

OFDM SAR Imaging

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Joint work with *Tianxian Zhang* from UESTC

- Background
- Range reconstruction problem
- CP-based OFDM range reconstruction
- OFDM pulse with arbitrary length
- OFDM pulse design
- Simulation results
- MIMO-OFDM radar
- Conclusion

- Synthetic aperture radar (SAR) was started in 1950s
 - It has tremendous military and commercial applications as an all weather and all time sensor
 - To achieve long distance imaging, a pulse with long enough time duration is used to carry enough energy
 - The received pulses from different scatters are overlapped each other and cause energy interference between different scatters
 - To mitigate the interference and achieve a good resolution, a transmitted pulse is coded using frequency or phase modulation, i.e., LFM and step frequency signals, or random noise type signals
 - Leads to LFM, step frequency, or random noise radars (or SAR)
 - **Coincides with the spread spectrum idea in communications**

Comparison Between Radar and Communications

- Radar (SAR)
 - Inter-scatter (range cell) interference (IRCI)

$$\sum g_i s(t - \tau_i)$$

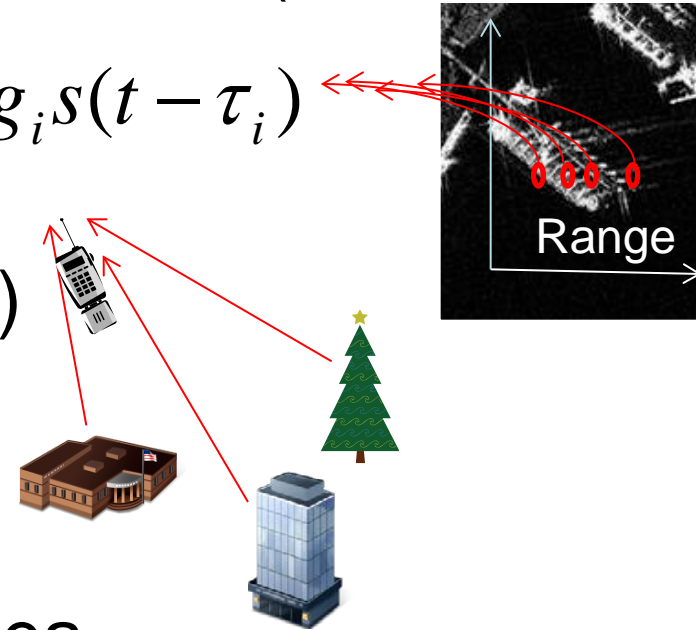
- Communications

- Inter-symbol interference (ISI)

- Solution

- Transmit: spread spectrum idea
- Receive: matched filtering

- To get g_i in radar
- To get s in communications (single user de-spreading in communications, when ISI is not too severe)



Comparison Between Radar and Communications



- How severe is IRCI in radar and/or ISI in communications?
 - In radar, it depends on the range resolution and the number of range cells in a swath (or a range line).
 - The range resolution depends on the transmitted signal bandwidth: the wider the signal bandwidth is; the higher range resolution is; and the more range cells a swath has.
 - The wider bandwidth is; the more IRCI is.
 - In communications, it depends on channel bandwidth
 - The wider bandwidth is; the more severe of the ISI is.

- The most important task in the past in physical layer communications is to deal with the ISI issue
 - In *wired systems*, such as from computer modems (time domain methods to deal with the ISI by using more bandwidth efficient coding called trellis coded modulation (TCM) and decision feedback equalizer (DFE)) to high speed cable modems (OFDM);
 - In *wireless systems*, it moves from the second and third generations of TDMA/CDMA to the forth generation of OFDM

Wired Computer Modems



< 9.6 kbs/s	equalization (Lucky 60s)	Squeeze more bits to a symbol
9.6 kbs/s	TCM +equalization (DFE)	
14.4 kbs/s	TCM + equalization	Use more bandwidth
28.8 kbs/s		
56 kbs/s		
Asymmetric Digital Subscriber Line (ADSL)	orthogonal frequency division multiplexing (OFDM)	
6 Mbs/s	or called discrete multi-tone (DMT)	

Wireless Communications Systems: Number of Multipath vs. Modulation Methods



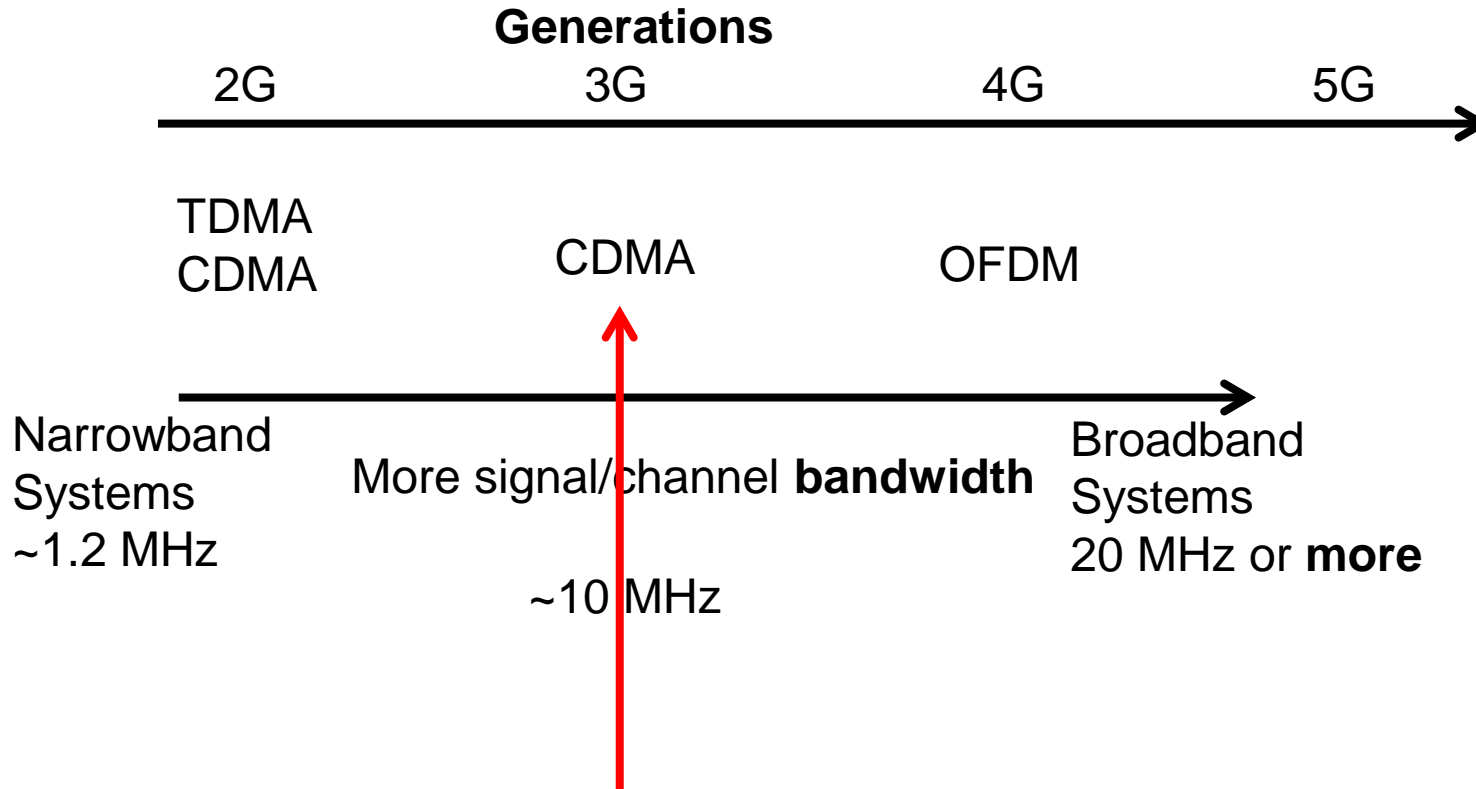
2G (IS-95)	1.23 MHz	Almost optimal for single path
3G (WCDMA CDMA2000)	< 11 MHz	6--8 multipath almost the break point to use CDMA
IEEE 802.11b (LAN)	similar to 3G	
IEEE 802.11a (LAN)	20 MHz	16 multipath OFDM
IEEE 802.11n (LAN)	20 & 40MHz	40MHz doubles everything in 20MHz OFDM
4G LTE	20 MHz	16 multipath OFDM and SC-FDE
5G	100 MHz	OFDM

Digital Wireless Standards vs. Bandwidth (#of Multipaths)



- A standard is determined by a bandwidth (so far)
- 2G: 1.23MHz, almost the highest for non-ISI
 - Both TDMA and CDMA (DS spread spectrum) work well
- 3G: ~10 MHz, a few multipaths
 - Due to the ISI and wireless varying channels, time domain equalization may not work well, TDMA is not used, but CDMA (DS spread spectrum) is used in all standards since it is good to resist a few chip level time delays (RAKE receiver --- matched filtering to all the multipaths)
- 4G: 20 MHz, more multipaths
 - **Even CDMA RAKE receiver (matched to all paths) may not work well due to non-ideal sidelobes of codes/waveforms**
 - **OFDM is adopted (down link)**
 - 25% data overhead for the cyclic prefix (CP) is used to deal with the multipath
 - The **key** of OFDM is to convert an ISI channel to multiple ISI-free subchannels, when a sufficient cyclic prefix (CP) is added

What Has Happened in Communications



Only a few multipaths exist

Spread spectrum idea

LFM/step-frequency radar is like frequency hopping
Random noise radar is like direct sequence

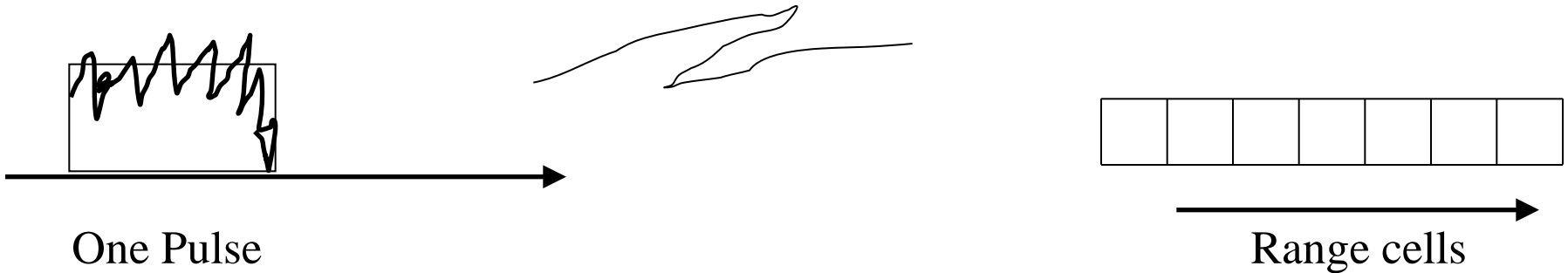
- It still uses the spread spectrum idea
 - Transmitter: LFM, step frequency (frequency hopping), random noise signals (direct sequence)
 - Receiver: Matched filtering (corresponding to the single user detection in CDMA systems in 2G)
 - The 2G and 3G technology in communications
 - **It works well only when there are not too many range cells in a swath (similar to that CDMA works only for a few multipaths)**
- Is a radar signal bandwidth large enough to use OFDM?
 - A good range resolution may require a high signal bandwidth
 - leads to have too many range cells in a swath
 - non-ideal sidelobes of a radar signal in the matched filtering (range compression) **cause** IRCI in a SAR image
 - The sidelobe level is about \sqrt{N} for a length N pulse/signal (or the range compression gain M)



- A high resolution SAR requires a high bandwidth → more severe IRCI → motives us to adopt OFDM signals
- There have been many OFDM signalings in radar already
Levanon'00, Franken'06, Garmatyuk'08, Sturm et al'09, Sen-Nehorai'09, Wang-Hou-Lu'09, Berger et al'10, Wu-Rishk-Glisson'10, Sit et al'12, Riche et al'12, Kim et al'13

 - For most of them, OFDM signals are just treated as a different kind of signals at the transmitter and the conventional matched filtering is used at the receiver.
 - The matched filter is optimal in terms of SNR where the IRCI is treated as the signal part but is clearly not desired.
 - The matched filtering may not be optimal in terms of less IRCI.
 - **The key of the OFDM that converts an ISI channel to multiple ISI free subchannels is not used.**
 - There still exists IRCI across range cells among a swath.
 - Yes, we can! → **IRCI free range reconstruction.**

