A Family of Distributed Space-Time Trellis Codes With Asynchronous Cooperative Diversity

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***Abstract*—In current cooperative communication schemes, to achieve cooperative diversity, synchronization between terminals is usually assumed, which may not be practical since each terminal has its own local oscillator. In this paper, based on the stack construction proposed by Hammons and El Gamal, we first construct a family of space-time trellis codes for BPSK modulation scheme that is characterized to possess the full cooperative diversity order without the synchronization assumption. We then generalize this family of the space–time trellis codes from BPSK to higher order QAM and PSK modulation schemes based on the unified construction proposed by Lu and Kumar. Some diversity product properties of space–time trellis codes are studied and simplified decoding methods are discussed. Simulation results are given to illustrate the performance of the newly proposed codes.**

***Index Terms*—Asynchronous cooperative diversity, multiple-input multiple-output (MIMO), sensor networks, space–time trellis codes.**

I. Introduction

In wireless communication systems, to combat fading, multiple antennas may be equipped at the transmitter and/or the receiver, where multiple antennas may provide spatial diversity gain as well as multiplexing gain. However, in cellular systems or sensor networking systems, it may be hard for a mobile station or a sensor terminal to equip with multiple antennas because of their limited sizes and also the cost. If a system is a single user point-to-point communication system, no spatial diversity gain can be exploited without multiple antennas. By realizing that a cellular or sensor networking system usually has multiple users, the idea of making different users to communicate cooperatively to achieve the spatial diversity gain has been proposed in, for example, [1]–[4], and such spatial diversity is called *cooperative diversity*.

In [3] and [4], different cooperative protocols are devised and their outage performances are analyzed. In [4], the problem of space–time code design for the cooperative communication is also proposed, and the orthogonal space–time block codes are used. In [5], to achieve cooperative diversity, transmitted symbols are estimated but not detected in relays, then the relays forward their estimated symbols to the destination according to the structure of orthogonal matrices. It is shown that this scheme can achieve diversity order of $M/2$, where M is the number of relays involved. In [6] and [7], distributed channel codes are proposed to achieve the cooperative diversity. In [9] and , linear dispersion space–time codes are used and the achieveable diversity order is analyzed. In all these schemes, synchronization is assumed as an *a priori* condition. But different from the spatial diversity provided by an antenna array in one terminal where only one local oscillator is used, the cooperative diversity is provided by different antennas in different terminals, where each terminal has its own local oscillator. Thus, the cooperative diversity is asynchronous in nature.

The method to achieve the cooperative diversity where synchronization between relays is not a required condition is discussed in [10]. In [10], intentional delays are introduced in different terminals. At the destination receiver, minimum mean square error (MMSE) estimator is used to exploit the cooperative diversity. Although some diversity gain can be achieved in [10], full diversity order is not guaranteed. The full diversity order means that the diversity order equals to the number of involved relays. The goal of this paper is to present a systematic construction of space–time trellis codes that can achieve the full cooperative diversity order in asynchronous cooperative communications for any number of involved relays.

In this paper, based on the stack construction in [14], a family of space–time trellis codes for BPSK modulation scheme is constructed. When this family of space–time trellis codes is used to exploit the cooperative diversity without the symbol synchronization requirement, the full diversity order is guaranteed/proven. Based on the unified construction proposed in [15] and [16], this family of space–time trellis codes is generalized to higher order modulation schemes, such as QAM and PSK. Some diversity product properties of the space–time trellis codes are studied in asynchronous cooperative communications. Our simulation results show that when relative timing errors/differences are known at the destination receiver and the optimum decoding method is used, the newly proposed space–time trellis codes perform even better when there are relative timing errors/differences, i.e., asynchronous case, than when there is no relative timing errors/differences, i.e., synchronous case, which differs from the existing space–time codes. Simplified decoding methods, such as M-algorithm, are discussed, and it is shown that we can trade (negligible) performance loss for (significantly) reduced decoding complexity.

