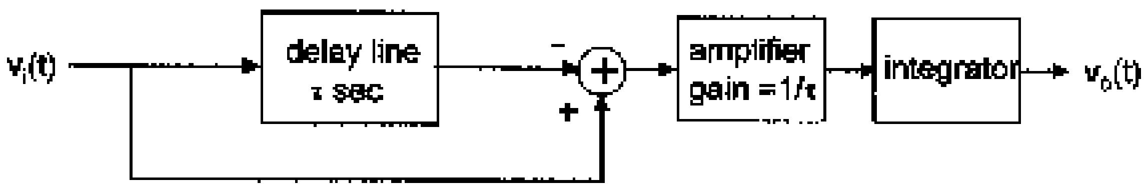
ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #2 due Monday September 22

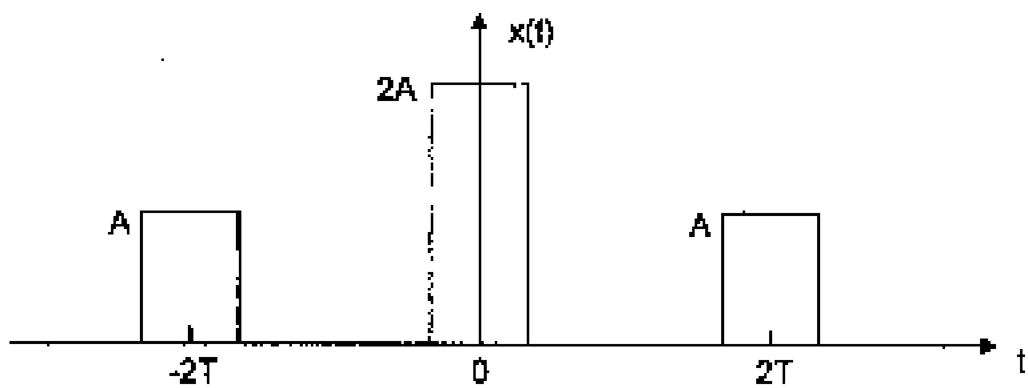
1. A delay line and integrating circuit as shown below are one example of a "holding circuit" that was commonly used in radar work, sampled-data servo systems, and pulse-modulation systems.

   a) Tracing through the circuit step-by-step, determine the transfer function $H(f) = V_o(f)/V_i(f)$.

   b) Let $v_i(t)$ be a rectangular pulse of width τ seconds, determine the output $v_o(t)$.



2. Using the superposition (linearity) and time-shift theorems for Fourier transforms, find the transform of the signal shown below. The width of each of the rectangles is τ.



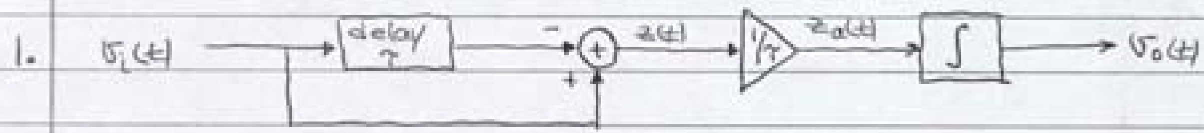
3. Given the energy signal, $x(t)=e^{-\alpha t}u(t)$, $\alpha>0$, compute the Fourier transform of the following expression (* denotes convolution)

$$y(t) = \beta_1 x(t-t_0) + \beta_2 x(t) * x(t) + \delta(t-t_0).$$

4. Haykin Problem P2.3 #7

SOLUTIONS TO ASSIGNMENT #1

1.



a.) $z(t) = v_i(t) - v_i(t-\tau)$; $z_a(t) = \frac{1}{\tau} z(t)$

$$v_o(t) = \int z_a(t)\,dt = \frac{1}{\tau}\int \left[v_i(t) - v_i(t-\tau)\right] dt$$

$$\therefore\ Z(f) = V_i(f) - V_i(f)\,e^{-j2\pi f\tau}$$

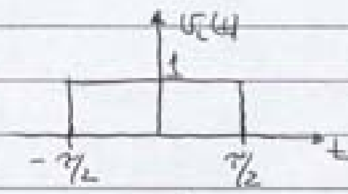
$$Z_a(f) = \frac{1}{\tau} V_i(f)\left[1 - e^{-j2\pi f\tau}\right]$$

$$V_o(f) = \frac{1}{j2\pi f} Z_a(f) = V_i(f)\,\frac{1 - e^{-j2\pi f\tau}}{j2\pi f\tau}$$

$$\therefore\ H(f) = \frac{V_o(f)}{V_i(f)} = e^{-j2\pi f\tau/2}\left[\frac{e^{j2\pi f\tau/2} - e^{-j2\pi f\tau/2}}{2j\,(\pi f\tau)}\right]$$

$$= e^{-j\pi f\tau}\ \underbrace{\frac{\sin \pi f\tau}{\pi f\tau}}_{\text{sinc } f\tau}$$

