

Problem 6.34

Coherent M -ary PSK requires exact knowledge of the carrier frequency and phase for the receiver to be accurately synchronized to the transmitter. When carrier recovery at the receiver is impractical, we may use differential encoding based on the phase difference between successive symbols at the cost of some degradation in performance. If the incoming data are encoded by a phase-shift rather than by absolute phase, the receiver performs detection by comparing the phase of one symbol with that of the previous symbol, and the need for a coherent reference is thereby eliminated. This procedure is the same as that described for binary DPSK. The exact calculation of probability of symbol error for the differential detection of differential M -ary PSK (commonly referred to as M -ary DPSK) is much too complicated for $M > 2$. However, for large values of E/N_0 and $M \geq 4$, the probability of symbol error is approximately given by

$$P_e \approx \operatorname{erfc}\left(\sqrt{\frac{2E}{N_0}} \sin\left(\frac{\pi}{2M}\right)\right), \quad M \geq 4 \quad (1)$$

For coherent M -ary PSK, the corresponding formula for the average probability of symbol error is approximately given by

$$P_e \approx \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}} \sin\left(\frac{\pi}{M}\right)\right) \quad (2)$$

(a) Comparing the approximate formulas of Eqs. (1) and (2), we see that for $M \geq 4$ an M -ary DPSK system attains the same probability of symbol error as the corresponding coherent M -ary PSK system provided that the transmitted energy per symbol is increased by the following factor:

$$k(M) = \frac{\sin^2\left(\frac{\pi}{M}\right)}{2 \sin^2\left(\frac{\pi}{2M}\right)}, \quad M \geq 4$$

(b) For example, $k(4) = 1.7$. That is, differential QPSK (which is noncoherent) is approximately 2.3 dB poorer in performance than coherent QPSK.