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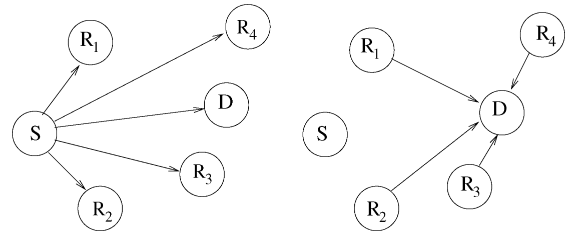

(a) Phase I (b) Phase II

Fig. 1. System architecture.

This paper is organized as follows. In Section II, the system model is described and the problem of interest is formulated. In Section III, the space–time trellis code family is constructed. In Section IV, some diversity product properties of space–time trellis codes are studied. In Section V, some simplified decoding methods are discussed and some simulation results are presented.

II. System Model and Problem Formulation

In our system, we assume that there are $M+2$ terminals that communicate cooperatively. The system is shown in Fig. 1. We assume that S is the source terminal, D is the destination terminal, and R_i, $i=1,2,\ldots,M$, are the potential relays. As in the analysis carried out in [3] and [4], we assume that there are two phases during the cooperative communication. In Phase I [Fig. 1(a)], S broadcasts its information to potential relays R_i, $i=1,2,\ldots,M$, and the destination D. In Phase II [Fig. 1(b)], S stops transmission, and the potential relays start to transmit. There are two different transmission schemes for a potential relay [3], [4]: one is *amplify-and-forward*, i.e., the relays just amplify the received noisy signal and transmit it to the destination; the other scheme is *decode-and-forward*, where each potential relay detects the source information first, and if it can successfully detect the source information, then it will be enrolled in Phase II transmission. In this paper, we adopt the second scheme, i.e., *decode-and-forward*. During Phase I, each potential relay receives

$$y_{r_i}(n)=h_{s,r_i}(n)x_s(n)+w_{r_i}(n)$$

where we assume that the channel is quasi-static Rayleigh flat fading, $h_{s,r_i}(n)$ is the channel coefficient between S and R_i and is Rayleigh distributed with unit power. We also assume that $h_{s,r_i}(n)$ is known at the receiver. $w_{r_i}(n)$ is the AWGN at R_i and has zero mean and variance σ^2 per real dimension. $x_s(n)$ is the transmitted symbol by S. During Phase II, first, R_i demodulates the received signal and does CRC check [18] to see whether the detected information is correct or not. We assume that those can pass the CRC check do not have any errors in their detected information. We use $\mathcal{R}_s$ to denote the set of potential relays that can successfully detect the source information during a packet/frame from S, and use M_s to denote the cardinality of the set $\mathcal{R}_s$, i.e., $M_s=|\mathcal{R}_s|$. Then, those $R_i\in\mathcal{R}_s$ will be enrolled in the transmission of Phase II. Clearly, the elements and the cardinality of set $\mathcal{R}_s$ depend on the channel quality between the source and the potential relays. It is usually assumed that M_s is a random variable [4]. As analyzed in [4], the protocol that relays transmit space–time coded signals on the overlapped channels performs better than the protocol that relays just repeat their detected information on the orthogonal channels. Therefore, in this paper, we assume that a space–time coded transmission is used during Phase II. In Phase II, if the enrolled relays are symbol synchronized, the destination receives

$$y_d(n)=\sum_{1\le i\le M \text{ and } r_i\in\mathcal{R}_s} h_{r_i,d}(n)x_{r_i}(n)+w_d(n). \tag{1}$$

