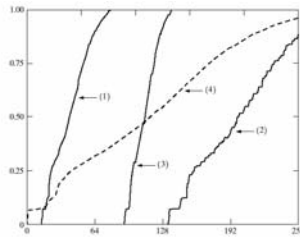


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Histogram Matching Mapping

a b c

FIGURE 3.19
(a) Graphical interpretation of mapping from r_k to s_k via $T(r)$.
(b) Mapping of z_q to its corresponding value v_q via $G(z)$.
(c) Inverse mapping from s_k to its corresponding value of z_k .

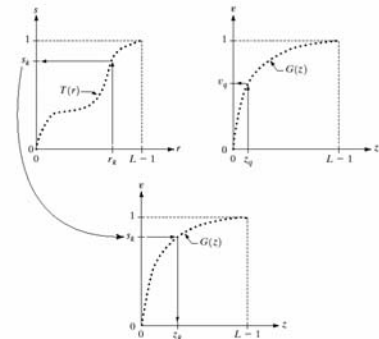


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Histogram Matching

- To control performance, map to a specified distribution
 - Desired PDF: $p_z(z)$, CDF: $P_z(z)$
 - Measured image PDF: $p_r(r)$, CDF: $P_r(r)$
- Set $u = G(z) = P_z(z)$ and $v = T(r) = P_r(r)$
 - Both uniformly distributed
- Set mapping as

$$z = G^{-1}[T(r)]$$

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Histogram Matching Example

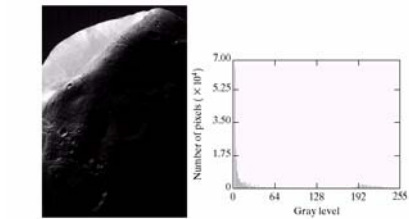


FIGURE 3.20 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

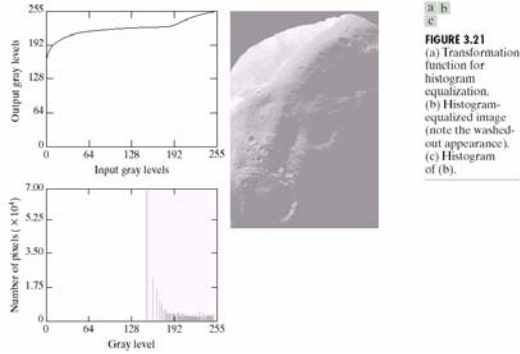
- Observation image has poorly distributed histogram
 - Concentration causes problems in standard histogram equalization

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Histogram Equalization Result



Local Enhancement

- Statistics are not uniform across an image
- Local statistic calculations yield spatially adaptive enhancement
 - Risk: nonuniform or blocky appearance

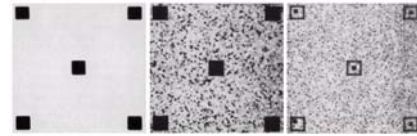


FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

Histogram Matching Result

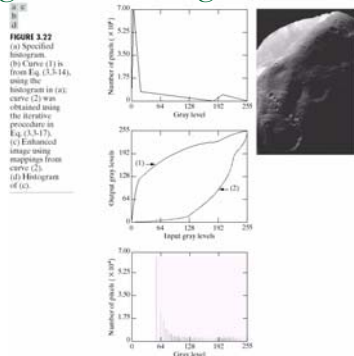


Image Statistics

- Gray-levels: $r_0, r_2, \dots, r_{L-1}$

■ Mean:

$$m = E[r] = \sum_{i=0}^{L-1} r_i p(r_i)$$

- n^{th} moment

$$\mu_n(r) = E[(r - m)^n] = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

- Assume statistics are stationary

- Local evaluations can be utilized
- Calculate over neighborhood

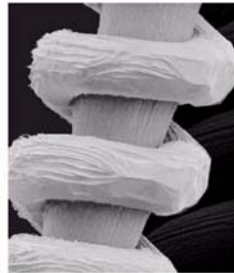
Heuristic Statistic-Based Enhancement

- Defined heuristics to enhance:

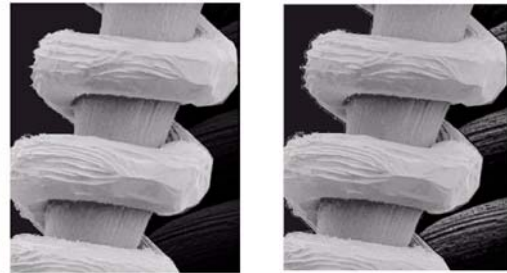
- Dark background object
- Edges and details

$$g(x, y) = \begin{cases} E \cdot f(x, y) & \text{if } m_{s_{xy}} \leq k_0 M_G \\ & \text{AND } k_1 D_G \leq \sigma_{s_{xy}} \leq k_2 D_G \\ f(x, y) & \text{otherwise} \end{cases}$$

