1. [30 pts] Consider the modulation of a message $m(t)$ with bandwidth 20KHz utilizing DSB-SC employing carrier frequency 400KHz. The transmission channel is corrupted by colored noise modeled as $S_N(f) = k_0 + k_1|f|$ Wats/Hz, where $k_0 = 10^{-6}$ and $k_1 = 10^{-12}$. What is the average noise power at the demodulator output? Assuming that the average message power is 2W and the modulating carrier has amplitude $A_c = 5$, determine the SNR at the output.

2. [30 pts] Consider a baseband binary modulation scheme that transmits the following waveforms:

message bit is a 1 message bit is a 0

Determine the matched filter for this system. Give an expression for the matched filter and plot its impulse response. Plot the response of the matched filter to the waveforms for a 1 and 0 message bit. What are the peak values at the sampling instant, T. What decision threshold should be used to make a decoder decision?

3. [40 pts] Consider a hybrid ASK–PSK modulation scheme where the transmitted signal is given by

$$s_{i,j}(t) = \sqrt{\frac{2E_0}{T}} a_i \cos\left(2\pi f_c + \frac{2\pi}{M}(j-1)\right) \qquad 0 \le t \le T$$

for $i = 1, 2, \ldots, K$ and $j = 1, 2, \ldots, M$. Also, it is assumed that $1 = a_1 < a_2 < \cdots < a_K$.

- What is the dimension of the signal space?
- Determine a set of orthonormal basis functions for the signal space and express $s_{i,j}(t)$ in terms of the basis functions. Give a general expression for the message point vector $\mathbf{s}_{i,j}$.
- Draw the signal space diagram for $K = 3$ and $M = 4$.
- Assuming all messages are equally likely, what is the average energy of $s_{i,j}(t)$?
- Assume the transmission is corrupted by low levels of AWGN with PSD $N_0/2$ and that symbol errors are dominated by nearest neighbor errors. Determine values for $a_2, a_3, \ldots, a_K$ such that all nearest neighbor errors are equally likely. Determine an approximation for the probability of a single nearest neighbor error.