In the scenario of exploiting the user cooperative diversity, we usually assume that the channel is quasi-static, i.e., we assume that the channel $h_{r_i,d}$ keeps constant during the transmission of one packet/frame, and then changes independently in the next packet/frame. Assuming the packet/frame length is L, (1) can be written in matrix form as

$$\mathbf{y}_d=\mathbf{h}_{r,d}X_r+\mathbf{w}_d \tag{2}$$

where $\mathbf{y}_d\in\mathbb{C}^{1\times L}$, $\mathbf{h}_{r,d}\in\mathbb{C}^{1\times M_s}$, $\mathbf{w}_d\in\mathbb{C}^{1\times L}$, and $X_r\in\mathbb{C}^{M_s\times L}$ is the space–time coded signal matrix of dimension $M_s\times L$

$$X_r=\begin{bmatrix} x_{r_1}(1) & x_{r_1}(2) & \cdots & x_{r_1}(L)\\ x_{r_2}(1) & x_{r_2}(2) & \cdots & x_{r_2}(L)\\ \vdots & \vdots & \ddots & \vdots\\ x_{r_{M_s}}(1) & x_{r_{M_s}}(2) & \cdots & x_{r_{M_s}}(L)\end{bmatrix}.$$

Different rows in X_r are transmitted by different relay terminals. $\mathbb{C}$ is the set of all the complex numbers, i.e., the complex plane. There are two major differences between the conventional space–time codes [12], and the space–time codes in the cooperative communication. One is that the row number M_s in X_r is a random variable instead of a constant in the conventional space–time codes which equals to the number of co-located transmit antennas. The other is that each row in matrix X_r may not be symbol aligned, and the relative timing errors between different relays may be random. For example, X_r can be equal to X_r^a in (3), shown at the bottom of the page.

In the following, we call X_r^a as an asynchronous version of X_r. This is due to the asynchronous nature of the cooperative

$$X_r^a=\begin{bmatrix} \star & x_{r_1}(1) & x_{r_1}(2) & \cdots & x_{r_1}(L) & \star & \star\\ \star & \star & x_{r_2}(1) & x_{r_2}(2) & \cdots & x_{r_2}(L) & \star\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ x_{r_{M_s}}(1) & x_{r_{M_s}}(2) & \cdots & x_{r_{M_s}}(L) & \star & \star & \star\end{bmatrix} \tag{3}$$

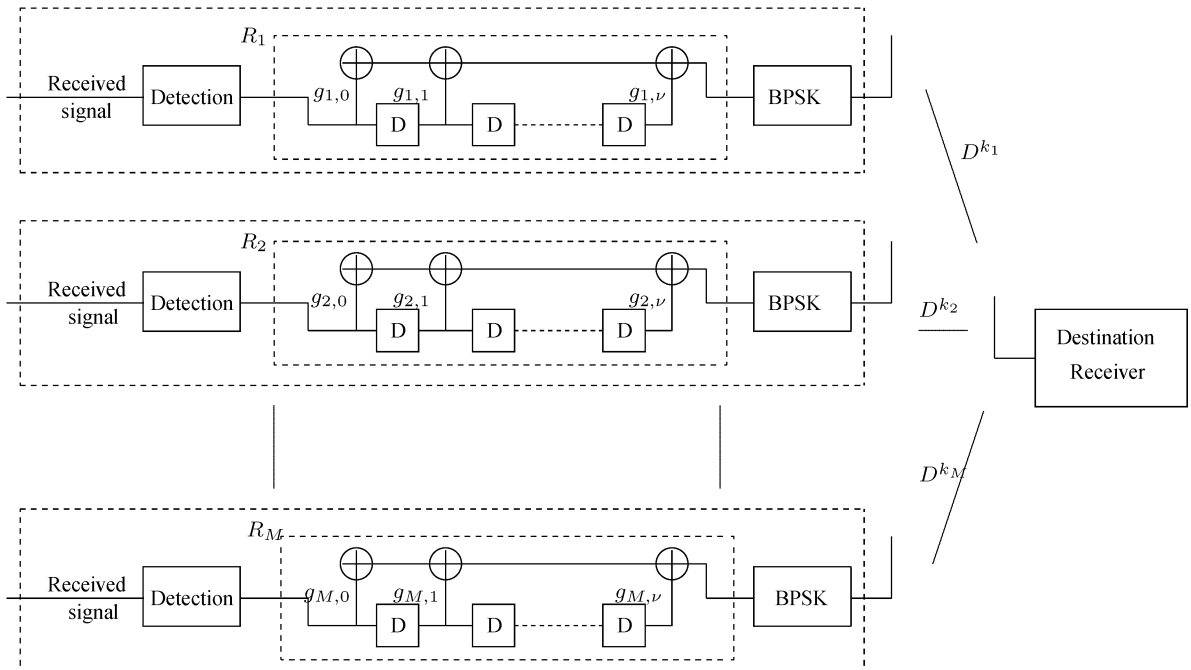

Fig. 2. Code construction.