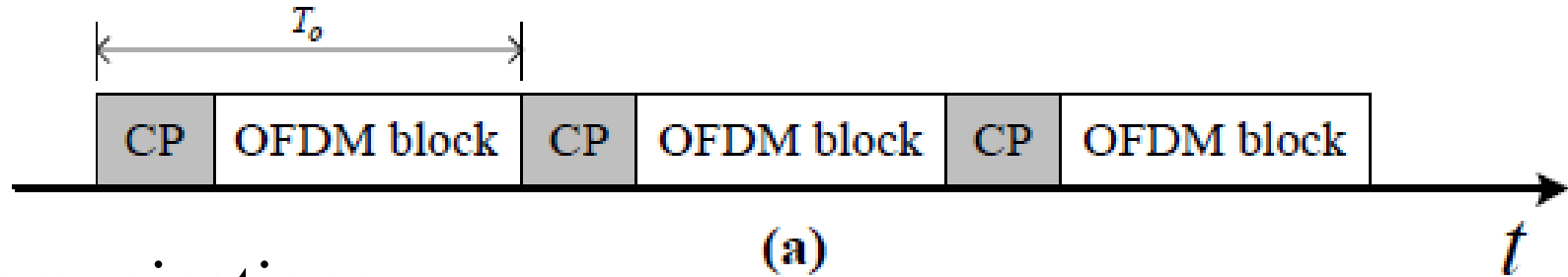
Range Cells vs. Multipaths



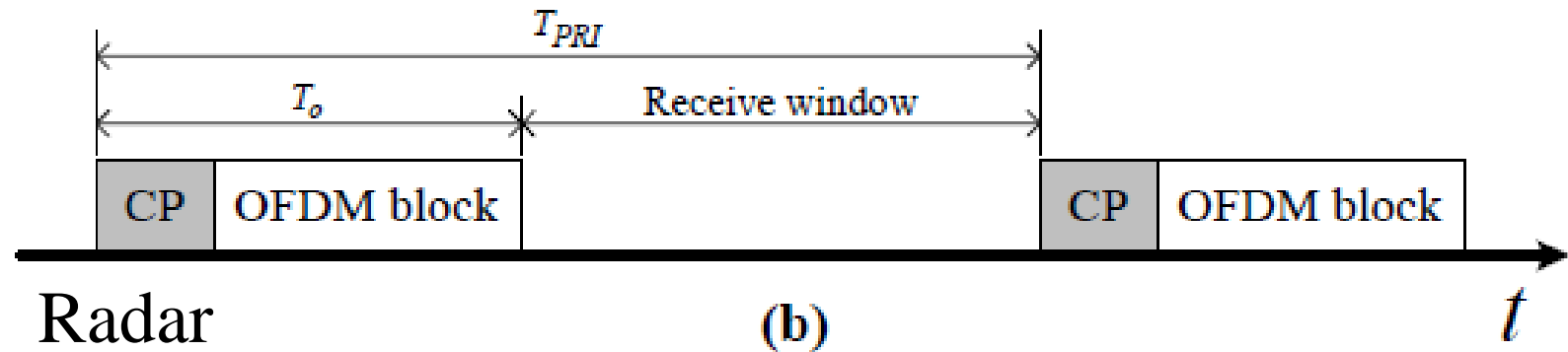
One range cell
in a swath



One path in
communications

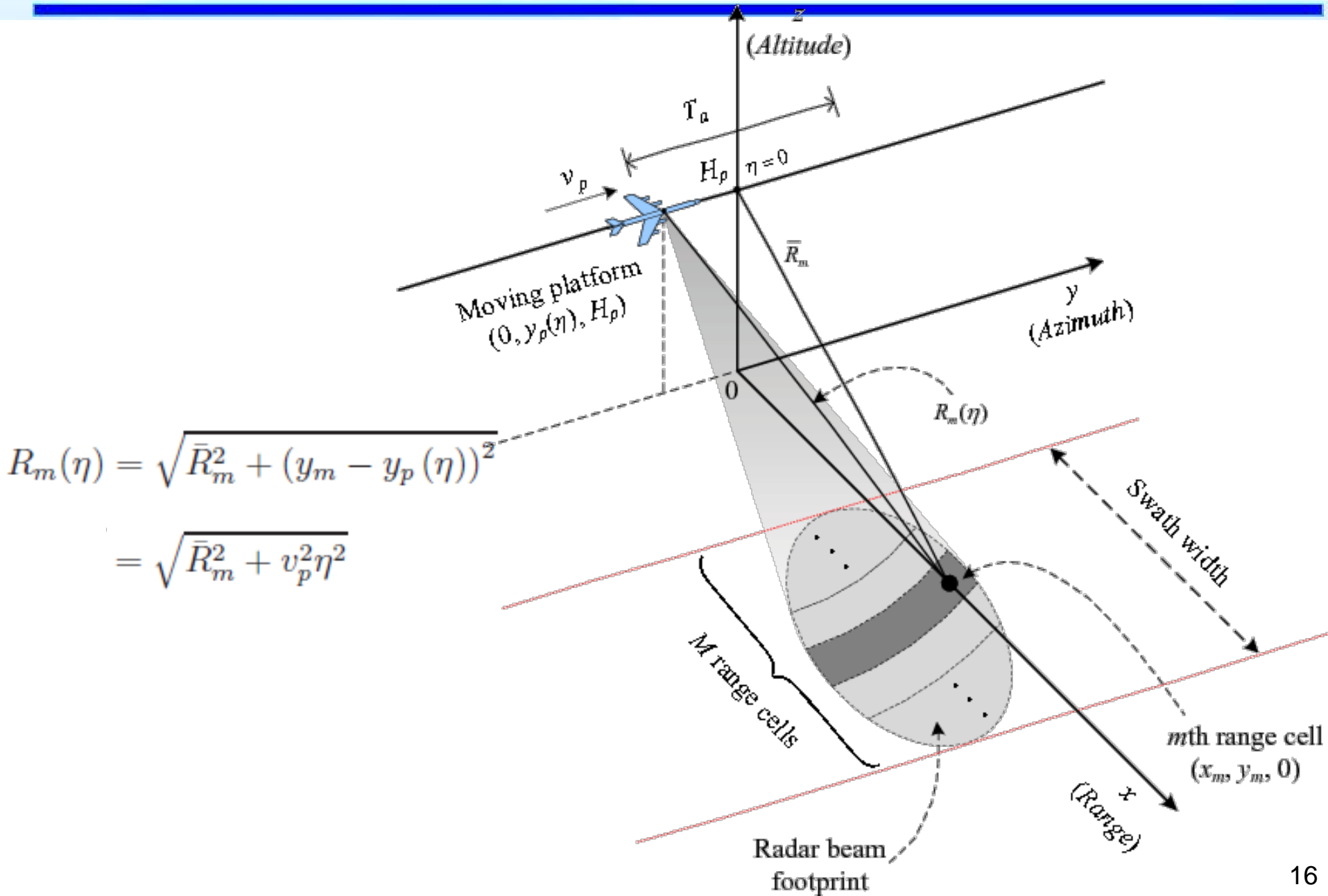


Communications



Radar

SAR Geometry



Transmit and Receive Signal Models

Radar transmitted signal

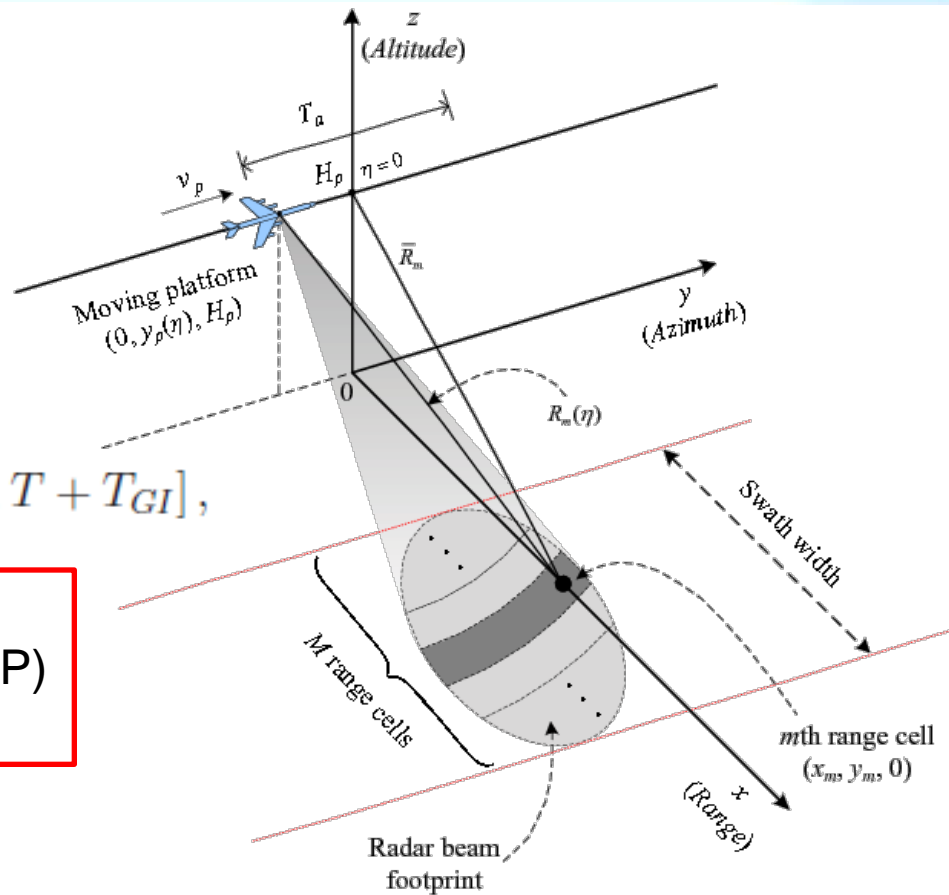
$$s_1(t) = \text{Re} \left\{ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp \{ j2\pi f_k t \} \right\}, t \in [0, T + T_{GI}],$$

T_{GI} is the guard interval length
(the analog time length of the CP)
And will be specified later

Radar return signal from the m th range cell

$$u_m(t, \eta) = g_m w_a(\eta) \exp \left\{ -j4\pi f_c \frac{R_m(\eta)}{c} \right\} \\ \times \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp \left\{ \frac{j2\pi k}{T} \left[t - \frac{2R_m(\eta)}{c} \right] \right\} + v(t, \eta), t \in \left[\frac{2R_m(\eta)}{c}, \frac{2R_m(\eta)}{c} + T + T_{GI} \right]$$

Radar return signal from a swath:
$$u(t, \eta) = \sum_m u_m(t, \eta)$$

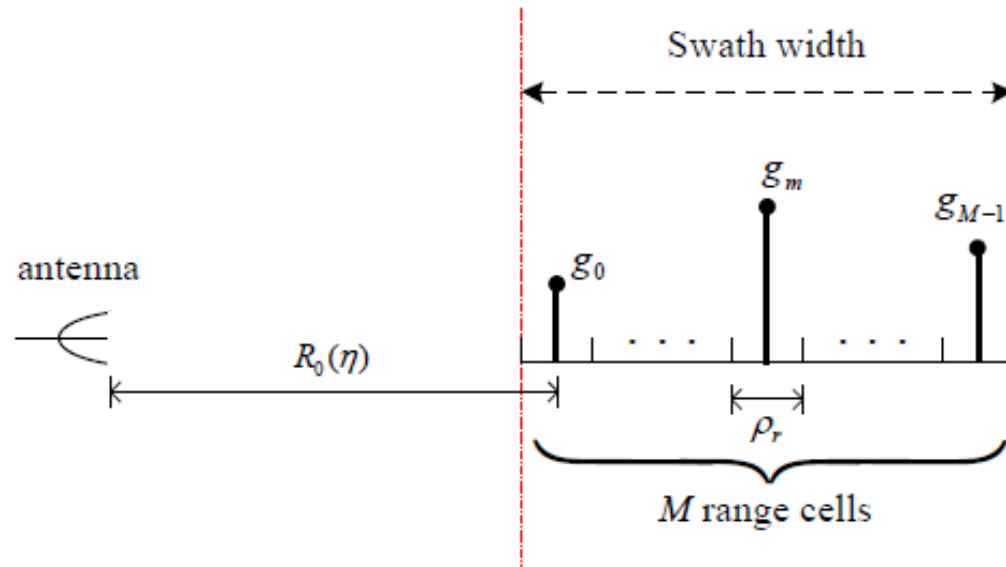


Discrete Received Signal Model

$$u(t, \eta) = \sum_m u_m(t, \eta)$$

$$t - \frac{2R_m(\eta)}{c} = t - \frac{2(R_0(\eta) + m\rho_r)}{c}$$

$$= t - t_0(\eta) - mT_s,$$



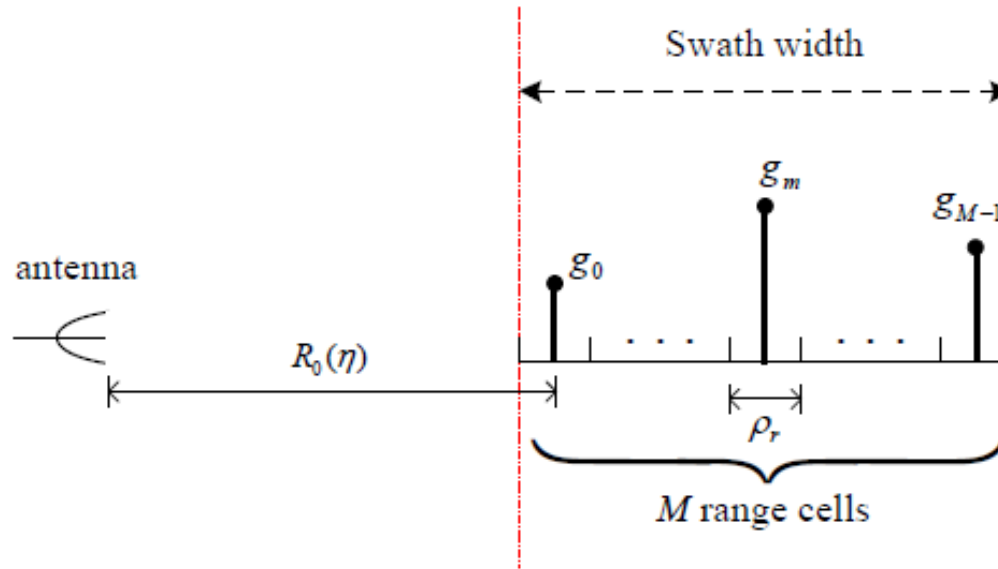
$$u_i = \sum_{m=0}^{M-1} d_m s_{i-m} + v_i, \quad i = 0, 1, \dots, N + 2M - 3,$$

$$d_m = g_m w_a(\eta) \exp \left\{ -j4\pi f_c \frac{R_m(\eta)}{c} \right\},$$

Radar Cross Section (RCS)
Coefficients related

$$s_i = s(iT_s) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp \left\{ \frac{j2\pi ki}{N} \right\}, \quad i = 0, 1, \dots, N + M - 2,$$

