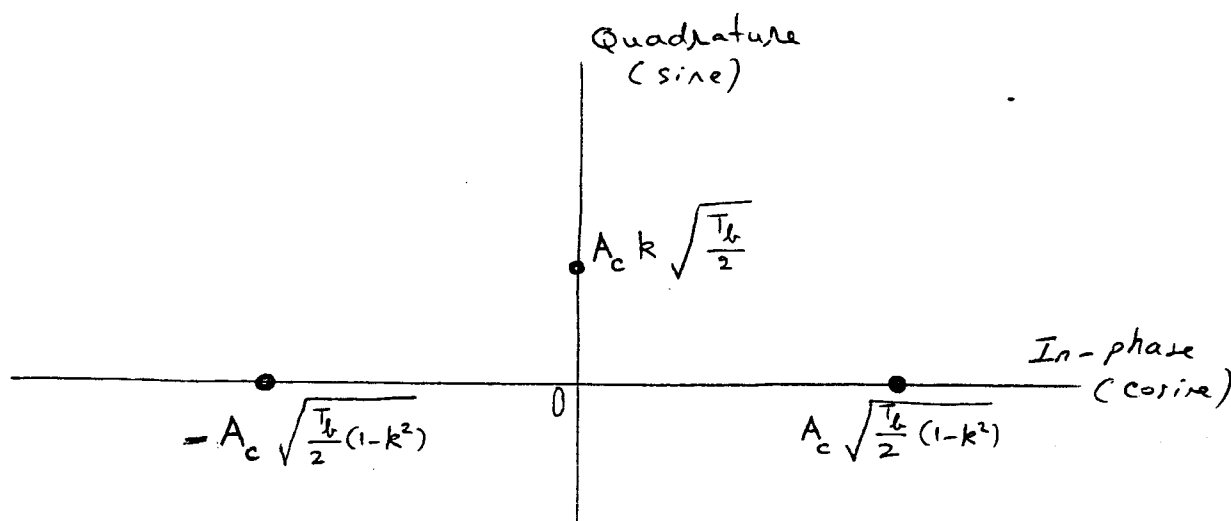


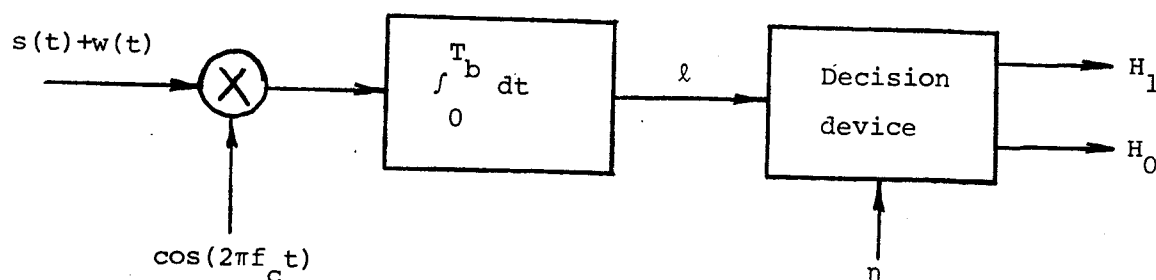
Problem 6.4

(a) The signal-space diagram of the scheme described in this problem is two-dimensional, as shown by



This signal-space diagram differs from that of the conventional PSK signaling scheme in that it is two-dimensional, with a new signal point on the quadrature axis at $A_c k \sqrt{T_b/2}$. If k is reduced to zero, the above diagram reduces to the same form as that shown in Fig. 8.14.

(b)



The signal at the decision device input is

$$\ell = \pm \frac{A_c}{2} \sqrt{1-k^2} T_b + \int_0^{T_b} w(t) \cos(2\pi f_c t) dt \quad (1)$$

Therefore, following a procedure similar to that used for evaluating the average probability of error for a conventional PSK system, we find that for the system defined by Eq. (1) the average probability of error is

$$P_e = \frac{1}{2} \operatorname{erfc}(\sqrt{E_b(1-k^2)}/N_0)$$

where $E_b = \frac{1}{2} A_c^2 T_b$.

(c) For the case when $P_e = 10^{-4}$ and $k^2 = 0.1$, we get

$$10^{-4} = \frac{1}{2} \operatorname{erfc}(u)$$

where $u^2 = \frac{0.9 E_b}{N_0}$

Using the approximation

$$\operatorname{erfc}(u) \approx \frac{\exp(-u^2)}{\sqrt{\pi} u}$$

we obtain

$$\exp(-u^2) - 2\sqrt{\pi} \times 10^{-4} u = 0$$

The solution to this equation is $u = 2.64$. The corresponding value of E_b/N_0 is

$$\frac{E_b}{N_0} = \frac{(2.64)^2}{0.9} = 7.74$$

Expressed in decibels, this value corresponds to 8.9 dB.

(d) For a conventional PSK system, we have

$$P_e = \frac{1}{2} \operatorname{erfc}(\sqrt{E_b}/N_0)$$

In this case, we find that

$$\frac{E_b}{N_0} = (2.64)^2 = 6.92$$

Expressed in decibels, this value corresponds to 8.4 dB. Thus, the conventional PSK system requires 0.5 dB less in E_b/N_0 than the modified scheme described herein.