communication, where the transceivers are distributed in different terminals and a central local oscillator is lacked. In the previous *asynchronous cooperative communication*, although the symbol synchronization is not required, we assume that each relay terminal is packet/frame synchronized, i.e., the start and the end of each packet/frame in different enrolled relays are aligned, which can be implemented by using some signaling feedback from the destination node. When a relay terminal is waiting for a packet/frame synchronizing flag, the dumb signal $\star$ is transmitted. We also assume that the relative timing errors between different relays are integers of the symbol duration and a fractional timing error can be absorbed in the channel dispersion. We further assume that these relative timing errors are known at the receiver but not at the transmitter. The timing errors can be estimated by using some random sequences transmitted periodically by the relay nodes. For example, in the WiMAX system [22], the relative timing error of the mobile stations are estimated by the base station during the initial ranging and periodic ranging period. The maximum relative timing error is assumed to be L_e. So the actual transmitted space–time code matrix is of dimension $M_s \times L'$, where $L \leq L' \leq L + L_e$. In each row, totally $L' - L$ dumb symbols $\star$ are padded to the beginning and/or the ending of a packet/frame transmission. Similar to the conventional space–time code design, to achieve good performance, we need to have the full *diversity order* and a good *diversity product* as shown in [11] and [12], while the following two differences must be considered:

- number of rows in the space–time code matrix is random;
- rows in the space–time code matrix are not symbol-aligned.

The first one is, in fact, not too difficult to deal with since every space–time code of dimension $M \times L$ designed to achieve full diversity order, M, also has full diversity order, M_s, if any $M - M_s$ rows in X_r^a are deleted, where it is assumed that the frame/packet length $L \geq M$. However, the second difference is not easy to handle. For example, the delay diversity codes that are designed to ensure full diversity order in the conventional space–time codes [12], [17] do not have the full diversity property in the asynchronous cooperative communication. Also, the existing space–time block codes, for example, the orthogonal space–time codes and the lattice based space–time block codes, do not have the full diversity order property when the transmission is not synchronized. The objective of this paper is to design space–time codes with full diversity order in the asynchronous cooperative communication, i.e., X_r^a in (3) has full diversity order for any symbol-wise timing errors within a maximal range L_e.

III. Code Construction

In this section, we introduce a family of space–time trellis codes that can achieve full diversity order in the asynchronous cooperative communication. We first design the space–time trellis codes where each element in X_r is BPSK modulated based on the stack construction in [14], and then we generalize the construction to QAM and PSK symbols by using the unified construction in [16].

Our space–time trellis code construction, when BPSK modulation scheme is used, is shown in Fig. 2. The source information bits are detected in a relay R_i, $i = 1, 2, \ldots, M$. If they are correct during a packet/frame, they are passed through a tapped delay line with tapped coefficients $[g_{i,0}, g_{i,1}, \ldots, g_{i,\nu}]$, where $g_{i,d} \in \mathbb{F} \triangleq \{0, 1\}$ for $d = 0, 1, 2, \ldots, \nu$, and ν is the maximal delay. We denote $g_i(D) \triangleq g_{i,0} + g_{i,1}D + \cdots + g_{i,\nu}D^{\nu}$ and

$G_M(D) = [g_1(D), g_2(D), \ldots, g_M(D)]$, where and in what follows D denotes the delay unit. The coefficient matrix of $G_M(D)$ is defined as

$$G_M = \begin{bmatrix} g_{1,0} & g_{1,1} & \cdots & g_{1,\nu} \\ g_{2,0} & g_{2,1} & \cdots & g_{2,\nu} \\ \vdots & \vdots & \vdots & \vdots \\ g_{M,0} & g_{M,1} & \cdots & g_{M,\nu} \end{bmatrix}.$$