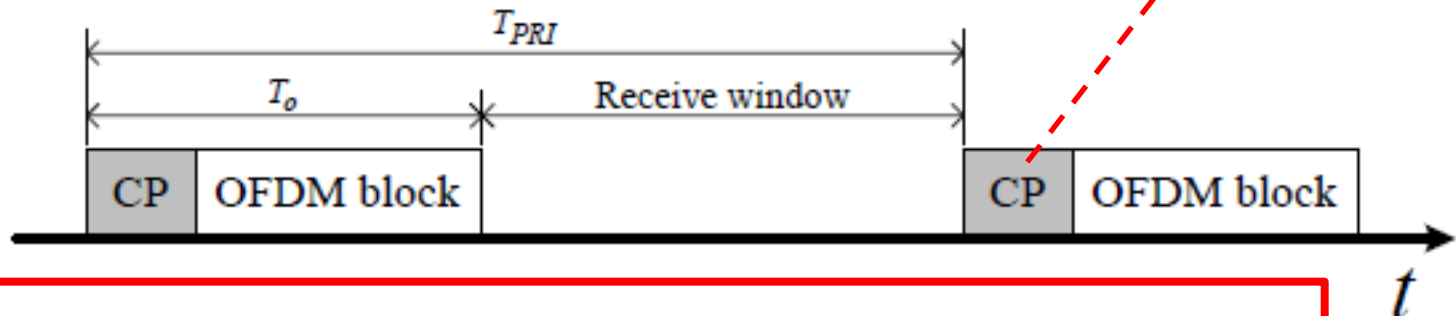
Cyclic Prefix (CP) Length



M range cells \leftarrow --- \rightarrow M paths

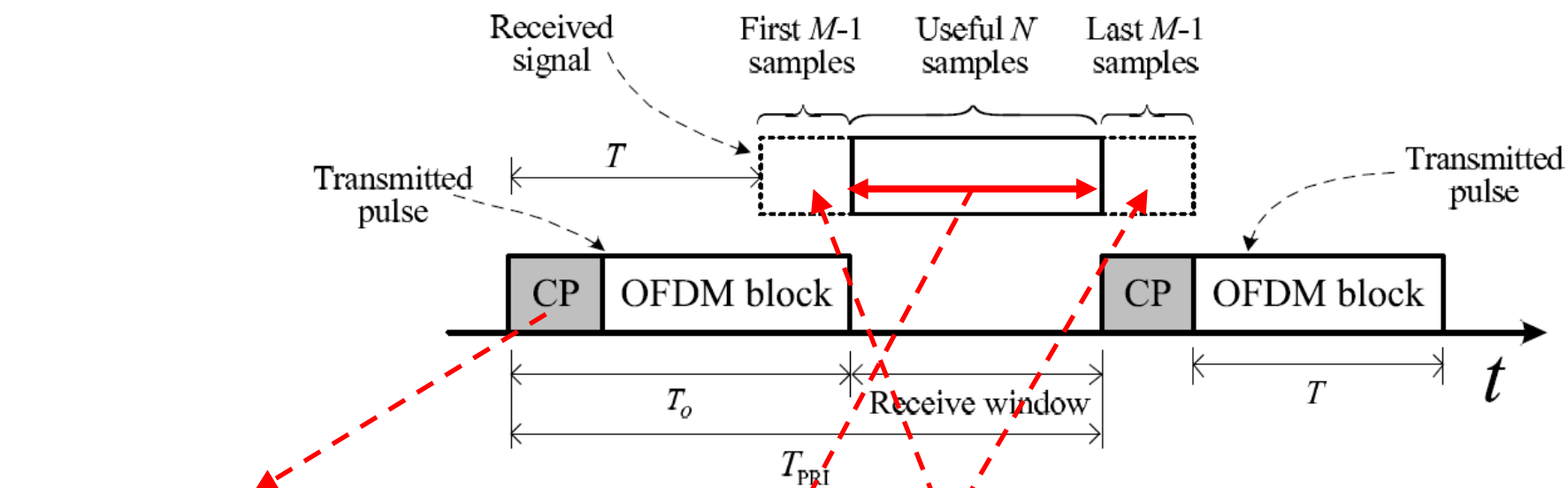
At least $M-1$

Radar



For a wide swathwidth, the CP length needs to be maximized, i.e., $N-1$, for N subcarrier OFDM

CP Removal at Receiver



$M-1$ samples

Remove $M-1$ samples

$$u_n = d_n \otimes s'_n$$

Linear convolution becomes
Cyclic convolution

s'_n is a shifted version of s_n of amount $M-1$

Range Reconstruction



N -point FFT:

$$U_k = D_k S'_k + W_k, \quad k = 0, 1, \dots, N - 1$$

$$D_k = \sum_{m=0}^{M-1} d_m \exp \left\{ \frac{-j2\pi mk}{N} \right\}$$

Frequency domain RCS coefficient estimation

$$\hat{D}_k = \frac{U_k}{S'_k} = \frac{U_k}{S_k \exp \left\{ \frac{j2\pi k(M-1)}{N} \right\}} = D_k + \frac{W_k}{S_k} \exp \left\{ \frac{-j2\pi k(M-1)}{N} \right\}$$

RCS coefficient estimation

$$\hat{d}_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{D}_k \exp \left\{ \frac{j2\pi mk}{N} \right\}$$

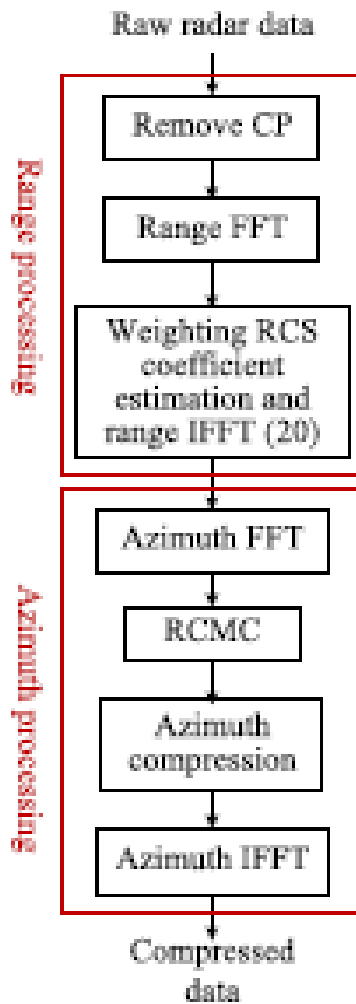
In order not to enhance the noise, the subcarrier weights S_k should have constant module.

$$\hat{d}_m = \sqrt{N} d_m + \tilde{w}'_m, \quad m = 0, \dots, M - 1 \quad \text{-----} \rightarrow \text{IRCI free}$$

An ideal zero sidelobe can be achieved.

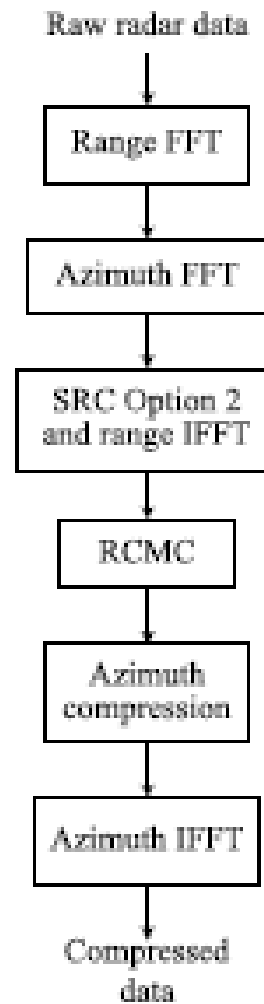
- The CP based OFDM range reconstruction/compression is not the same as the matched filtering
- Although the matched filtering is optimal when IRCI is also treated as a signal part, which may not be optimal when IRCI is considered as non-desired interference that is the case here

SAR Imaging Comparison



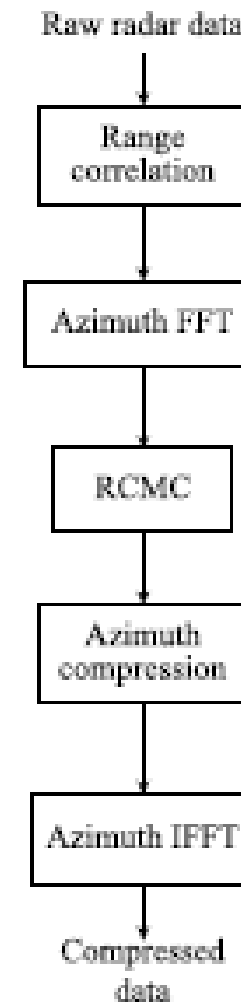
(a)

CP Based OFDM SAR



(b)

LFM SAR



(c) Random noise and the conventional OFDM SAR

Simulation Results: Some Parameters



PRF = 800 Hz,

bandwidth is $B = 150$ MHz,

carrier frequency $f_c = 9$ GHz,

sampling frequency $f_s = 150$ MHz,

platform velocity is $v_p = 150$ m/sec,

platform height of the antenna is $H_p = 5$ km.

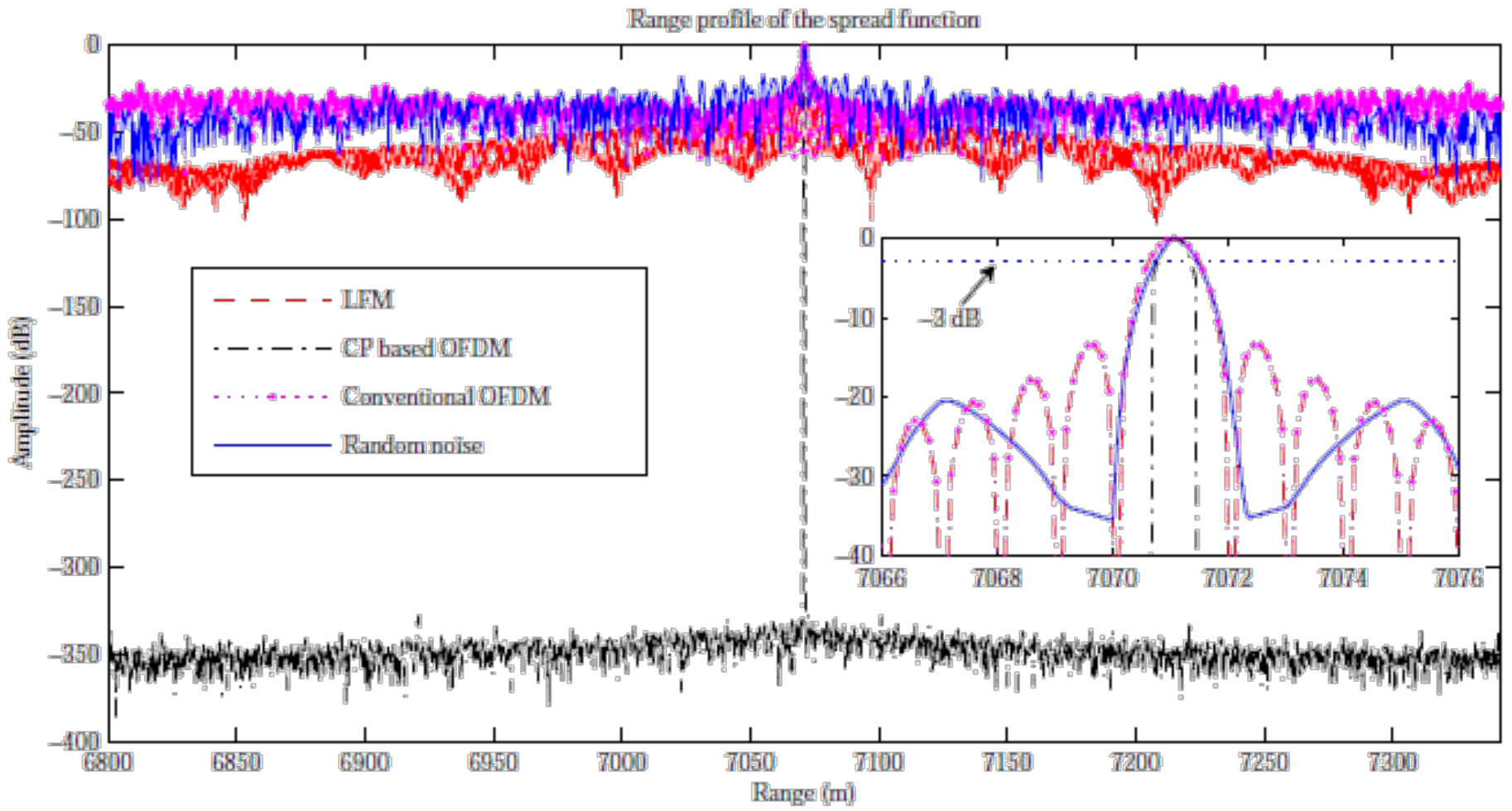
range cells is $M = 96$ in a swath

CP length is 95 that is sufficient and the CP time duration is $T_{GI} = \frac{95}{150} \mu s \approx 0.63 \mu s$.