- Note: image depended *ad hoc* procedure



Observation and Enhancement Results



Heuristic Enhancement Local Statistics and Mask

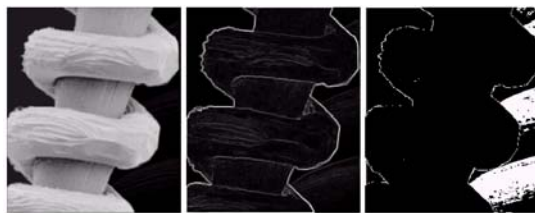
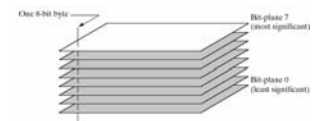


FIGURE 3.25

(a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

Image Subtraction

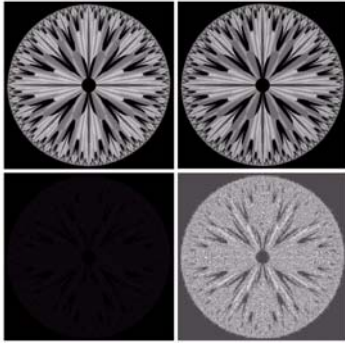
- Simple pixel-wise operation
 - Removes reference to show details
- Can be applied on bit-planes or independently captured reference
 - Histogram equalization improves detail visualization



Bit-Plane Application Example

FIGURE 3.28

(a) Original fractal image.
(b) Result of setting the four lower-order bit planes to zero.
(c) Difference between (a) and (b).
(d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Bonds, Swarthmore College, Swarthmore, PA).



Noise Reduction

- Simple observation model: $g=f+\eta$
- Reduce noise by averaging across (fixed) images
 - $\bar{g} = \frac{1}{K} \sum_{i=1}^K g_i$
- Note that
 - $E\{\bar{g}\} = f$
 - $\sigma_{\bar{g}}^2 = \frac{1}{K} \sigma_{\eta}^2$
- Images must be registered to avoid blurring

Mask Mode Radiography Example

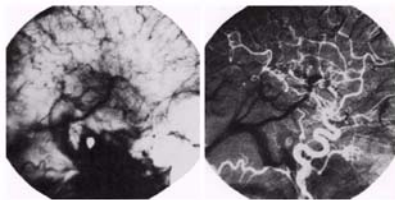
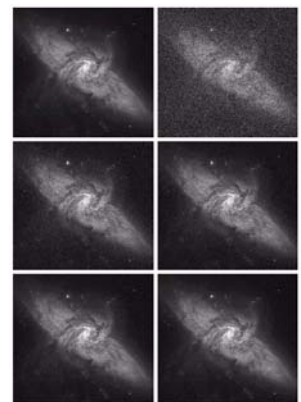


FIGURE 3.29
Enhancement by image subtraction.
(a) Mask image.
(b) An image taken after injection of a contrast medium into the bloodstream with mask subtracted out.

- Goal:
 - Observe arterial bloodstream pathways
- Procedure:
 - Reference (mask): x-ray image
 - Observation: x-ray image with contrast

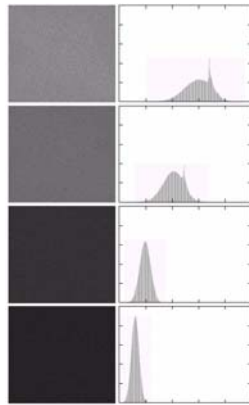
Astronomy Example

- Applicable for sensor noise
 - Additive Gaussian model
- Increasing the number of images averaged reduces the noise variance
 - Shown: $K=8, 16, 64$, and 128



Difference Images and Histograms

- Increasing K reduces the difference images
 - Mean
 - Variance
- Similar to that integrating characteristics of CCD sensors



Simple Smoothing Masks

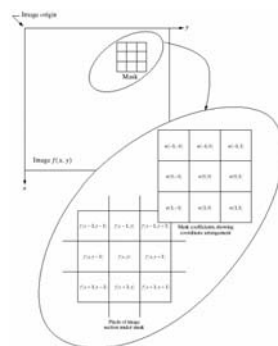
$\frac{1}{9} \times$	1	1	1
	1	1	1
	1	1	1