If the binary source information bits detected in the relays in one packet/frame is $\overline{u} \in \mathbb{F}^{L_u}$, then the binary output of the tapped delay lines is the set

$$\mathcal{C} = \{C(\overline{u}) \in \mathbb{F}^{M\times(\nu+L_u)} \mid C(\overline{u}) = [c_1(\overline{u}), c_2(\overline{u}), \ldots, c_M(\overline{u})]^T, \overline{u} \in \mathbb{F}^{L_u}\}$$

where $c_i(\overline{u})$ for $i = 1, 2, \ldots, M$ is shown as follows:

$$c_i(\overline{u}) = [u_1, u_2, \ldots, u_{L_u}] \times \begin{bmatrix} g_{i,0} & g_{i,1} & \cdots & g_{i,\nu} & 0 & \cdots & 0 \\ 0 & g_{i,0} & g_{i,1} & \cdots & g_{i,\nu} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & g_{i,0} & g_{i,1} & \cdots & g_{i,\nu} \end{bmatrix}_{L_u\times(L_u+\nu)}. \quad (4)$$

The *space–time code generated by* $G_M(D)$ is defined as the set $\mathcal{X} =$

$$\{X_r(\overline{u}) \in \mathbb{C}^{M\times(\nu+L_u)} \mid (X_r(\overline{u}))_{m,n} = (-1)^{(C(\overline{u}))_{m,n}}, C(\overline{u}) \in \mathcal{C}\}.$$

The above space–time codes have trellis structure. For example, if $G_2(D) = [1 + D^2,\ \ 1 + D + D^2]$, then the trellis structure is shown in Fig. 3, where in the representation x/yz, x is the input binary bit, y is the tapped delay binary output for R_1, and z is the tapped delay binary output for R_2. In this construction, if the maximum timing error is L_e, the number information bits in one packet/frame is L_u, and BPSK modulation scheme is used, then the rate of the space–time code $\mathcal{X}$ generated from $G_M(D)$ is $L_u/(L_u + \nu + L_e)$ bits/s/Hz. For long packet/frame, the rate approaches 1 bit/s/Hz. The previous construction in general is the same as the one obtained by Hammons and El Gamal [14]. In the following, to achieve the full diversity order in asynchronous cooperative communications, we investigate conditions on the generating matrix G_M.

Assume the timing error of relay R_i is k_i, then k_i dumb symbols $\star$ are padded to the left of the ith row of every matrix X in the set $\mathcal{X}$. The resulting asynchronous version matrix is X_r^a and the set is $\mathcal{X}^a$. If the dumb symbol $\star = 1 = (-1)^0$, then it is equivalent to that k_i many 0's are padded to the left of the ith row of binary matrix C in $\mathcal{C}$. These matrices can be generated by $G^a(D) = [g_1^a(D),\ g_2^a(D),\ \ldots, g_M^a(D)]$, where $g_i^a(D) = D^{k_i} g_i(D)$. For example, if $G_2(D) = [1 + D^2,\ 1 + D + D^2]$ and the second relay R_2 has one timing error, i.e., $k_2 = 1$, then the equivalent trellis structure is shown in Fig. 3. To ensure the full diversity order in the asynchronous cooperative communication, there are requirements for the tapped coefficients $g_{i,d}$ for $i = 1, 2, \ldots, M$ and $d = 1, 2, \ldots, \nu$, which are stated in Theorem 1 and the proof is based on the stack construction in [14], which is stated in the following lemma.

