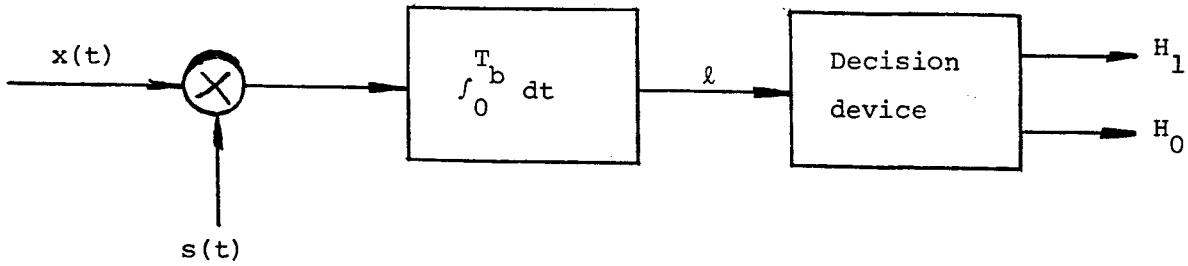


CHAPTER 6

Problem 6.1

(a) ASK with coherent reception



Denoting the presence of symbol 1 or symbol 0 by hypothesis H_1 or H_0 , respectively, we may write

$$H_1: x(t) = s(t) + w(t)$$

$$H_0: x(t) = w(t)$$

where $s(t) = A_c \cos(2\pi f_c t)$, with $A_c = \sqrt{2E_b/T_b}$. Therefore,

$$\ell = \int_0^{T_b} x(t) s(t) dt$$

If $\ell > E_b/2$, the receiver decides in favor of symbol 1. If $\ell < E_b/2$, it decides in favor of symbol 0.

The conditional probability density functions of the random variable L , whose value

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is denoted by ℓ , are defined by

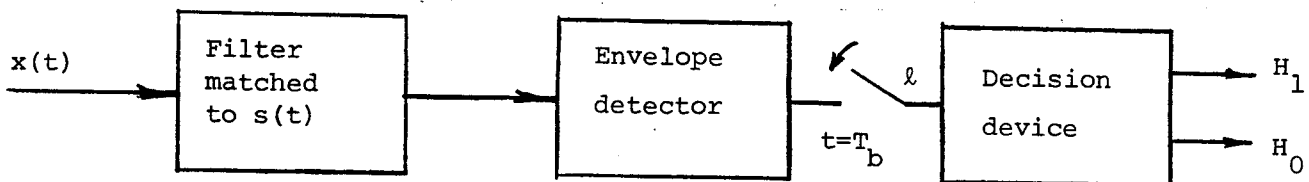
$$f_{L|0}(\ell|0) = \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left(-\frac{\ell^2}{N_0 E_b}\right)$$

$$f_{L|1}(\ell|1) = \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left[-\frac{(\ell - E_b)^2}{N_0 E_b}\right]$$

The average probability of error is therefore,

$$\begin{aligned} P_e &= p_0 \int_{E_b/2}^{\infty} f_{L|0}(\ell|0) d\ell + p_1 \int_{-\infty}^{E_b/2} f_{L|1}(\ell|1) d\ell \\ &= \frac{1}{2} \int_{E_b/2}^{\infty} \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left(-\frac{\ell^2}{N_0 E_b}\right) d\ell + \frac{1}{2} \int_{-\infty}^{E_b/2} \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left[-\frac{(\ell - E_b)^2}{N_0 E_b}\right] d\ell \\ &= \frac{1}{\sqrt{\pi N_0 E_b}} \int_{E_b/2}^{\infty} \exp\left(-\frac{\ell^2}{N_0 E_b}\right) d\ell \\ &= \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2} \sqrt{E_b/N_0}\right) \end{aligned}$$

(b) ASK with noncoherent reception



In this case, the signal $s(t)$ is defined by

$$s(t) = A_c \cos(2\pi f_c t + \theta)$$

where $A_c = \sqrt{2 E_b/T_b}$, and

$$f_{\theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta \leq 2\pi \\ 0, & \text{otherwise} \end{cases}$$

For the case when symbol 0 is transmitted, that is, under hypothesis H_0 , we find that the random variable L , at the input of the decision device, is Rayleigh-distributed:

$$f_{L|0}(\ell|0) = \frac{4\ell}{N_0 T_b} \exp\left(-\frac{2\ell^2}{N_0 T_b}\right)$$

For the case when symbol 1 is transmitted, that is, under hypothesis H_1 , we find that the

random variable L is Rician-distributed:

$$f_{L|1}(\ell|1) = \frac{4\ell}{N_0 T_b} \exp\left(-\frac{\ell^2 + A_c^2 T_b^2/4}{N_0 T_b/2}\right) I_0\left(\frac{2\ell A_c}{N_0}\right)$$

where $I_0(2\ell A_c/N_0)$ is the modified Bessel function of the first kind of zero order.

