1. [35 pts] Consider the modulation of a dual tone message,

$$m(t) = A_{m_1}\cos(2\pi f_{m_1} t) + A_{m_2}\sin(2\pi f_{m_2} t)$$

using a carrier with frequency $f_c >> f_{m_2}$ where $f_{m_2} \geq f_{m_1}$.

- [15 pts] For AM, DSB–SC, SSB, and FM modulation determine the expressions for the time domain, $s(t)$, and frequency domain, $S(f)$, representations of the modulated signal. Plot $S(f)$ and several cycles of $s(t)$. For AM modulation assume 50% modulation. For FM, comment on the differences between narrow and wide band modulation and use approximate representations, where necessary.
- [5pts] What is the bandwidth of each scheme?
- [10 pts] What is the power of the modulated signal in each scheme? For AM, DSB–SC, and SSB cases, what percentage of the total power is due to the modulated message?
- [5 pts] Briefly discuss the advantages and disadvantages of each scheme.

2. [35 pts] Consider the mixer below that outputs the resultant signal $m(t)$. Suppose the signals $m_l(t)$ and $m_r(t)$ have the following spectrums:

$$M_l(f) = \begin{cases} A_l & |f| \leq W \\ 0 & \text{otherwise} \end{cases}$$

and

$$M_r(f) = \begin{cases} A_r(1 - |f|/W) & |f| \leq W \\ 0 & \text{otherwise} \end{cases}$$

- [15 pts] Determine the signal $m(t)$ and draw the spectrum $M(f)$. Label the various components of the spectrum and indicate what signal components they correspond to and what role they play in the mixer modulation/demodulation.
- [10 pts] Draw the block diagram of a demultiplexer that recovers the desired signals.
- [10 pts] Suppose $m(t)$ is input into an FM modulator and noise with PSD $N_0/2$ is added into the communications channel. Draw the PSD of the FM demodulated signal. Do the components of the demodulated $m(t)$ have equal noise power? Why or why not. From a noise interference perspective, can the mixer be redesigned to minimize noise interference? If so how?
- [0 pts] Complete the following line from the famous Steely Dan 1978 hit entitled *FM*:

"No _ _ _ _ _ _  _ _  _ _ _ FM"

3. [30 pts]

- [20 pts] Mathematically prove the following identity:

$$\alpha(t)\cos(2\pi f_c t) - \beta(t)\sin(2\pi f_c t) = a(t)\cos(2\pi f_c t + \phi(t))$$

where

$$a(t) = \sqrt{\alpha^2(t) + \beta^2(t)} \quad \text{and} \quad \phi(t) = \tan^{-1}\left(\frac{\beta(t)}{\alpha(t)}\right)$$

- [10 pts] Explain what roles $a(t)$ and $\phi(t)$ play in AM and FM modulation.