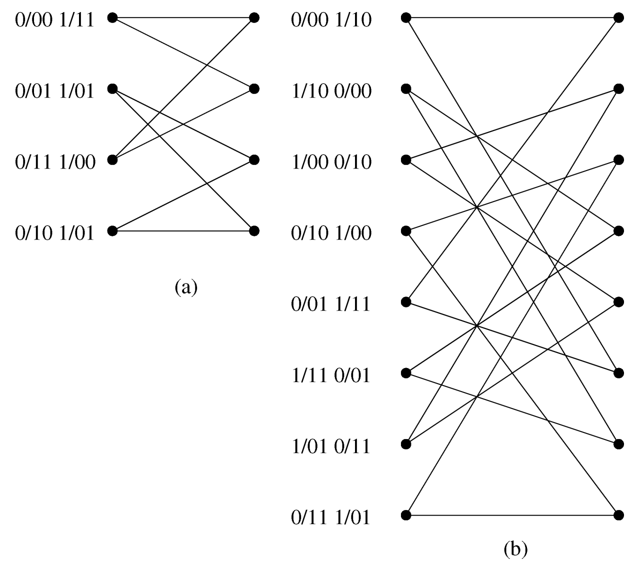

Fig. 3. (a) Trellis structure of $G_2(D) = [1 + D^2,\ \ 1 + D + D^2]$. (b) Trellis structure of $G_2^a(D)$ for $k_1 = 0, k_2 = 1$.

Lemma 1 [Hammons and El Gamal]: Let $T_1, T_2, \ldots, T_M$ be linear vector-space transformations from $\mathbb{F}^k$ to $\mathbb{F}^L$, where $\mathbb{F} = \{0, 1\}$, and let $\mathcal{C}$ be the set of $M \times L$ codeword matrices

$$C_r(\overline{u}) = \begin{bmatrix} T_1(\overline{u}) \\ T_2(\overline{u}) \\ \vdots \\ T_M(\overline{u}) \end{bmatrix}$$

where $\overline{u}$ denotes an arbitrary k-tuple of information bits and $L \geq M$. Then, the space–time code $\mathcal{X}$, which consists of the matrices $X_r(\overline{u})$ with its entries $(X_r(\overline{u}))_{m,n} = (-1)^{(C_r(\overline{u}))_{m,n}}$, achieves the full diversity order M if and only if $T_1, T_2, \ldots, T_M$ have the property that $\forall a_1, a_2, \ldots, a_M \in \mathbb{F}$: $T = a_1T_1 \oplus a_2T_2 \oplus \cdots \oplus a_MT_M$ is nonsingular unless $a_1 = a_2 = \cdots = a_M = 0$, where $\oplus$ is the modular 2 addition.

Based on this lemma, we have the following theorem.

Theorem 1: The space–time code generated by $G_M(D) = [g_1(D), g_2(D), \ldots, g_M(D)]$ has full diversity order in the asynchronous cooperative communication if and only if the coefficient matrix G_M^a of any asynchronous version

$$G_M^a(D) = [g_1^a(D), g_2^a(D), \ldots, g_M^a(D)] = [D^{k_1} g_1(D),\ D^{k_2} g_2(D), \ldots, D^{k_M} g_M(D)]$$

of $G_M(D)$

$$G_M^a = \begin{bmatrix} \overline{g}_1^a \\ \overline{g}_2^a \\ \vdots \\ \overline{g}_M^a \end{bmatrix} = \begin{bmatrix} g_{1,0}^a & g_{1,1}^a & \cdots & g_{1,\nu+L_e}^a \\ g_{2,0}^a & g_{2,1}^a & \cdots & g_{2,\nu+L_e}^a \\ \vdots & \vdots & \vdots & \vdots \\ g_{M,0}^a & g_{M,1}^a & \cdots & g_{M,\nu+L_e}^a \end{bmatrix}$$

has full rank, M, in the binary field $\mathbb{F}$ for arbitrary $k_1, k_2, \ldots, k_M$, where $L_e = \max_{1\leq i\leq M} k_i$.

Proof: According to the construction, the matrix that transforms the vector space $\mathbb{F}^{L_u}$ to $\mathbb{F}^{L_u+\nu+L_e}$ in the potential relay R_i is

$$T_i = \begin{bmatrix} g^a_{i,0} & g^a_{i,1} & \cdots & g^a_{i,\nu+L_e} & 0 & \cdots & 0 \\ 0 & g^a_{i,0} & g^a_{i,1} & \cdots & g^a_{i,\nu+L_e} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & g^a_{i,0} & g^a_{i,1} & \cdots & g^a_{i,\nu+L_e} \end{bmatrix}$$

for $i = 1, 2, \ldots, M$, where L_u is the length of the packet/frame before encoding.

