ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #3 due Monday September 29

1. Haykin Problem P2.4 #16

2. Haykin Problem P2.7 #22

3. Consider the following modulator: The input $x(t) = \text{sinc}^2(1000t)$ is multiplied by a cosine with center frequency $f_c = 0.5$ kHz and then passed through an ideal lowpass filter with bandwidth $W = 0.5$ kHz (amplitude = 1).

   a) Find $X(f)$, the Fourier transform of $x(t)$.
   b) Find $Z(f)$, the Fourier transform of $x(t)\cos(2\pi f_c t)$.
   c) Find $Y(f)$, the Fourier transform of the filter output.
   d) Find $y(t)$.

4. Find the convolution of two time-scaled rectangles, $\text{rect}(t/T_1)$ and $\text{rect}(t/T_2)$, in terms of T_1 and T_2 for $T_1 > T_2$.

SOLUTIONS TO ASSIGNMENT #3

1. Haykin Problem #2.16

$$g(t) = \frac{1}{\sqrt{2\pi}\,\tau} \exp\left(-\frac{t^2}{2\tau^2}\right)$$

We know that

$$\exp(-\pi t^2) \longleftrightarrow \exp(-\pi f^2) \qquad \text{(Problem \#12)}$$

But $g(t)$ is this time function with a scaling factor $a = 1/\sqrt{2\pi}\,\tau$. Therefore, from the time-scaling property of Fourier transforms

$$\exp\left(-\frac{t^2}{2\tau^2}\right) \longleftrightarrow \sqrt{2\pi}\,\tau\, \exp(-2\pi f^2 \tau^2)$$

$$\therefore\ F\{g(t)\} = e^{-2\pi f^2 \tau^2}$$

provides a measure of the time duration of the Gaussian pulse

$$= \exp\left[-\frac{f^2}{2(1/4\pi^2\tau^2)}\right]$$

So, a corresponding measure for the bandwidth is

$$\underbrace{\frac{1}{2}}_{}\sqrt{\frac{1}{4\pi^2\tau^2}} = \frac{1}{4\pi\tau}$$

accounts for use of positive frequencies only

Therefore, the time-bandwidth product of the Gaussian pulse is $1/4\pi$.

2. Haykin Problem #2.22

a.) $g(t) = \text{sinc}(200t)$

This sinc pulse corresponds to a bandwidth $W = 100$ Hz. Hence, the Nyquist rate is 200 Hz and the Nyquist interval is 1/200 seconds (5 ms)

b.) $g(t) = \text{sinc}^2(200t)$

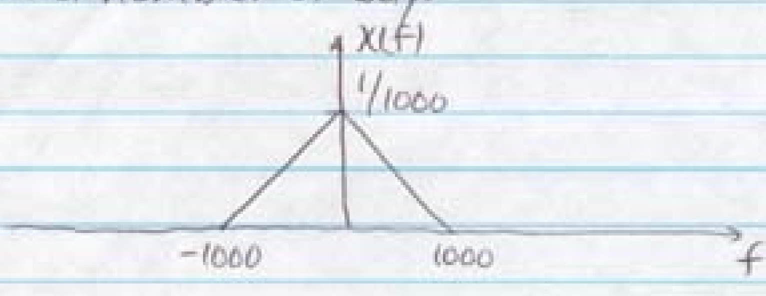
This signal may be viewed as the product of the sinc pulse sinc(200t) with itself. Since multiplication in the time domain corresponds to convolution in the frequency domain, this signal has a bandwidth equal to twice that of the sinc(200t), i.e. 200 Hz. The Nyquist rate is 400 Hz and the Nyquist interval is 2.5 ms.

c.) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

The bandwidth of g(t) is determined by the highest frequency component of sinc(200t) or $\text{sinc}^2(200t)$, whichever is the largest. Therefore, the Nyquist rate is 400 Hz and the Nyquist interval is 2.5 ms

3. a.) X(f)

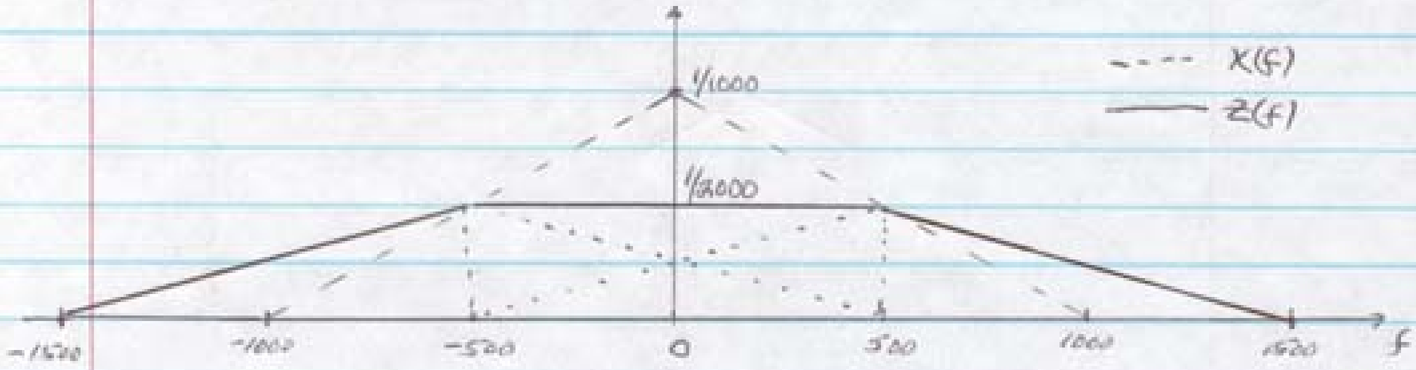
The Fourier transform of $\text{sinc}^2(1000t)$ is the triangle extending from −1000 Hz to 1000 Hz. This is easily derived in a number of ways



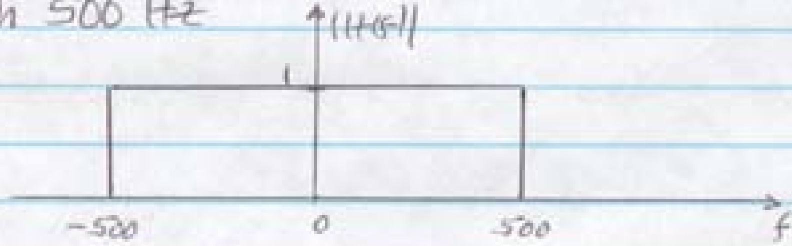
2.cont) b.) $z(t) = x(t)\cos 2\pi f_c t$ $\quad \underbrace{f_c = 500\text{ Hz}}$

less than the bandwidth of the signal

$$Z(f) = \tfrac{1}{2}X(f-f_c) + \tfrac{1}{2}X(f+f_c)$$



c.) The filter is an ideal lowpass filter with amplitude 1 and bandwidth 500 Hz



$Y(f) = H(f)Z(f) \Rightarrow$ retain flat part of spectrum

$$= \begin{cases} 1/2000, & |f| < 500\text{ Hz} \\ 0, & |f| > 500\text{ Hz} \end{cases}$$

d.) $Y(f) = \dfrac{1}{2000}\,\text{rect}\left(\dfrac{f}{1000}\right)$

$\therefore\ y(t) = \dfrac{1}{2000}\cdot 1000\cdot \text{sinc}(1000t) = \dfrac{1}{2}\,\text{sinc}(1000t)$

4. See derivation on pages 32-34 of the notes ($\tau_1 \to T_1$, $\tau_2 \to T_2$).
($A_1 = A_2 = 1$)