Before we can obtain a solution for the error performance of the receiver, we have to determine a value for the threshold. Since symbols 1 and 0 occur with equal probability, the minimum probability of error criterion yields:

$$\exp\left(-\frac{A_c^2 T_b^2}{2N_0}\right) I_0\left(\frac{2\ell A_c}{N_0}\right) \underset{H_0}{\overset{H_1}{\gtrless}} 1 \quad (1)$$

For large values of E_b/N_0 , we may approximate $I_0(2\ell A_c/N_0)$ as follows:

$$I_0\left(\frac{2\ell A_c}{N_0}\right) \approx \frac{\exp(2\ell A_c/N_0)}{\sqrt{4\pi\ell A_c/N_0}}$$

Using this approximation, we may rewrite Eq. (1) as follows:

$$\exp\left[-\frac{A_c(4 - A_c T_b)}{2N_0}\right] \underset{H_0}{\overset{H_1}{\gtrless}} \sqrt{\frac{4\pi\ell A_c}{N_0}}$$

Taking the logarithm of both sides of this relation, we get

$$\ell \underset{H_0}{\overset{H_1}{\gtrless}} \frac{A_c T_b}{4} + \frac{1}{2} \sqrt{\frac{\pi\ell N_0}{A_c}}$$

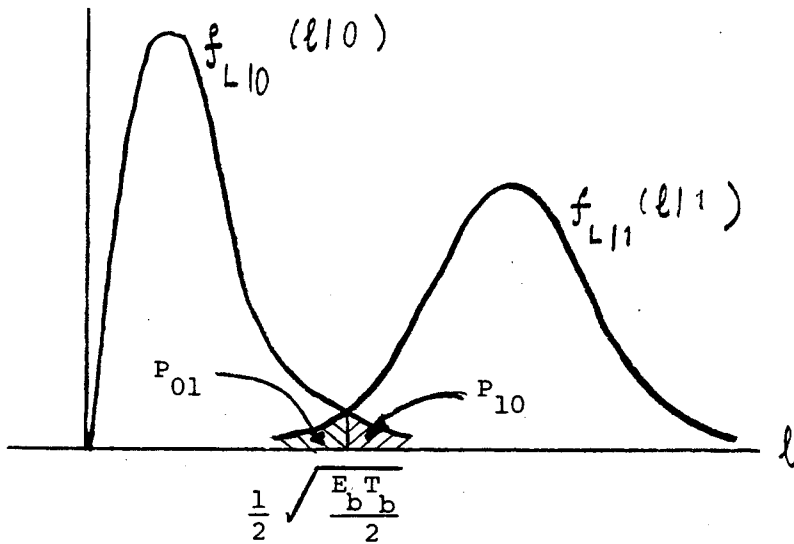
Neglecting the second term on the right hand side of this relation, and using the fact that

$$E_b = \frac{A_c^2 T_b^2}{2}$$

we may write

$$\ell \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{2} \sqrt{\frac{E_b T_b}{2}}$$

The threshold $\frac{1}{2} \sqrt{\frac{E_b T_b}{2}}$ is at the point corresponding to the crossover between the two probability density functions, as illustrated below.



The average probability of error is therefore

$$P_e = p_0 P_{10} + p_1 P_{01}$$

where

$$\begin{aligned} P_{10} &= \int_{\sqrt{E_b T_b}/2}^{\infty} f_{L|0}(\ell|0) d\ell \\ &= \int_{\sqrt{E_b T_b}/2}^{\infty} \frac{4\ell}{N_0 T_b} \exp\left(-\frac{2\ell^2}{N_0 T_b}\right) d\ell \\ &= \left[-\exp\left(-\frac{2\ell^2}{N_0 T_b}\right) \right]_{\sqrt{E_b T_b}/2}^{\infty} \\ &= \exp\left(-\frac{E_b}{4N_0}\right) \end{aligned}$$

$$\begin{aligned} P_{01} &= \int_0^{\sqrt{E_b T_b}/2} f_{L|1}(\ell|1) d\ell \\ &= \int_0^{\sqrt{E_b T_b}/2} \frac{4\ell}{N_0 T_b} \exp\left(-\frac{\ell^2 + A_c^2 T_b^2/4}{N_0 T_b/2}\right) I_0\left(\frac{2\ell A_c}{N_0}\right) d\ell \\ &= \int_0^{\sqrt{E_b T_b}/2} \frac{4\ell}{N_0 T_b} \exp\left(-\frac{\ell^2 + A_c^2 T_b^2/4}{N_0 T_b/2}\right) \cdot \frac{\exp(2\ell A_c/N_0)}{\sqrt{4\pi\ell A_c/N_0}} d\ell \end{aligned}$$

$$= \int_0^{\sqrt{E_b T_b}/2\sqrt{2}} \sqrt{\frac{2\ell}{A_c T_b}} \sqrt{\frac{2}{\pi N_0 T_b}} \exp\left[-\frac{(\ell - A_c T_b/2)^2}{N_0 T_b/2}\right] d\ell \quad (2)$$

The integrand in Eq. (2) is the product of $\sqrt{2\ell/A_c T_b}$ and the probability density function of a Gaussian random variable of mean $A_c T_b/2$ and variance $N_0 T_b/4$. For high values of E_b/N_0 , the standard deviation $\sqrt{N_0 T_b/4}$ is much less than the threshold $\sqrt{E_b T_b}/2\sqrt{2}$. Consequently, the area under the portion of the curve from 0 to $\sqrt{E_b T_b}/2\sqrt{2}$ is quite small, that is, $P_{01} \approx 0$. Then, we may approximate the average probability of error as

$$P_e \approx p_0 P_{10} \\ = \frac{1}{2} \exp\left(-\frac{E_b}{4N_0}\right)$$

where it is assumed that symbols 0 and 1 occur with equal probability.