$T = \frac{512}{150} \mu s \approx 3.41 \mu s$, then the number of subcarriers for the OFDM signal is $N = 512$.

the time duration of an OFDM pulse is $T_o = \frac{607}{150} \mu s \approx 4.05 \mu s$.

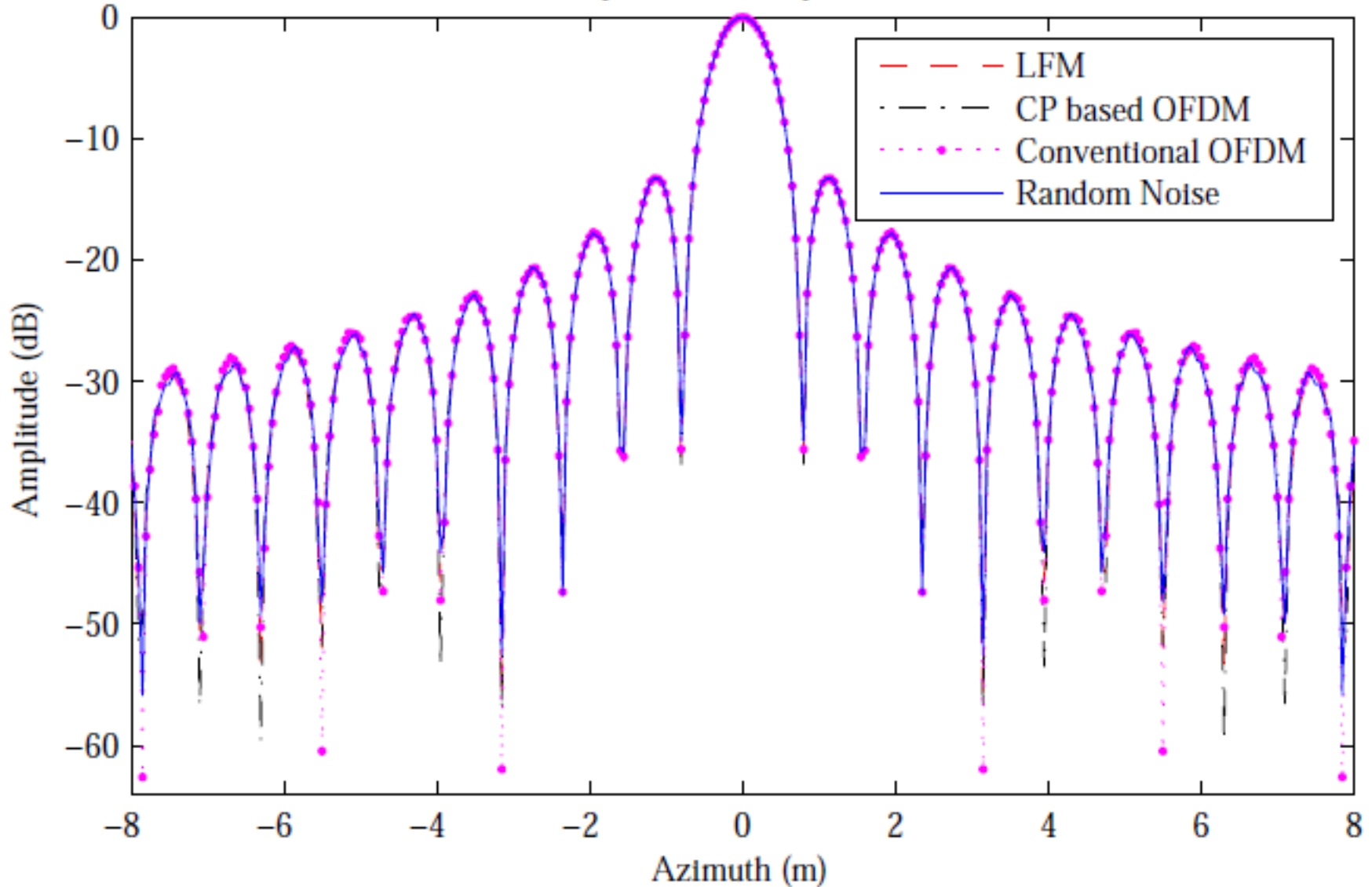
Simulation Results



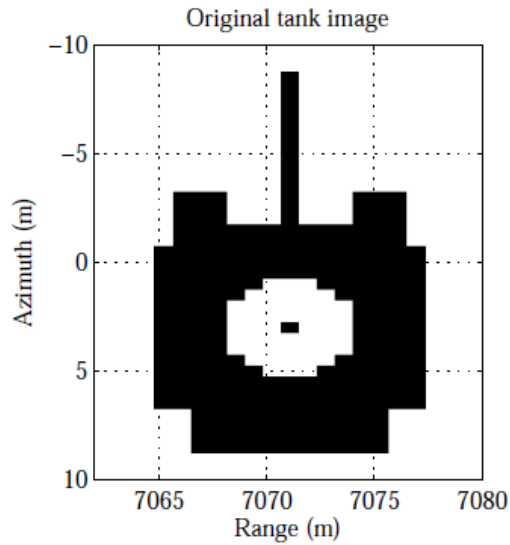
Range profile of a point target

Simulation Results

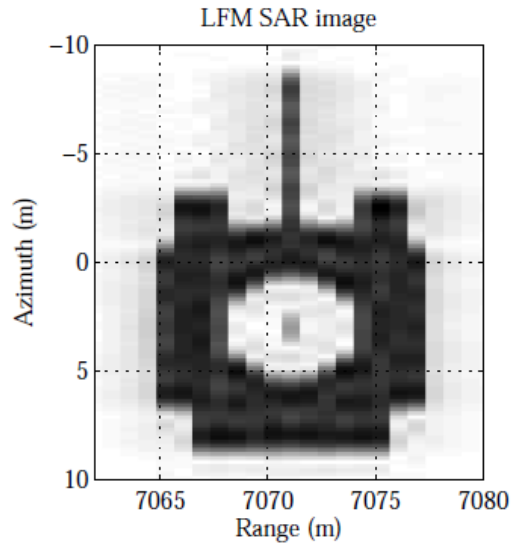
Azimuth profile of the spread function



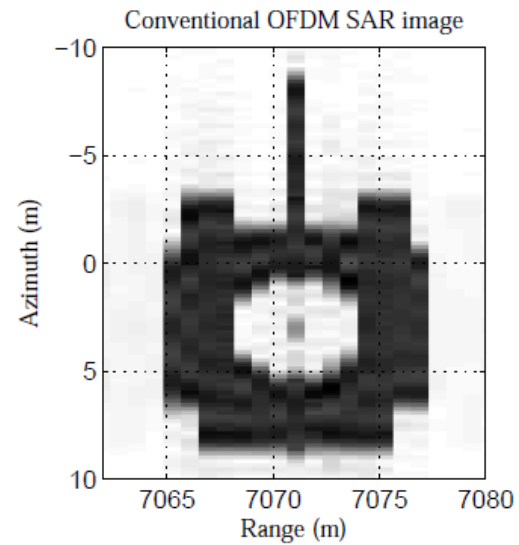
Simulation Results



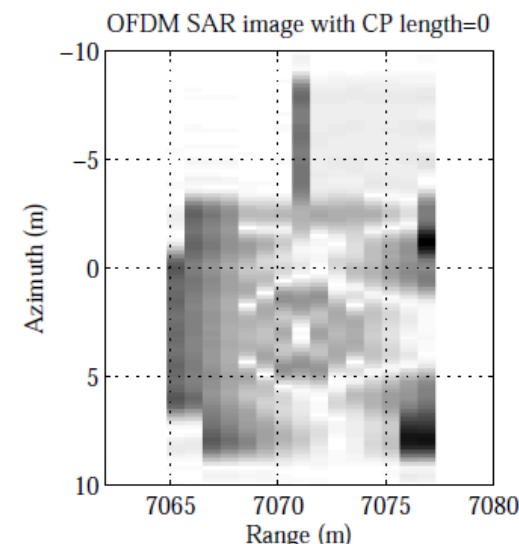
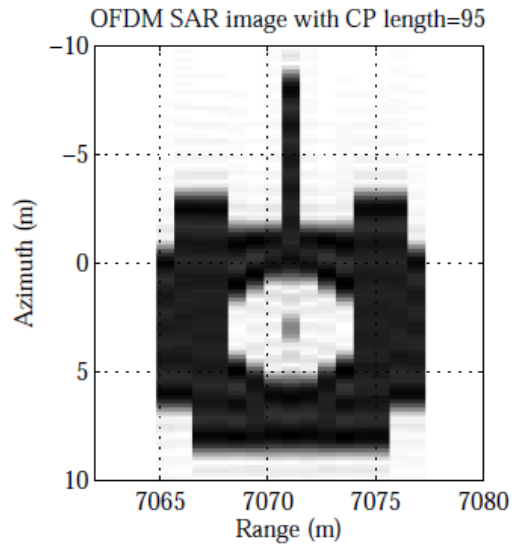
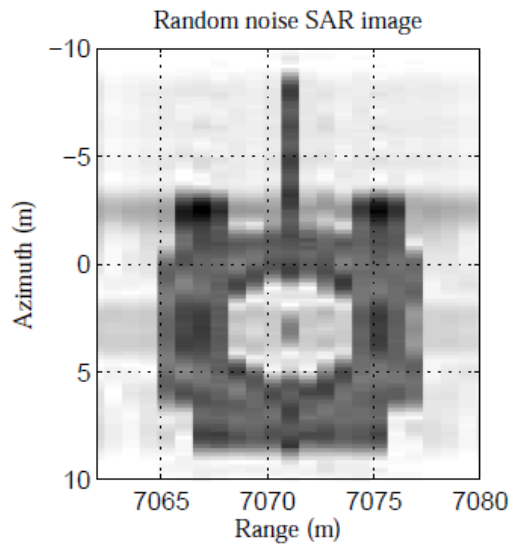
(a)



(b)

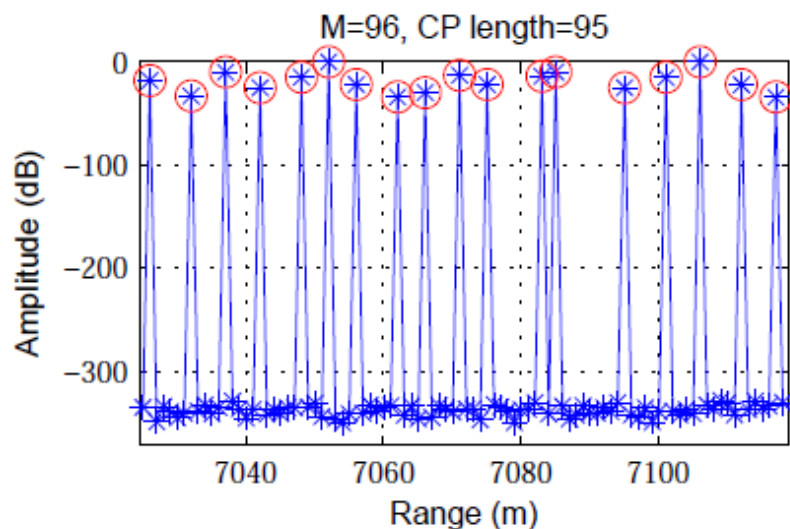


(e)