$\frac{1}{16} \times$	1	2	1
	2	4	2
	1	2	1

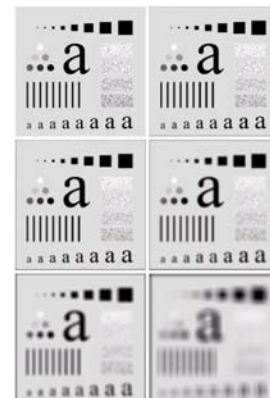
- Simplest a linear filter: spatial average
 - Reduces noise, but introduces blurring
- Distance weight samples
 - Centrally located samples are more important
 - Reduces blurring (somewhat)
 - Example above: simple integer arithmetic
- Alternative approach: utilize spectral characteristics

Spatial Filtering

- Spatial filtering is based on a moving window or mask
- In the linear filtering case:
 - $g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$
- More compactly:
 - $R = \sum_{i=1}^m w_i z_i$
 - Must take into account boarder affects



Smoothing Example



Noise Reduction and Object Detection Example

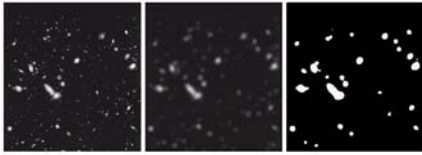


FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15×15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

- Hubble Space Telescope image
 - Averaging reduces noise
 - Thresholding localizes the most significant objects

Salt-and-Pepper Noise Example

- Bad sensors and bit errors yield salt-and-pepper noise
 - Heavy tailed noise distribution
 - Other examples: Laplacian, Cauchy, α -Stable
 - Probability of corruption: p
 - Outliers (generally) located in the extremes of the ordered set
 - Do not affect median

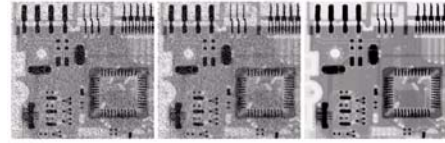


FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, LVA, Inc.)

Order-Statistic Filters

- Linear (weighted sum) filters
 - Blur edges and details
 - Are susceptible to outliers
- Order-statistic filters
 - Preserve edges and details
 - Are less susceptible to outliers
- Spatially ordered samples: $z_1, z_2, \dots, z_N$
- Rank ordered samples: $z_{(1)}, z_{(2)}, \dots, z_{(N)}$

$$z_{(1)} \leq z_{(2)} \leq \dots \leq z_{(N)}$$
- $\text{Med}[z_1, z_2, \dots, z_N] = z_{((N+1)/2)}$
 - Selection-type filter

Maximum Likelihood Estimation and the Generalized Gaussian Distribution

$$f(x) = \frac{k}{2\sigma^2 \Gamma(1/k)} \exp\left(-\frac{|x-\mu|}{\sigma}\right)^k,$$

where $\Gamma(x) = \int_0^\infty t^{x-1} \exp(-t) dt$, σ , and k denote the Gamma function, scale, and tail parameter respectively. The distribution tail decays slower as k decreases.

Special Cases:

➤ $k = 2$: The standard Gaussian distribution,

$$f_G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

➤ $k = 1$: The Laplacian distribution,

$$f_L(x) = \frac{1}{2\lambda} \exp\left(-\frac{|x-\mu|}{\lambda}\right).$$

Probability Density Function Plots

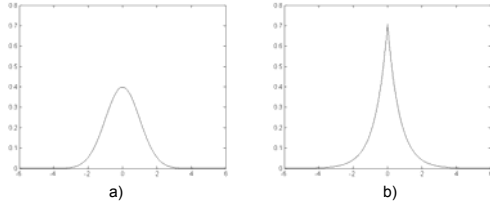


Figure: Probability Density Functions with unit variances: a) Gaussian density function and b) Laplacian density function.

- The Laplacian distribution has a lower tail decay rate
 - Greater probability of observing outliers

ML Estimate: Gaussian Case

- Under the i.i.d *Gaussian* distributed statistics assumption:

$$f(\mathbf{x} | \mu) = \prod_{i=1}^N \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma_i^2}}.$$

$$\begin{aligned} \ln(f(\mathbf{x} | \mu)) &= -N \ln(\sqrt{2\pi}) - \sum_{i=1}^N \ln(\sigma_i) - \sum_{i=1}^N \frac{(x_i - \mu)^2}{2\sigma_i^2}, \\ &= -N \ln(\sqrt{2\pi}) - \sum_{i=1}^N \ln(\sigma_i) - \sum_{i=1}^N \frac{x_i^2}{2\sigma_i^2} - \mu^2 \sum_{i=1}^N \frac{1}{2\sigma_i^2} + \mu \sum_{i=1}^N \frac{x_i}{\sigma_i^2}. \end{aligned}$$

