ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #1 due Monday September 15

1. Determine whether each of the following signals is periodic and, if so, find the fundamental period T_0.

   a) $j\exp(j10t)$
   b) $\exp[(-1+j)t]$
   c) $\cos(10t+1) - \sin(4t-1)$
   d) $[\cos(2t-\pi/3)]^2$

2. Consider a filter with input x(t) and output y(t). For each of the following input-output relationships, determine whether the corresponding filter is linear, time-invariant, or both.

   a) $y(t) = t^2x(t-1)$
   b) $y(t) = x^2(t-2)$
   c) $y(t) = x(t+1) + x(t-1)$
   d) $y(t) = x(\sin(t))$

3. Haykin Problem P2.1 #1

4. Haykin Problem P2.1 #2

SOLUTIONS TO ASSIGNMENT #1

#1 a.) periodic

$$x(t) = je^{j10t} = e^{j(\frac{\pi}{2}+10t)}$$

– repeats such that $10t = 2k\pi$

$$t = \frac{k\pi}{5},\ k=1,2,3,\ldots$$

T_0 = fundamental period = $\frac{\pi}{5}$

b.) not periodic

$$x(t) = e^{(-1+j)t} = \underbrace{e^{-t}}_{\substack{\text{monotonically}\\ \text{decreasing with } t}} e^{jt}$$

c.) periodic

$$x(t) = \cos(10t+1) - \sin(4t-1)$$

↳ sum of two sinusoids with fundamental periods $T_{0,c} = \frac{2\pi}{10} = \frac{\pi}{5}$ and $T_{0,s} = \frac{2\pi}{4} = \frac{\pi}{2}$

The fundamental period of the composite signal is the Least Common Multiple of $T_{0,c}$ and $T_{0,s}$ which is equal to $T_0 = \pi$

d.) periodic

$$x(t) = [\cos(2t - \tfrac{\pi}{3})]^2$$

$$= \tfrac{1}{2}[\underset{\substack{\uparrow\\ \text{constant}}}{1} + \underset{\substack{\uparrow\\ \text{periodic}}}{\cos(4t - \tfrac{2\pi}{3})}]$$

The fundamental period $T_0 = 2\pi/4 = \pi/2$

#2 a.) Linear/time-varying

To test for linearity, we calculate the output when the input is equal to $\hat{x}(t) = \alpha x_1(t) + \beta x_2(t)$. In this case,

$$\hat{y}(t) = \alpha t^2 x_1(t-1) + \beta t^2 x_2(t-1)$$
$$= \alpha y_1(t) + \beta y_2(t) \Rightarrow \therefore \text{ linear}$$

To check for time invariance, we apply the signal $x_d(t) = x(t-\tau)$ to the system input. The output is then

$$y_d(t) = t^2 x_d(t-1)$$
$$= t^2 x(t-1-\tau) \neq (t-\tau)^2 x(t-1-\tau) = y(t-\tau)$$
$$\Rightarrow \therefore \text{ time varying}$$

b.) Not linear/time-invariant

$$\hat{y}(t) = \hat{x}^2(t) = [\alpha x_1(t) + \beta x_2(t)]^2$$
$$\neq \alpha x_1^2(t) + \beta x_2^2(t) = \alpha y_1(t) + \beta y_2(t)$$
$$\therefore \text{ not linear}$$

$$y_d(t) = x_d^2(t-2) = x^2(t-2-\tau) = y(t-\tau)$$
$$\therefore \text{ time invariant}$$

c.) Linear/time-invariant

$$\hat{y}(t) = \alpha x_1(t+1) + \beta x_2(t+1) + \alpha x_1(t-1) + \beta x_2(t-1)$$
$$= \alpha x_1(t+1) + \alpha x_1(t-1) + \beta x_2(t+1) + \beta x_2(t-1)$$
$$= y_1(t) + y_2(t) \Rightarrow \therefore \text{ linear}$$

$$y_d(t) = x_d(t+1) + x_d(t-1)$$
$$= x(t+1-\tau) + x(t-1-\tau) = y(t-\tau) \Rightarrow \therefore \text{ time invariant}$$

