

Problem 6.2

The transmitted binary PSK signal is defined by

$$s(t) = \begin{cases} \sqrt{E_b}\phi(t), & 0 \leq t \leq T_b, & \text{symbol 1} \\ -\sqrt{E_b}\phi(t), & 0 \leq t \leq T_b, & \text{symbol 0} \end{cases}$$

where the basis function $\phi(t)$ is defined by

$$\phi(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

The locally generated basis function in the receiver is

$$\begin{aligned} \phi_{\text{rec}}(t) &= \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \varphi) \\ &= \sqrt{\frac{2}{T_b}} [\cos(2\pi f_c t) \cos \varphi - \sin(2\pi f_c t) \sin \varphi] \end{aligned}$$

where φ is the phase error. The correlator output is given by

$$y = \int_0^{T_b} x(t) \phi_{\text{rec}}(t) dt$$

where

$$x(t) = s_k(t) + w(t), \quad k = 1, 2$$

Assuming that f_c is an integer multiple of $1/T_b$, and recognizing that $\sin(2\pi f_c t)$ is orthogonal to $\cos(2\pi f_c t)$ over the interval $0 \leq t \leq T_b$, we get

$$y = \pm \sqrt{E_b} \cos \varphi + W$$

when the plus sign corresponds to symbol 1 and the minus sign corresponds to symbol 0, and W is a zero-mean Gaussian variable of variance $N_0/2$. Accordingly, the average probability of error of the binary PSK system with phase error φ is given by

$$P_e = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b \cos \varphi}{N_0}} \right)$$

When $\varphi = 0$, this formula reduces to that for the standard PSK system equipped with perfect phase recovery. At the other extreme, when $\varphi = \pm 90^\circ$, P_e attains its worst value of unity.