$$\frac{\partial(\ln(f(\mathbf{x} | \mu)))}{\partial \mu} = -\mu \sum_{i=1}^N \frac{1}{\sigma_i^2} + \sum_{i=1}^N \frac{x_i}{\sigma_i^2} = 0. \quad \longrightarrow \quad \mu_{ML} = \frac{\sum_{i=1}^N \frac{1}{\sigma_i^2} x_i}{\sum_{i=1}^N \frac{1}{\sigma_i^2}},$$

which is a normalized version of a FIR filter output,

$$y = \sum_{i=1}^N w_i x_i,$$

with $w_i = 1/\sigma_i^2$.

Maximum Likelihood (ML) Estimate

- Consider a set of N observations forming a vector, $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$, we assume that the RVs come from a known density, $f(\cdot)$, that has unknown (but fixed) parameter μ . The ML estimate is given by,

$$\mu_{ML}(\mathbf{x}) = \arg \max_{\mu} f(\mathbf{x} | \mu),$$

where $f(\mathbf{x} | \mu) = f(x_1 | \mu) f(x_2 | \mu) \cdots f(x_N | \mu)$ is the Likelihood function. The ML estimate of μ is obtained as the solution to,

$$\left. \frac{\partial}{\partial \mu} f(\mathbf{x} | \mu) \right|_{\mu = \mu_{ML}} = 0,$$

or,

$$\left. \frac{\partial}{\partial \mu} \ln(f(\mathbf{x} | \mu)) \right|_{\mu = \mu_{ML}} = 0.$$

ML Estimate: Laplacian Case

- Under the i.i.d *Laplacian* distributed statistics assumption:

$$f(\mathbf{x} | \mu) = \prod_{i=1}^N \frac{1}{2\lambda_i} e^{-\frac{|x_i - \mu|}{\lambda_i}}.$$

$$\ln(f(\mathbf{x} | \mu)) = -\sum_{i=1}^N \ln(2\lambda_i) - \sum_{i=1}^N \frac{|x_i - \mu|}{\lambda_i}.$$

$$\frac{\partial(\ln(f(\mathbf{x} | \mu)))}{\partial \mu} = -\frac{\partial}{\partial \mu} \left(\sum_{i=1}^N \frac{|x_i - \mu|}{\lambda_i} \right) = 0. \quad \longrightarrow \quad \mu_{ML} = \text{MED} \left(\frac{1}{\lambda_i} \diamond x_i \right)_{i=1}^N,$$

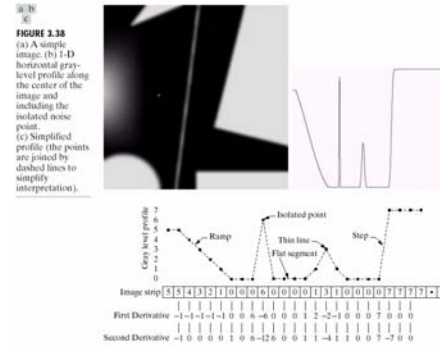
where $\diamond$ denotes the replication operator defined as $w_i \diamond x_i = \underbrace{x_i, x_i, \dots, x_i}_{w_i \text{ many}}$, with $w_i = 1/\lambda_i$. Hence, the weighted median filter output is,

$$y = \text{MED} \left(w_i \diamond x_i \right)_{i=1}^N.$$

Sharpening Filters

- Objective: Highlight and enhance fine detail
 - Details may have been blurred in acquisition process
- Method: utilize first- and second-order derivative
 - Derivatives identify signal changes (details/features)
- Derivative is not unique – impose requirements:
 - Must be zero in flat regions
 - Must be nonzero at the onset of ramps and steps
 - Must be nonzero along ramps
- Second-derivative requirements:
 - Must be zero in flat regions
 - Must be nonzero at the onset and end of ramps and steps
 - Must be zero along ramps of constant slope

Scan Line Derivative example



Derivatives

- Utilize difference equations:
 - First derivative:
 - $\frac{\partial f}{\partial x} = f(x+1) - f(x)$
 - Second derivative:
 - $\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$
- Extend to 2D

Derivative Observations

- First-order derivatives generate thick edges
- Second-order derivatives
 - Have stronger response to details
 - Produce a double response at step changes
 - Order of response strength
 - Point, line, step
- Second-order derivative is therefore preferred for enhancement
 - Use isotropic (rotation invariant) formulation

Laplacian

- Laplacian is the simplest isotropic derivative:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