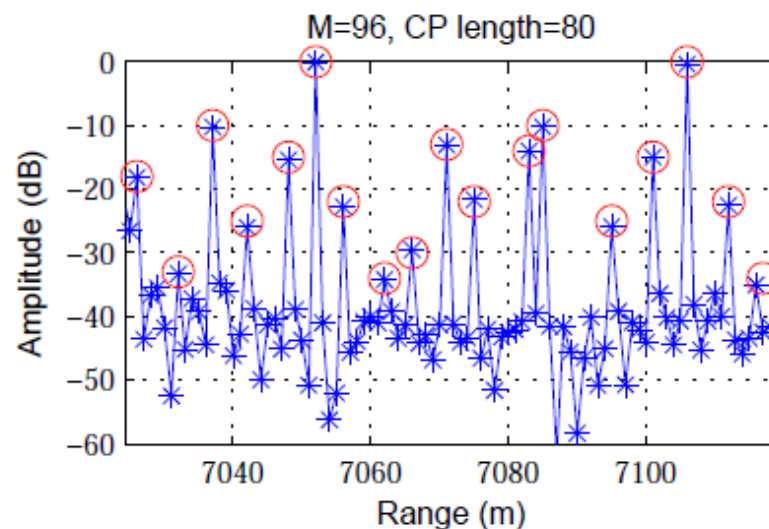


(f)

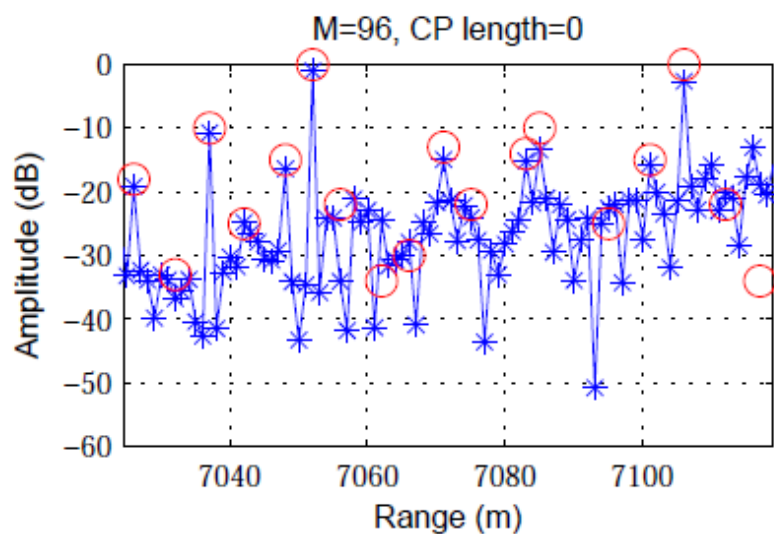
Simulation Results



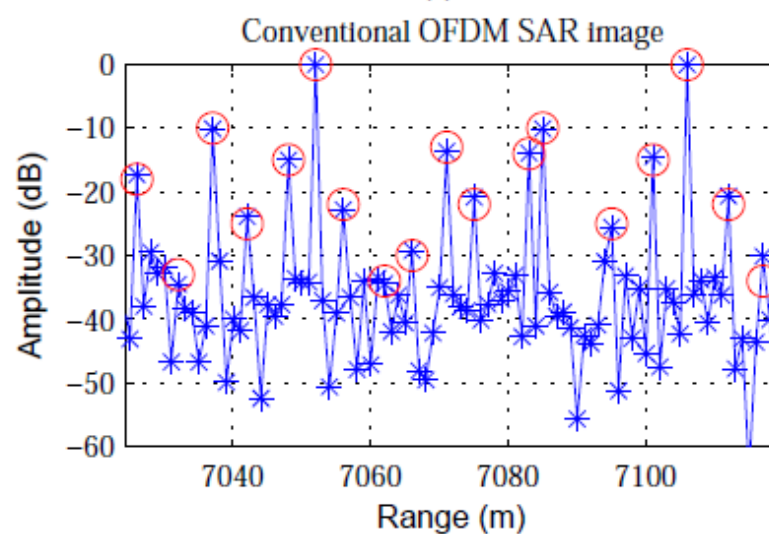
(a)



(b)



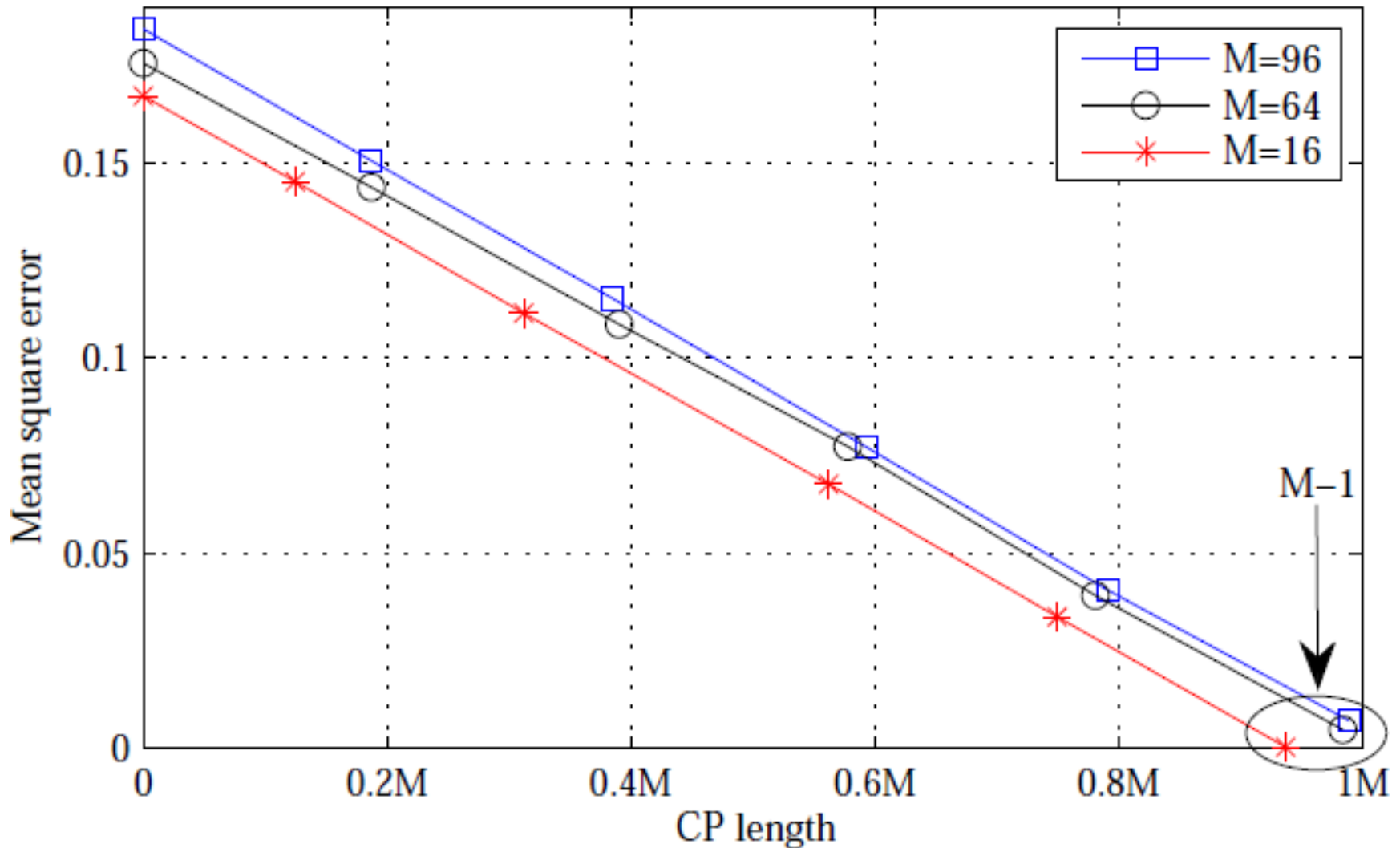
(c)



(d)

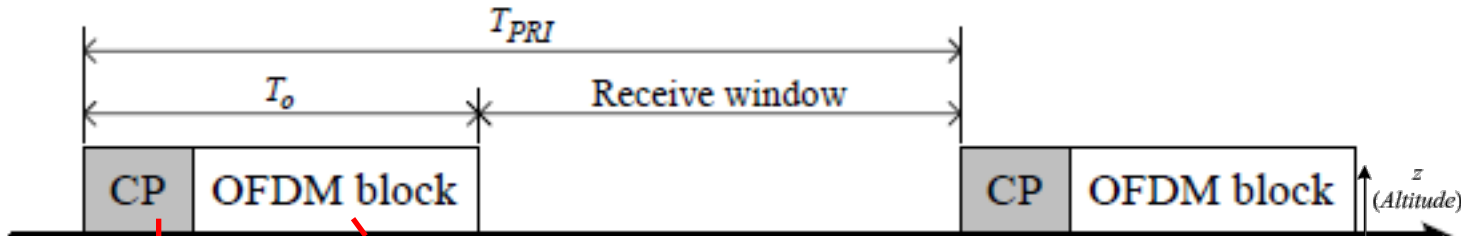
A range line imaging with different CP lengths.

Simulation Results



The mean square errors for insufficient CP lengths.

Pulse Length Problem for Wide Swath SAR



(b)

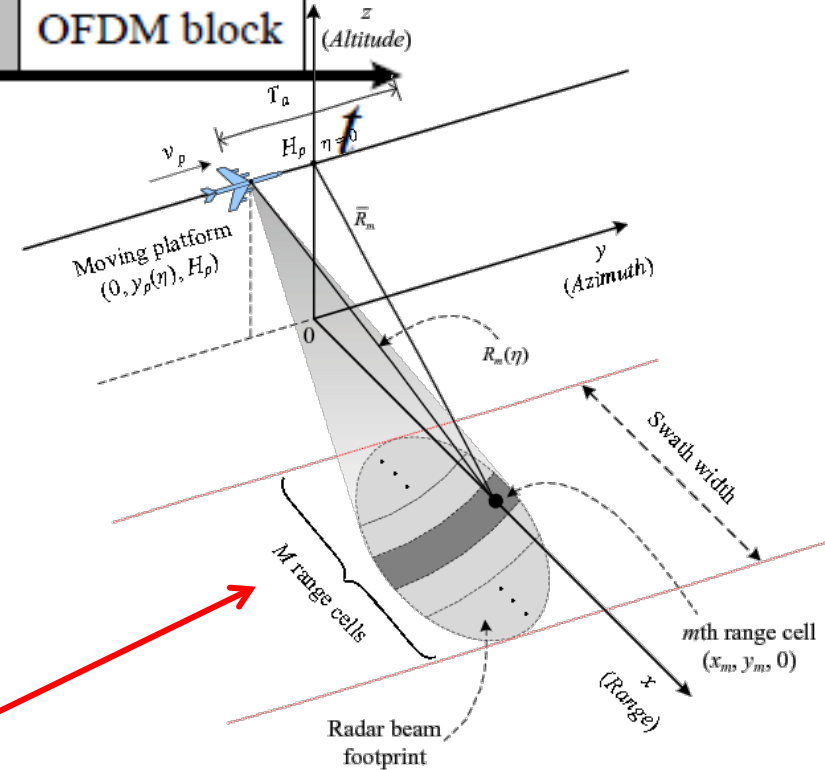
At least
 $M-1$

At least
 M

OFDM transmitted pulse length is at least $2M - 1$



Too long

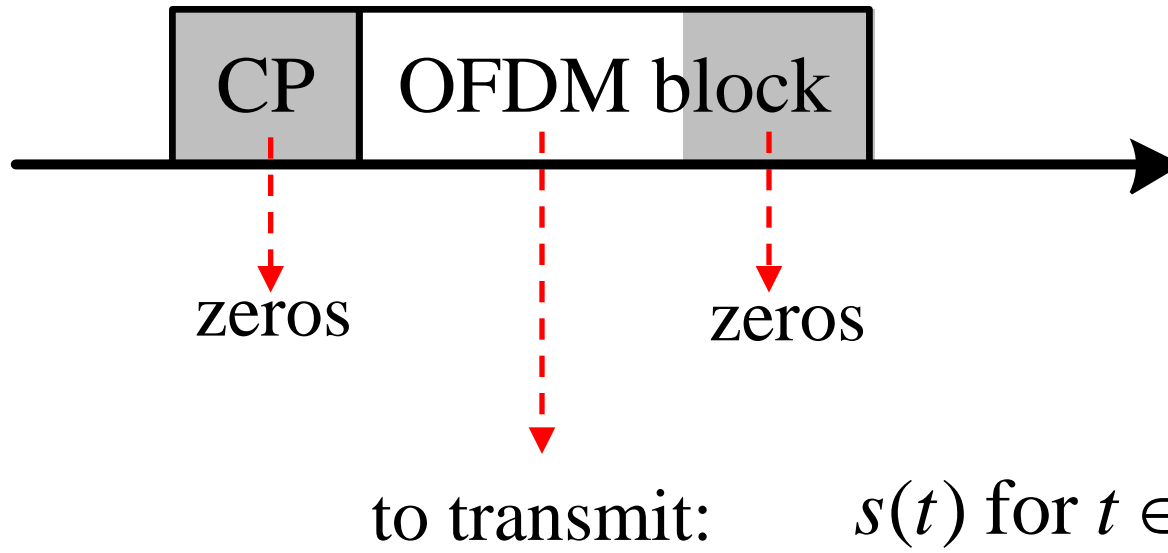


M is the number of range cells in a swath that
Could have thousands of range cells

Question: Can we have arbitrary length CP based OFDM pulses?