First, we show that if G^a_M is full rank, then $T = a_1T_1 \oplus a_2T_2 \oplus \cdots \oplus a_MT_M$ is full rank unless $a_1 = a_2 = \cdots = a_M = 0$. If G^a_M is full rank, then $\overline{g} = a_1\overline{g}^a_1 \oplus a_2\overline{g}^a_2 \oplus \cdots \oplus a_M\overline{g}^a_M$ is not equal to zero, unless $a_1 = a_2 = \cdots = a_M = 0$. Correspondingly, $\overline{g}$ equals to the first row of T after the last $L_u - 1$ zeros in the row are deleted. Because of the diagonal structure of T, we have T is full rank if $\overline{g}$ is not equal to zero.

Second, we show that, if $T = a_1T_1 \oplus a_2T_2 \oplus \cdots \oplus a_MT_M$ is full rank unless $a_1 = a_2 = \cdots = a_M = 0$, then the corresponding G^a_M is full rank. Again, based on the diagonal structure of T, if T is full rank then every row of T is not an all zero vector. Since the first row of T after deleting the last L_u-1 zeros is equal to the linear combination of the rows of G^a_M with the same coefficients $a_1, a_2, \ldots, a_M$, we have that the linear combination of the rows of G^a_M is not equal to zero, which is equivalent to say that G^a_M is full rank. ■

Clearly, in Theorem 1, G^a_M is a row-shifted version of G_M, and for each row i, $i = 1, 2, \ldots, M$, the shift amount k_i is arbitrary. The importance of Theorem 1 is that, in order to construct space–time code $\mathcal{X}$ generated by $G_M(D)$ with full diversity order in asynchronous cooperative communication, we need to and only need to construct the generating matrix $G_M(D)$ such that any row-shifted version G^a_M of its coefficient matrix G_M has full rank. We next present some constructions of such $G_M(D)$. The following theorem gives some sufficient conditions for $G_M(D)$ to ensure that their asynchronous versions $G^a_M(D)$ have full rank for some values of M.

Theorem 2:

1) $M = 2$: If the coefficient matrix of $G_2(D)$ has the structure

$$G_2 = \begin{bmatrix} 1 & * & \cdots & \cdots \\ 1 & * & \cdots & \cdots \end{bmatrix}$$

and full rank in $\mathbb{F}$, then the space–time code generated by $G_2(D)$ has full diversity order in the asynchronous cooperative communication, where in G_2, $*$ denotes an arbitrary element in $\mathbb{F}$.

2) $M = 3$: If the coefficient matrix of $G_3(D)$ has the structure

$$G_3 = \begin{bmatrix} 1 & * & \cdots & \cdots & * & 1 \\ 1 & * & \cdots & \cdots & * & 1 \\ 1 & * & \cdots & \cdots & * & 1 \end{bmatrix}$$

and full rank in $\mathbb{F}$, then the space–time code generated by $G_3(D)$ has full diversity order in the asynchronous cooperative communication.

3) $M = 4$: If the coefficient matrix of $G_4(D)$ has the structure

$$G_4 = \begin{bmatrix} 1 & * & \cdots & \cdots & * & 1 \\ 1 & * & \cdots & \cdots & * & 1 \\ 1 & * & \cdots & \cdots & * & 1 \\ 1 & * & \cdots & \cdots & * & 1 \end{bmatrix}$$

and full rank in $\mathbb{F}$, and after two rows of G_4 are shifted together with an arbitrary shift amount, the modular two sum of each column of the shifted matrix is not equal to zero, then the space–time code generated by $G_4(D)$ has full diversity order in asynchronous cooperative communication.