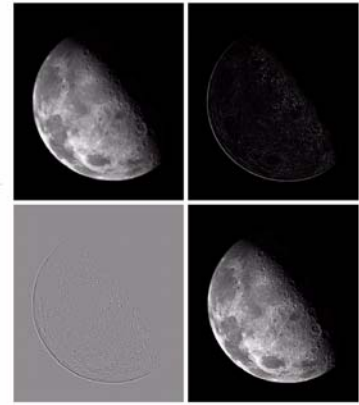
- Discrete realization:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

- Isotropic to 90° rotations
- Add diagonal derivatives to make it 45° isotropic

Example

FIGURE 3.40
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)



Mask Implementations & Enhancement

- Similar definition produces a sign change
- Enhancement adds (subtracts) derivative and observed image

0	1	0	1	1	1
1	-4	1	1	-4	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	4	-1
0	-1	0	-1	-1	-1

FIGURE 3.39
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).
(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive} \end{cases}$$

Composite Mask & Example

0	-1	0
-1	3	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

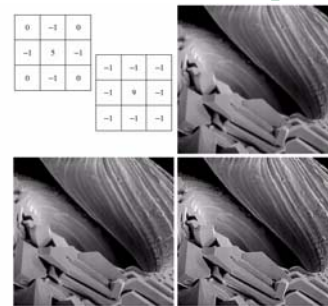


FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Unsharp Masking

■ Unsharp masking process:

- $f_s(x, y) = f(x, y) - \tilde{f}(x, y)$
- Originally a darkroom process combining a blurred negative and positive film

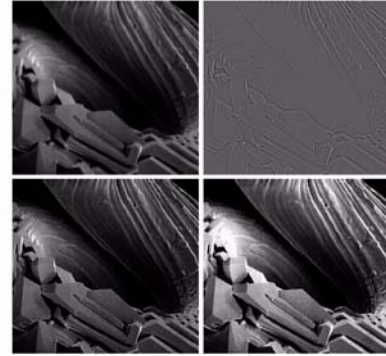
■ Generalization: high-boost filtering

- $f_{hb}(x, y) = Af(x, y) - \tilde{f}(x, y)$
- $f_{hb}(x, y) = (A-1)f(x, y) + f_s(x, y)$
- Using the Laplacian to obtain the sharp image $f_s(x, y)$
- $$f_{hb} = \begin{cases} Af(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ Af(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive} \end{cases}$$

High-Boost Example

(a) (b)
(c) (d)

FIGURE 3.43
(a) Same as Fig. 3.41(c), but darker.
(b) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$.
(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



Mask Implementation

0	-1	0	-1	-1	-1
-1	$A+4$	-1	-1	$A+8$	-1
0	-1	0	-1	-1	-1

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

- Standard Laplacian sharpening: $A=1$
- Increasing A reduces the level of sharpening
 - Scales (brightens) original image

First Derivatives

■ 2D first derivative:

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

■ Magnitude:

$$\nabla f = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

■ For computational simplicity:

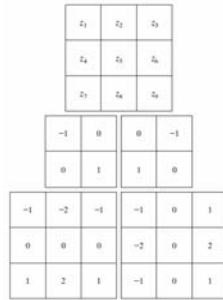
$$\nabla f \approx |G_x| + |G_y|$$

Sobel Operators

- 3x3 Sobel operator:

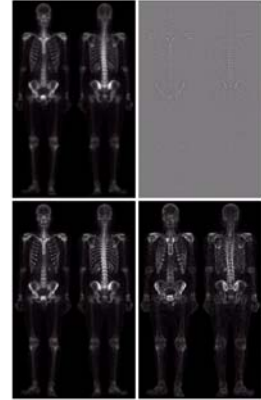
$$\nabla f \approx [(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)] \\ + [(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)]$$

- Utilizes x and y directional masks
- Distance weighting minimizes smoothing
- Extensions:
 - Increased window size
 - Diagonal directions



Composite Operations Example I

- Sharpen
 - Add original & Laplacian
- Identify edges
 - Sobel operator



Sobel Example

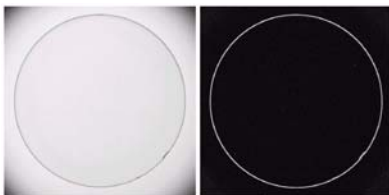


FIGURE 3.45
Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

- Common application: edge detection
 - Threshold Sobel output
 - Binary edge mask

Composite Operations Example II

- Thicken identified edges
 - Smooth Sobel output
- Form mask
 - Product of smoothed Sobel and sharpened image
- Sharpened image
 - Addition of original and mask
 - Only sharpen edges
- Final display
 - Apply power law transformation