To make the CP part all zero-valued:

$$s(t) = 0 \text{ for } t \in [0, T_{GI}) \text{ and } t \in (T, T + T_{GI}]$$



T_{GI} is determined by the swath width, but T can be arbitrary, so the OFDM pulse length $T - T_{GI}$ can be arbitrary

Arbitrary Length OFDM Pulses: Zero Head and Tail Property



Since the range reconstruction only depends on the discrete subcarrier weights S_k in the discrete frequency domain, or equivalently discrete time domain sequence,

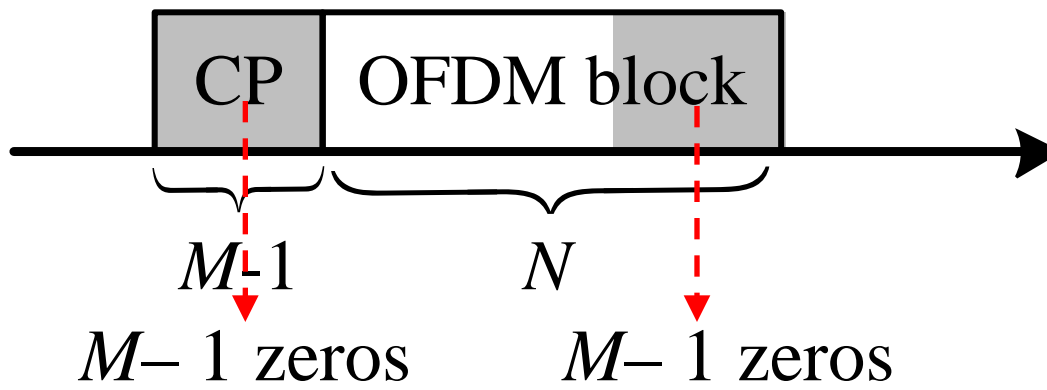
$$s' = [s_0, s_1, \dots, s_{N+M-2}]^T$$

we only need to generate a discrete sequence

$$s' = [0, \dots, 0, s_{M-1}, \dots, s_{N-1}, 0, \dots, 0]^T$$

to correspond to the zero CP OFDM waveform, i.e., the following **zero head and tail property**:

$$[s_0, \dots, s_{M-2}]^T = [s_N, \dots, s_{N+M-2}]^T = 0_{(M-1) \times 1}$$



- If a pulse $s(t)$ has its discrete sequence s_n to satisfy the zero head and tail property, it will be equivalent to a pulse with the zero CP property, and is also equivalent to an OFDM pulse such that its sampled version is

$$s_n = s(nT_s) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp \left\{ \frac{j2\pi kn}{N} \right\}, \quad n = 0, 1, \dots, N + M - 2,$$

- From this property, to design an arbitrary length OFDM pulse, we only need to design subcarrier weights S_k in the frequency domain such that its time domain discrete sequence s_n satisfies the zero head tail property

- To design an OFDM pulse, to design subcarrier weights S_k , $k=0, \dots, N-1$, in frequency domain and to design time domain sequence s_n , $n=0, \dots, N-1$, are equivalent
- Design criteria
 - The time domain sequence $\{s_n\}$ satisfies the zero head and tail property
 - In order to achieve the maximum SNR after the range reconstruction, the frequency domain sequence $\{S_k\}$ should be as constant module as possible (otherwise, the noise will be enhanced in the RCS coefficient estimation)
 - The analog time domain waveform $s(t)$ should have as low peak-to-average power ratio (PAPR) as possible for radar to implement easier, otherwise a delta $\delta(t)$ pulse would serve the above 2 criteria perfectly
- There is, unfortunately, no closed-form solution for the above design problem

Arbitrary Length OFDM Pulses: Design



Time domain: $\mathbf{s} = [\mathbf{0}^{(M-1) \times 1}, s_{M-1}, s_M, \dots, s_{N-1}]^T$

Frequency domain: $\mathbf{S} = [S_0, S_1, \dots, S_{N-1}]^T$

Oversampled time domain sequence of L times to measure the PAPR

$$\tilde{s}_n = \frac{1}{\sqrt{LN}} \sum_{k=0}^{N-1} S_k \exp \left\{ \frac{j2\pi nk}{LN} \right\}, \quad n = 0, \dots, LN - 1,$$

Maximize $S_{\min} = \min |S_k|$ ↔ Minimize

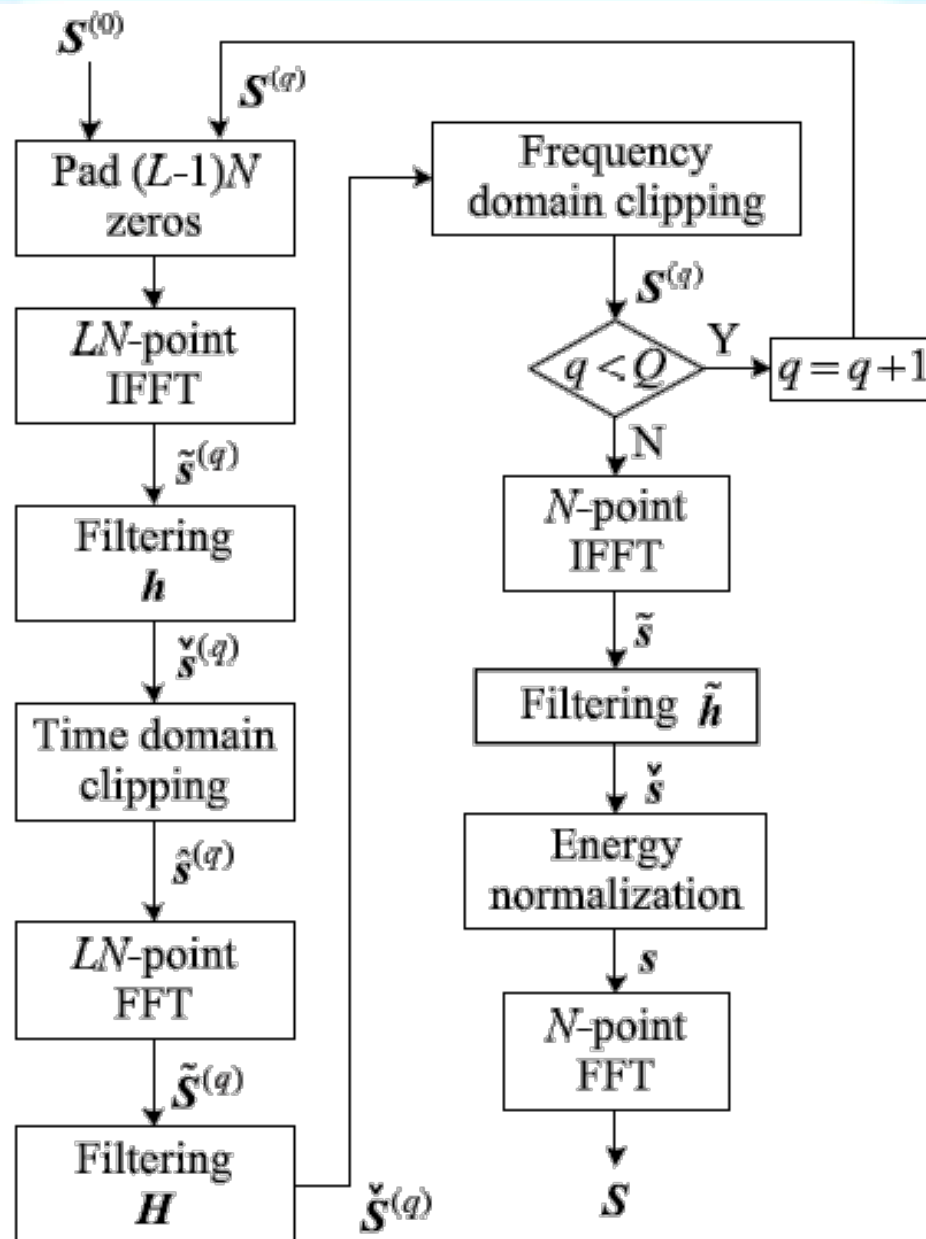
SNR degradation factor

$$\xi = \frac{\text{SNR}_m}{\text{SNR}_{\max, m}} = \frac{N^2}{\sum_{k=0}^{N-1} |S_k|^{-2}}$$

Minimize PAPR =
$$\frac{\max_{n=0, \dots, LN-1} |\tilde{s}_n|^2}{\frac{1}{LN} \sum_{n=0}^{LN-1} |\tilde{s}_n|^2}$$

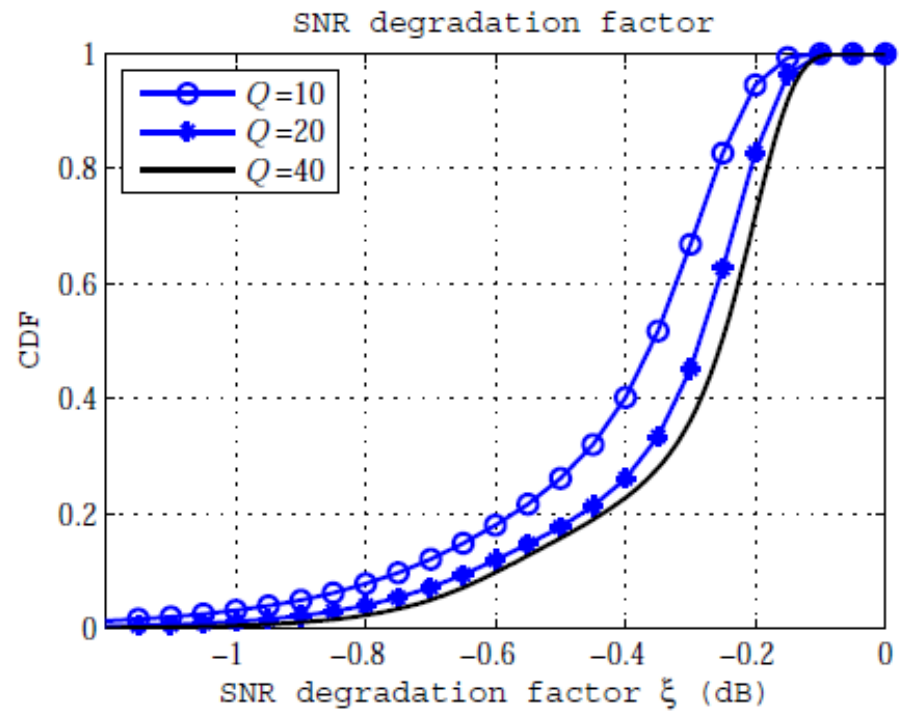
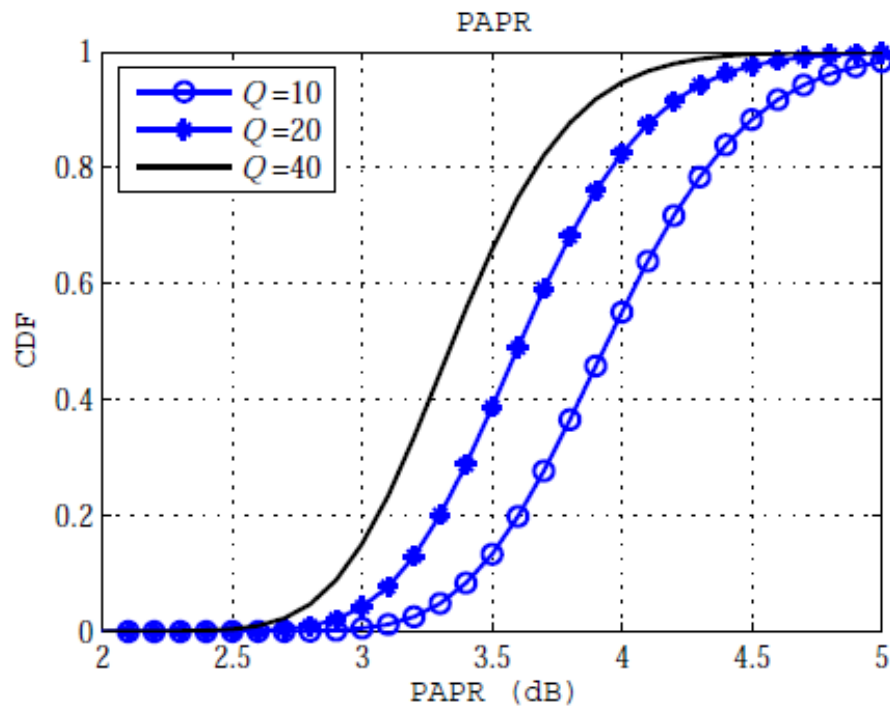
Arbitrary Length OFDM Pulses Design: An Iterative Method

Block diagram of the
OFDM sequence
design algorithm.