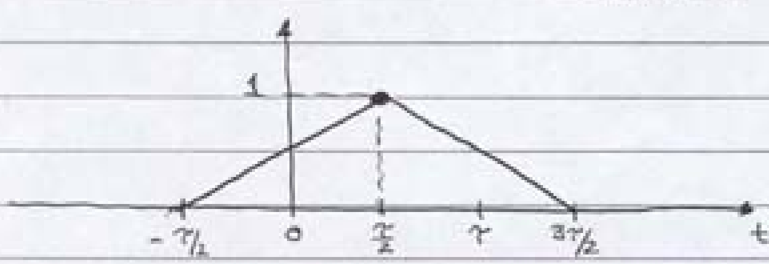
b.) $v_i(t)$ = rectangular pulse of width τ seconds



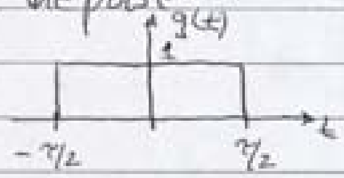
$$V_i(f) = \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-j2\pi f t}\,dt = \tau\,\frac{\sin \pi f\tau}{\pi f\tau}$$

$$V_o(f) = H(f)V_i(f) = \tau e^{-j\pi f\tau}\left[\frac{\sin \pi f\tau}{\pi f\tau}\right]^2$$

$v_o(t) = \mathcal{F}^{-1}\left[V_o(f)\right] \Rightarrow$ triangle of amplitude 1, shifted by $\tau/2$, duration 2τ



2. Define $g(t)$ as the pulse



$$G(f) = \mathcal{F}[g(t)] = \tau \frac{\sin \pi f \tau}{\pi f \tau}$$

Then,

$$x(t) = 2A\,g(t) + A\,g(t-2T) + A\,g(t+2T)$$

$$\begin{aligned} X(f) &= \mathcal{F}[x(t)] \\ &= 2A\,\mathcal{F}[g(t)] + A\,\mathcal{F}[g(t-2T)] + A\,\mathcal{F}[g(t+2T)] \quad \text{linearity} \\ &= 2A\,G(f) + A e^{-j2\pi f 2T} G(f) + A e^{-j2\pi f(-2T)} G(f) \end{aligned}$$

time-shift property

$$= 2A\left[1 + \frac{e^{-j2\pi f 2T} + e^{-j2\pi f(-2T)}}{2}\right] G(f)$$

$$= 2A(1 + \cos 4\pi f T)\,G(f)$$

$$X(f) = 2A\tau(1 + \cos 4\pi f T)\frac{\sin \pi f \tau}{\pi f \tau}$$

3. $x(t) = e^{-\alpha t} u(t)$

$$X(f) = \mathcal{F}[x(t)] = \int_0^\infty e^{-\alpha t} e^{-j2\pi f t}\,dt = \frac{1}{\alpha + j2\pi f}$$

$$y(t) = \beta_1 x(t-t_0) + \beta_2\, x(t) * x(t) + \delta(t-t_0)$$

$$\begin{aligned} Y(f) &= \beta_1 \mathcal{F}[x(t-t_0)] + \beta_2 \mathcal{F}[x(t)*x(t)] + \mathcal{F}[\delta(t-t_0)] \\ &= \beta_1 e^{-j2\pi f t_0} X(f) + \beta_2 X^2(f) + e^{-j2\pi f t_0} \\ &= e^{-j2\pi f t_0}\left[1 + \frac{\beta_1}{\alpha + j2\pi f}\right] + \beta_2\left[\frac{1}{\alpha + j2\pi f}\right]^2 \end{aligned}$$

4. Haykin Chapter 2 #7

a.) $\mathcal{F}[x(t)] = X(f)$

$$G_1(f) = \mathcal{F}[g_1(t)] = \mathcal{F}[x(\tfrac{t}{5})]$$
$$= 5\,X(5f) \qquad \text{time scaling property}$$
$$G_2(f) = \mathcal{F}[g_2(t)] = \mathcal{F}[x(5t)]$$
$$= \tfrac{1}{5}\,X(\tfrac{f}{5})$$

b.) $g_1(t) \rightarrow$ time expansion
$g_2(t) \rightarrow$ time compression

a.) ① $y(t) = a g_1(t) = a\,x(t/5)$
$$Y(f) = 5a\,X(5f)$$
For $Y(0) = X(0)$, $5a = 1 \Rightarrow a = 1/5$

② $y(t) = a g_2(t) = a\,x(5t)$
$$Y(f) = \tfrac{a}{5}\,X(f/5)$$
For $Y(0) = X(0)$, $\tfrac{a}{5} = 1 \Rightarrow a = 5$

4. Haykin Chapter 2 #7

a.) $\mathcal{F}[x(t)] = X(f)$

$G_1(f) = \mathcal{F}[g_1(t)] = \mathcal{F}[x(\frac{t}{5})]$

$= 5X(5f)$ time scaling property

$G_2(f) = \mathcal{F}[g_2(t)] = \mathcal{F}[x(5t)]$

$= \frac{1}{5}X(\frac{f}{5})$

b.) $g_1(t) \rightarrow$ time expansion

$g_2(t) \rightarrow$ time compression

a.) ① $y(t) = a g_1(t) = a\,x(t/5)$

$Y(f) = 5a\,X(5f)$

For $Y(0) = X(0)$, $5a = 1 \Rightarrow a = 1/5$

② $y(t) = a g_2(t) = a\,x(5t)$

$Y(f) = \frac{a}{5}X(f/5)$

For $Y(0) = X(0)$, $\frac{a}{5} = 1 \Rightarrow a = 5$