#2(cont.) d.) Linear/time not invariant

$$\hat{y}(t) = \alpha x_1(\sin(t)) + \beta x_2(\sin(t)) = \alpha y_1(t) + \beta y_2(t)$$
$\Rightarrow \therefore$ linear

$$y_d(t) = x_d(\sin(t)) = x(\sin(t) - \tau)$$
$$\neq x(\sin(t-\tau)) = y(t-\tau)$$
$\Rightarrow \therefore$ time varying

#3. Haykin Problem P2.1 #1

$g_p(t)$ = periodic sequence of raised cosine pulses

even $\Rightarrow \therefore$ only cosine terms

$$g_p(t) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos(\pi n t), \quad T_0 = 2 \text{ sec}$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\,dt = \frac{1}{2}\int_{-1/2}^{1/2}(1+\cos(2\pi t))\,dt = \frac{1}{2}$$

only nonzero over part of period

$$a_1 = \frac{1}{2}\int_{-1/2}^{1/2}(1+\cos(2\pi t))\cos\pi t\,dt$$
$$= \frac{1}{2}\int_{-1/2}^{1/2}\left[\cos\pi t + \frac{1}{2}\cos 3\pi t + \frac{1}{2}\cos\pi t\right]dt$$
$$= \frac{1}{2}\left[\frac{3\sin\pi t}{2\pi} + \frac{\sin 3\pi t}{6\pi}\right]\Bigg|_{-1/2}^{1/2}$$
$$= \frac{3}{2\pi} - \frac{1}{6\pi} = \frac{4}{3\pi}$$

$$a_2 = \frac{1}{2}\int_{-1/2}^{1/2}(1+\cos(2\pi t))\cos 2\pi t\,dt = \frac{1}{4} \quad \cdots$$

#4. Haykin Problem P2.1 #2

periodic pulsed RF waveform ($f_0 T_0 \gg 1 \Rightarrow$ center freq. $\gg$ fund. freq.)

$$g_p(t) = a_p(t)\cos(2\pi f_0 t)$$

where

$$a_p(t) = \begin{cases} A, & -T_0/4 \le t \le \frac{T_0}{4} \\ 0, & \text{remainder of period} \end{cases}$$

$$= \frac{1}{2} a_p(t) \underbrace{\left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t} \right]}_{\text{this just represents shift to right and left}}$$

only need spectrum of $a_p(t)$ and then shift

$$a_p(t) = \sum_{n=-\infty}^{\infty} c_n' e^{j\frac{2\pi n t}{T_0}}$$

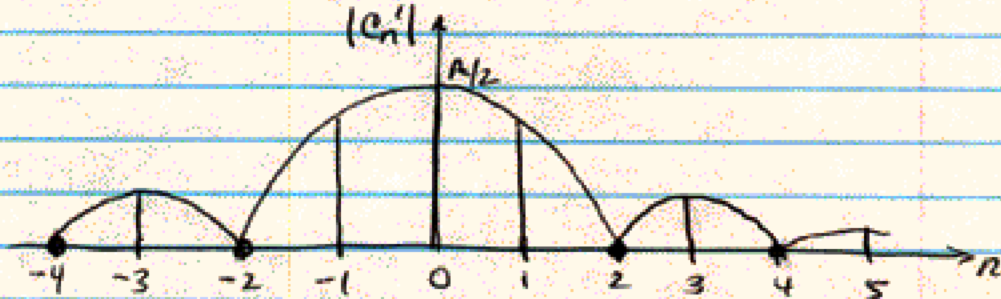
where

$$c_n' = \frac{A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right)$$ (eq. 2.17 in Haykin)

$\frac{T}{T_0}$ ½ for this waveform

$$= \frac{A}{2}\,\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{2}}$$

$$= \frac{A}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right)$$



Spectrum of $g_p(t)$, $|C_n|$, is this spectrum shifted up to f_0 and down to $-f_0$