Simulation Results

Q: Iteration number



Simulation Results

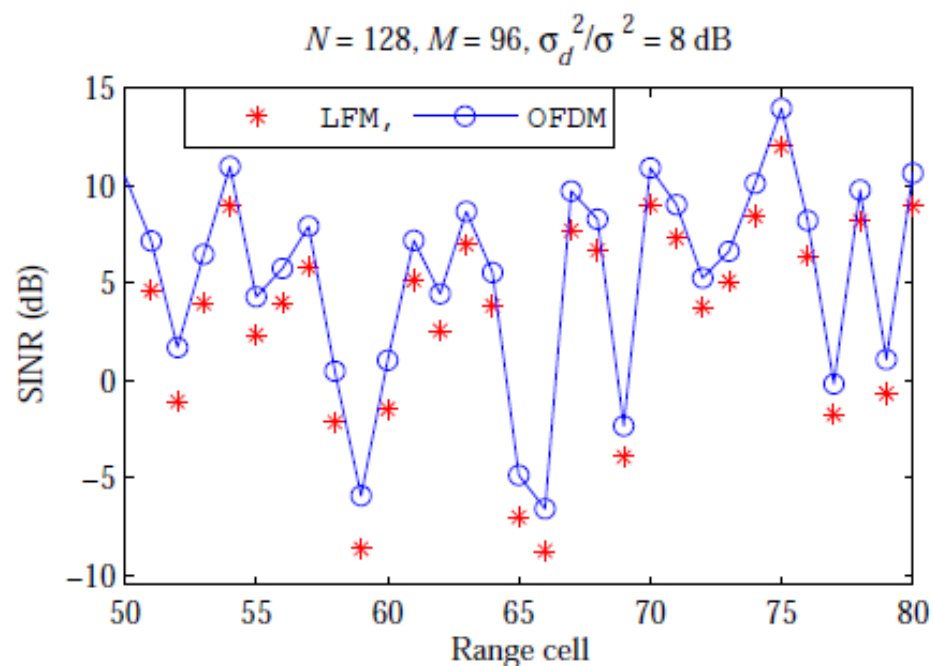
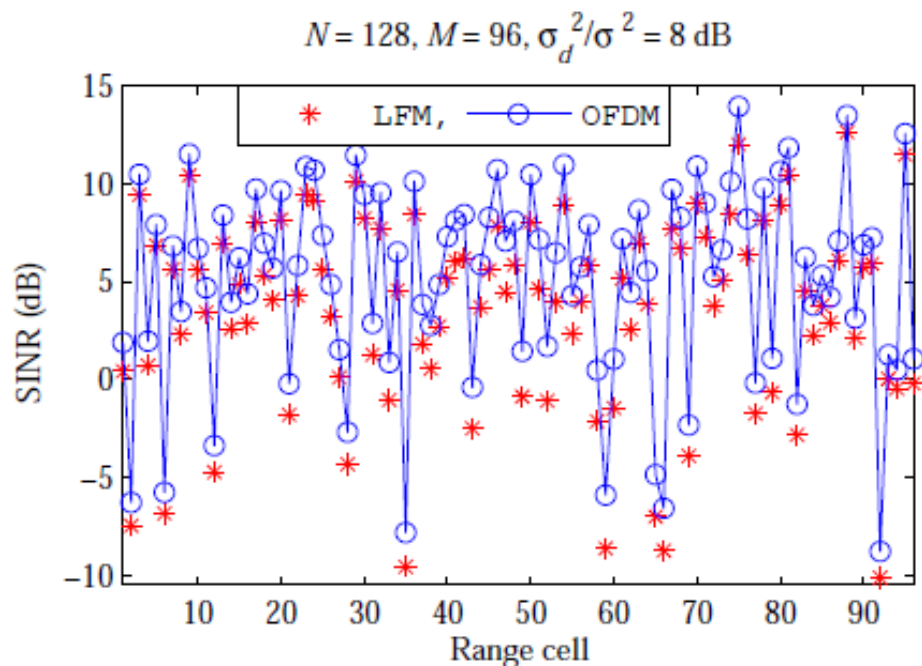
Number of subcarrier sequences $S_k, k=0, 1, \dots, N-1$.

	$\xi \geq -0.1$ dB	$\xi \geq -0.2$ dB	$\xi \geq -0.4$ dB
PAPR ≤ 2 dB	4	5	7
PAPR ≤ 2.5 dB	145	1511	2134
PAPR ≤ 3 dB	615	35036	69735
Total number of trials: 5×10^5			

SNR degradation
factor

$S_{min} \geq 0.88 \frac{1}{\sqrt{N}}$	$S_{min} \geq 0.85 \frac{1}{\sqrt{N}}$	$S_{min} \geq 0.8 \frac{1}{\sqrt{N}}$	$S_{min} \geq 0.5 \frac{1}{\sqrt{N}}$
7	371	14415	353782
Total number of trials: 5×10^5			

Simulation Results



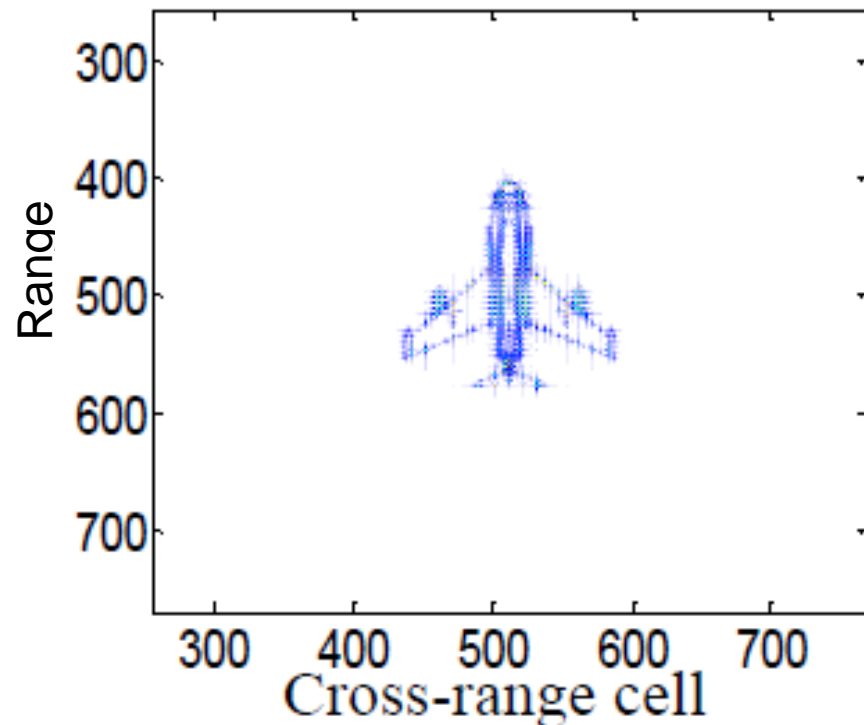
SINRs after the range reconstructions using an LFM pulse and a designed OFDM pulse

OFDM ISAR Imaging

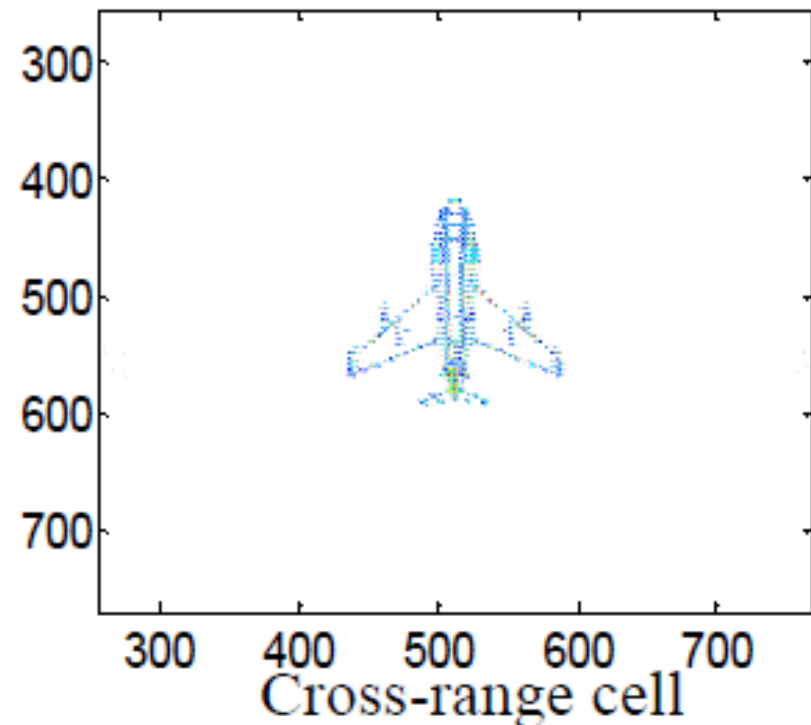
M. Wu, Xia, Xing 2015

R_0	10 km	N_{cp}	511
B	750 MHz	N	512
T_p	$0.7 \mu s$	f_c	9.6 GHz
PRF	200 Hz	f_s	750 MHz
		Ω	0.03 rad/s

LFM



OFDM



Target translational velocity and acceleration are 100 m/s and 5 m/s², respectively.

- Co-located MIMO Radar
 - Cannot achieve the spatial diversity (one antenna signal is poor, the others may not)
- Statistical MIMO Radar
 - Signal waveforms of the same bandwidth from different Tx antennas may have different time delays and thus may not be orthogonal in **time domain**
 - They may not achieve the spatial diversity if the range resolution is not reduced, i.e., all the waveforms from all the Tx antennas have the same bandwidth.
 - In order to have the orthogonality of the waveforms from different Tx antennas in **time domain**, the waveforms need to have non-overlapping frequency bands
 - Their bandwidths are reduced and thus their range resolution is reduced.

- Sufficient CP based OFDM radar can be extended to MIMO radar
 - Orthogonality of the transmit signals across multiple transmit antennas hold in the discrete frequency domain, which is not affected by the time delays from different transmit antennas in the time domain
 - All transmit signals from multiple transmit antennas may share the same bandwidth, and thus, the range resolution is not reduced
 - Subcarrier weights S_k in single transmit OFDM radar becomes subcarrier weight matrices \mathbf{S}_k
 - Constant module subcarrier weights S_k for single transmit OFDM radar with arbitrary pulse length are hard to achieve and thus lead to the SNR degradation after the IRCI free range reconstruction
 - Unitary subcarrier weight matrices \mathbf{S}_k are very easy to construct and can be parameterized and thus the SNR is not degraded

OFDM波形设计约束条件

发射脉冲大
发射能量冗余
最小作用距离大

CP为零

$$\left[s_{\alpha,0}^{(p)}, s_{\alpha,1}^{(p)}, \dots, s_{\alpha,\eta_{max}+M-2}^{(p)} \right]^T = \left[s_{\alpha,N}^{(p)}, s_{\alpha,N+1}^{(p)}, \dots, s_{\alpha,N+\eta_{max}+M-2}^{(p)} \right]^T = \left[\underbrace{0, 0, \dots, 0}_{\eta_{max}+M-1} \right]^T$$

(1) 发射脉冲间频域子载波正交性约束: $\mathbf{S}_k \mathbf{S}_k^+ = \mathbf{I}_T$

Not enhance noise

(2) 时域约束: **CP为零**

(3) 频域恒模约束: $\sum_{p=0}^{P-1} |s_{\alpha,k}^{(p)}|^2 = \frac{1}{NT}$, paraunitariness in time domain

Maximize the SNR after the imaging

(4) PAPR约束。

优先级



CP MIMO OFDM Waveforms



Orthogonal design based approach $\mathbf{S}_k \mathbf{S}_k^+ = \mathbf{I}_T$

Orthogonal Design: $\mathbf{X} \mathbf{X}^\dagger = (|x_1|^2 + |x_2|^2 + \dots + |x_P|^2) \mathbf{I}_T$

