Vector Orthogonal Frequency-Division Multiplexing

Vector orthogonal frequency-division multiplexing (VOFDM) was proposed by Xiang-Gen Xia in 2000 in [1] [2] for single transmit antenna systems. VOFDM replaces each scalar value in the conventional OFDM by a vector value and is a bridge between OFDM and the single carrier frequency domain equalizer (SC-FDE). When the vector size is 1, it is OFDM and when the vector size is at least the channel length and the FFT size is 1, it is SC-FDE.

In OFDM systems, assume N is the number of subcarriers, i.e., the FFT size, and a discrete time signal of length N, $x_n, 0 \leq n \leq N-1$, is to be transmitted. One takes the N-point inverse FFT (IFFT) of $x_n, 0 \leq n \leq N-1$, and obtains another discrete time signal of length N as $X_k, 0 \leq k \leq N-1$. Then, one adds a cyclic prefix (CP) of length Γ to this sequence as

$$X_0, X_1, \ldots, X_{N-1}, X_0, X_1, \ldots, X_{\Gamma-1},$$

which is transmitted at a transmit antenna sequentially.

Assume the channel in the z-transform domain is $H(z) = \sum_{k=0}^{L} h_k z^{-k}$where $h_k, 0 \leq k \leq L$, are the multipath channel coefficients. If the CP length is at least L, i.e., $\Gamma \geq L$, then at the receiver, after the CP is removed from the received signal and taking the N-point FFT, the received signal becomes

$$y_n = H_n x_n + w_n,\ 0 \leq n \leq N-1, \quad (1)$$

where $H_n = H(\exp(2\pi jn/N)) = \sum_{k=0}^{L} h_k \exp(-2\pi jkn/N)$ and w_n is the additive noise (it is assumed white noise here). From (1), one can see that the original intersymbol-interference (ISI) channel is converted to N many ISI free subchannels.

In VOFDM, assume M is the vector size, and each scalar-valued signal x_n in OFDM is replaced by a vector-valued signal $\mathbf{x}_n$of vector size M, $0 \leq n \leq N-1$. One takes the N-point IFFT of $\mathbf{x}_n, 0 \leq n \leq N-1$, component-wisely and gets another vector sequence of the same vector size M, $\mathbf{X}_k, 0 \leq k \leq N-1$. Similar to OFDM, one adds a vector CP of length Γ^* to this vector sequence as

$$\mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_{N-1}, \mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_{\Gamma^*-1}.$$

This vector sequence is then converted to a scalar sequence by sequentializing all the vectors of size M, which is transmitted at a transmit antenna sequentially.

Similar to OFDM, when the CP length satisfies $\Gamma^* \geq \lceil \frac{L}{M} \rceil$, then at the receiver, the received scalar sequence is first converted to the vector sequence of vector size M, after the vector CP is removed from the vector sequence and the N-point FFT is implemented component-wisely to the vector sequence of length N, one obtains

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{w}_n, \; 0 \leq n \leq N-1, \quad (2)$$

where $\mathbf{w}_n$ are additive white noise and $\mathbf{H}_n = \mathbf{H}(\exp(2\pi jn/N)) = \mathbf{H}(z)|_{z=\exp(2\pi jn/N)}$ and $\mathbf{H}(z)$ is the following $M \times M$ polyphase matrix of $H(z)$:

$$\mathbf{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{M-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{M-1}(z) & H_{M-2}(z) & \cdots & H_0(z) \end{bmatrix},$$

where $H_m(z) = \sum_l h_{Ml+m} z^{-l}$ is the mth polyphase component of the channel $H(z), 0 \leq m \leq M-1$. From (2), one can see that the original ISI channel is converted to N many vector subchannels of vector size M. There is no ISI across these vector subchannels but there is ISI inside each vector subchannel. In each vector subchannel, at most M many symbols are interfered each other. Clearly, when the vector size $M = 1$, the above VOFDM returns to OFDM and when $M > L$ and $N = 1$, it becomes the SC-FDE. The vector size M is a parameter that one can choose freely and properly in practice and controls the ISI level.

Note that there exist many other different generalizations/forms of OFDM, to see their essential differences, it is critical to see their corresponding received signal equations as (1) and (2). The above VOFDM is the earliest and the only one that achieves the received signal equation (2) and/or its equivalent form, although it may have different implementations at transmitter vs. different IFFT algorithms. It has been shown in [3] that applying the MMSE linear receiver to each vector subchannel (2), it achieves multipath diversity.

References

1. Xiang-Gen Xia, "Precoded OFDM systems robust to spectral null channels and vector OFDM systems with reduced cyclic prefix length," *Proceedings of IEEE International Conference on Communications (ICC)*, vol. 2, pp. 1110-1114, New Orleans, LA, USA, June 18-22, 2000.
2. Xiang-Gen Xia, "Precoded and vector OFDM robust to channel spectral nulls and with reduced cyclic prefix length in single transmit antenna systems," *IEEE Transactions on Communications*, vol. 49, no. 8, pp. 1361-1374, Aug. 2001.
3. Yabo Li, Ibo Ngebani, Xiang-Gen Xia, and Anders Host-Madsen, "On performance of vector OFDM with linear receivers," *IEEE Transactions on Signal Processing*, vol. 60, no. 10, pp. 5268-5280, Oct. 2012.