0, x_i , $-x_i$, x_i^* 和 $-x_i^*$

Alamouti code for 2 Tx antennas

$$\mathbf{X}_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$



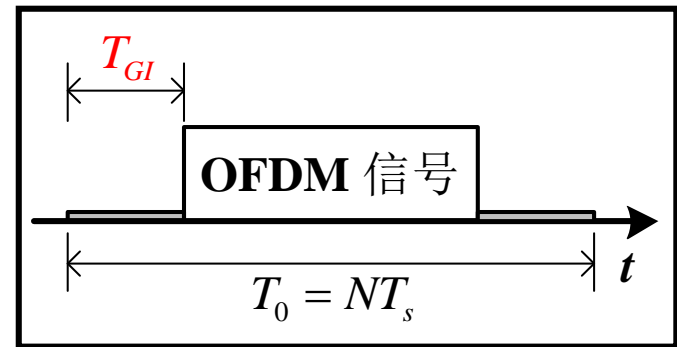
$$\mathbf{S}_k = \begin{bmatrix} S_{1,k}^{(0)} & S_{1,k}^{(1)} \\ S_{2,k}^{(0)} & S_{2,k}^{(1)} \end{bmatrix} = \begin{bmatrix} S_{1,k}^{(0)} & S_{1,k}^{(1)} \\ -(S_{1,k}^{(1)})^* & (S_{1,k}^{(0)})^* \end{bmatrix}$$

频域

$$\begin{aligned} S_{2,k}^{(0)} &= -(S_{1,k}^{(1)})^* \\ S_{2,k}^{(1)} &= (S_{1,k}^{(0)})^* \end{aligned}$$

时域

$$\begin{aligned} S_{2,i}^{(0)} &= -(S_{1,N-i}^{(1)})^* \\ S_{2,i}^{(1)} &= (S_{1,N-i}^{(0)})^* \end{aligned}$$



一个周期内时域翻转

CP为零

$$\left[S_{\alpha, N-\eta_{\max}-M+2}^{(p)}, S_{\alpha, N-\eta_{\max}-M+3}^{(p)}, \dots, S_{\alpha, N-1}^{(p)} \right]^T = \left[\underbrace{0, 0, \dots, 0}_{\eta_{\max}+M-2} \right]^T$$

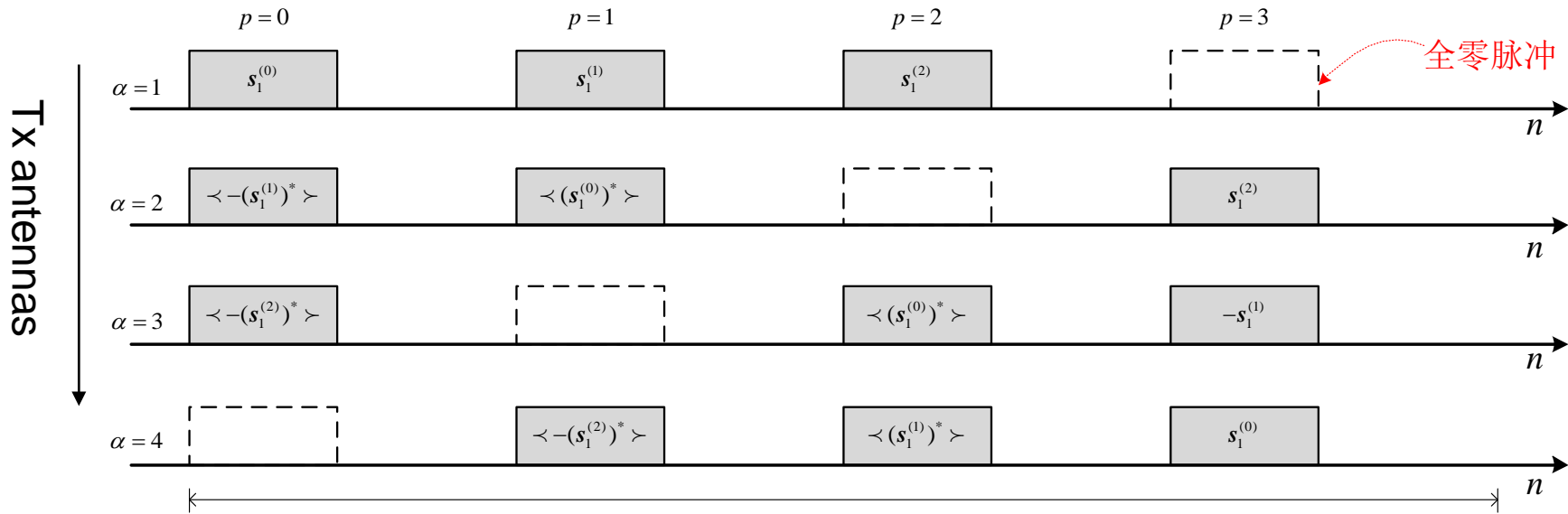
时域约束 (强化)

CP Based MIMO-OFDM Radar



Orthogonal design in discrete frequency domain

$$\mathbf{X}_4 = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ -x_3^* & 0 & x_1^* & -x_2 \\ 0 & -x_3^* & x_2^* & x_1 \end{bmatrix} \rightarrow \mathbf{S}_k = \begin{bmatrix} S_{1,k}^{(0)} & S_{1,k}^{(1)} & S_{1,k}^{(2)} & S_{1,k}^{(3)} \\ S_{2,k}^{(0)} & S_{2,k}^{(1)} & S_{2,k}^{(2)} & S_{2,k}^{(3)} \\ S_{3,k}^{(0)} & S_{3,k}^{(1)} & S_{3,k}^{(2)} & S_{3,k}^{(3)} \\ S_{4,k}^{(0)} & S_{4,k}^{(1)} & S_{4,k}^{(2)} & S_{4,k}^{(3)} \end{bmatrix} = \begin{bmatrix} S_{1,k}^{(0)} & S_{1,k}^{(1)} & S_{1,k}^{(2)} & 0 \\ -\left(S_{1,k}^{(1)}\right)^* & \left(S_{1,k}^{(0)}\right)^* & 0 & S_{1,k}^{(2)} \\ -\left(S_{1,k}^{(2)}\right)^* & 0 & \left(S_{1,k}^{(0)}\right)^* & -S_{1,k}^{(1)} \\ 0 & -\left(S_{1,k}^{(2)}\right)^* & \left(S_{1,k}^{(1)}\right)^* & S_{1,k}^{(0)} \end{bmatrix}$$



一个 CPI 内 $P = 4$ 个脉冲

12

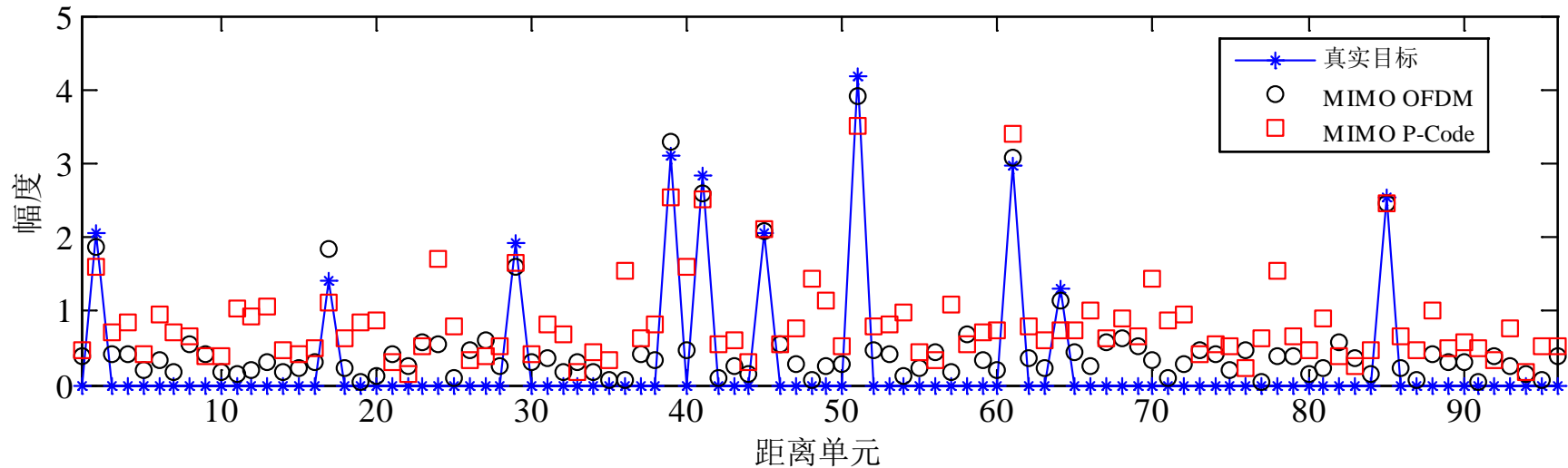
p is the CP OFDM pulse index

Here is 4 CP OFDM pulses together to form a block

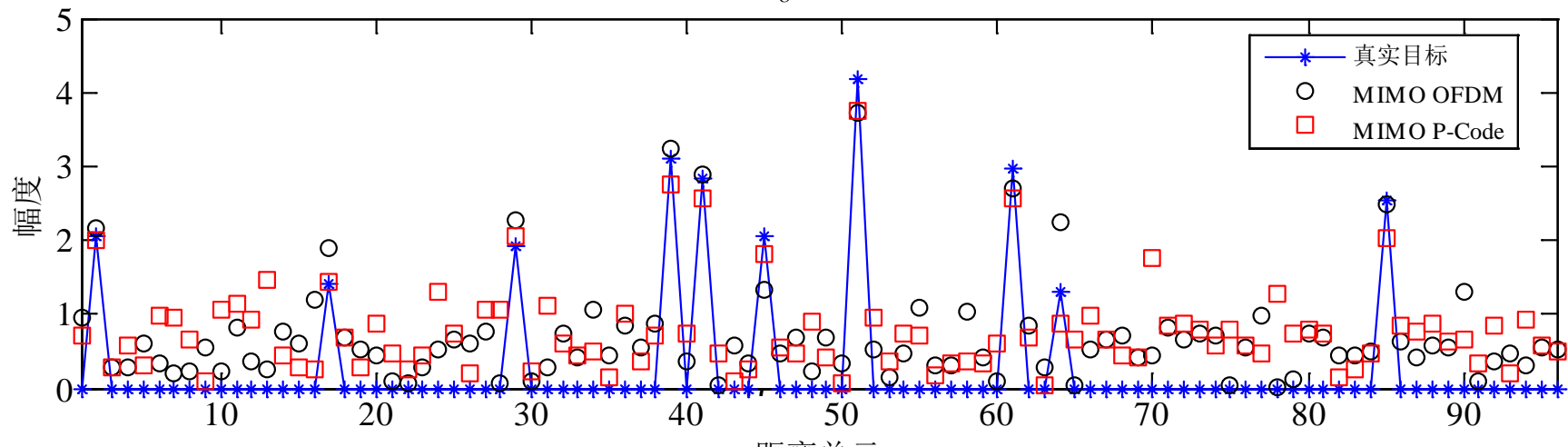
CP MIMO-OFDM Radar

2 Tx antennas, 309 subcarriers, 10 targets over 96 range cells, PAPR=2dB
Non-zero pulse length is $40T_s$, compared with polyphase code, p-code

(a). $\sigma_g^2/\sigma^2=12$ dB



(b). $\sigma_g^2/\sigma^2=8$ dB



Conclusions



- Sufficient CP based OFDM SAR may be a promising technique to improve range resolution with zero sidelobes (or IRCI free) in the range reconstruction
 - Adding sufficient CP at transmitter
 - Use a new range reconstruction (not matched filtering)
- Sufficient CP based MIMO-OFDM has advantages for MIMO radar to overcome the existing MIMO radar shortcomings, in particular for statistical MIMO radar
 - Can collect full spatial diversity
 - No range resolution is reduced
- 更容易做到通信雷达一体化
- In communications business, OFDM has become the most updated standard (5G, 4G) and the spread spectrum is already the past (2G and 3G)
 - We are still spread spectrum radar
 - What will happen to OFDM radar?

Some of Our Papers



- T. Zhang and X.-G. Xia, "OFDM synthetic aperture radar imaging with sufficient cyclic prefix," *IEEE Trans. on Geoscience and Remote Sensing*, Vol. 53, No. 1, pp. 394-404, Jan. 2015.
- T. Zhang, X.-G. Xia, and L. Kong, "IRCI free range reconstruction for SAR imaging with arbitrary length OFDM pulse," *IEEE Trans. on Signal Processing*, vol. 42, no. 21, pp. 5775-5786, Nov. 2014.
- X.-G. Xia, T. Zhang, and L. Kong, "MIMO OFDM radar IRCI free range reconstruction with sufficient cyclic prefix," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 51, no. 3, pp. 2276-2293, July 2015.
- Y.-H. Cao, X.-G. Xia, and S.-H. Wang, "IRCI free co-located MIMO radar based on sufficient cyclic prefix OFDM waveforms," *IEEE Trans. on Aerospace and Electronic Systems*, vo. 51, no. 3, pp. 2107-2120, July 2015.
- Y.-H. Cao and X.-G. Xia, IRCI free MIMO-OFDM SAR using circularly shifted Zadoff-Chu sequences, *IEEE Geoscience and Remote Sensing Letters*, vol. 12, no. 5, pp.1126-1130, May 2015.

Thanks!