Proof: It is easy to verify for 1) and 2) that if any relay is delayed with respect to others, then for $M = 2$, the coefficient matrix G^a_2 may have the following structure:

$$G^a_2 = \begin{bmatrix} 1 & * & \cdots & \cdots & \cdots \\ 0 & * & * & \cdots & \cdots \end{bmatrix},$$

and for $M = 3$, G^a_3 may have the following structure:

$$G^a_3 = \begin{bmatrix} 1 & * & \cdots & \cdots & * & * & 0 \\ 0 & * & * & \cdots & \cdots & * & 1 \\ 0 & * & * & \cdots & \cdots & * & 1 \end{bmatrix}.$$

We can see from the structure that the 1 in the first row and the first column can not be obtained from any linear combination of other rows. Furthermore, the other rows are linearly independent because of the full rank properties of G_2 and G_3. Thus, G^a_2 and G^a_3 have full rank. Based on Theorem 1, we have that the space–time codes generated by $G_2(D)$ and $G_3(D)$ have full diversity in the asynchronous cooperative communication.

For 3), there are two cases. The first case is that one row is shifted more with respected to others. In this case, G^a_4 may have the following structure:

$$G^a_4 = \begin{bmatrix} 1 & * & \cdots & \cdots & * & * & 0 \\ 0 & * & * & \cdots & \cdots & * & 1 \\ 0 & * & * & \cdots & \cdots & * & 1 \\ 0 & * & * & \cdots & \cdots & * & 1 \end{bmatrix}.$$

Because of 2), we can see that the last three rows are linearly independent. Since the 1 in the first row and the first column can not be generated by the zeros in other three rows, in this case G^a_4 is full rank.

The second case is that two rows are shifted the same amount, i.e., the structure of G^a_4 is

$$G^a_4 = \begin{bmatrix} 1 & * & \cdots & \cdots & * & * & 0 \\ 1 & * & \cdots & \cdots & * & * & 0 \\ 0 & * & * & \cdots & \cdots & * & 1 \\ 0 & * & * & \cdots & \cdots & * & 1 \end{bmatrix}.$$

In this case, we can see that either one of the first two rows can not be obtained by any linear combination of the last two, and the first two rows can not be obtained from each other, because G_4 is full rank. So, if the modular two sum of each column is not equal to zero, G^a_4 is full rank. Based on Theorem 1, we has that the space–time code constructed from $G_4(D)$ have full diversity order in the asynchronous cooperative communication.

The two columns of ones in G_3 and G_4 are used to guarantee the equivalence of two way shifting. ∎

Theorem 2 is a sufficient condition for the construction when M =2, 3, 4. We can see from Theorem 2 that, the minimum required number of columns in G_2 is 2, in G_3 is 4, and in G_4 is 5, respectively. The following theorem gives a sufficient condition for G_M to ensure the full diversity order of the corresponding space–time code in the asynchronous cooperative communication for arbitrary M.

Theorem 3: Let $w(\overline{v})$ denote the weight of binary vector $\overline{v}$, i.e., the number of 1's in $\overline{v}$. Assume that the binary matrix

$$G_M = \begin{bmatrix} \overline{g}_1 \\ \overline{g}_2 \\ \vdots \\ \overline{g}_M \end{bmatrix} = \begin{bmatrix} g_{1,0} & g_{1,1} & \cdots & g_{1,\nu} \\ g_{2,0} & g_{2,1} & \cdots & g_{2,\nu} \\ \vdots & \vdots & \vdots & \vdots \\ g_{M,0} & g_{M,1} & \cdots & g_{M,\nu} \end{bmatrix}$$

are row permuted such that $w(\overline{g}_1) \leq w(\overline{g}_2) \leq \cdots \leq w(\overline{g}_M)$, where $\overline{g}_i$ is the ith row vector of matrix G_M. If

$$w(\overline{g}_k) > \sum_{i=1}^{k-1} w(\overline{g}_i) \tag{5}$$

for $k = 2, 3, \ldots, M$, and G_M is used as the coefficient matrix of $G_M(D)$ to construct the space–time code $\mathcal{X}$, then $\mathcal{X}$ has the full diversity order in the asynchronous cooperative communication. In contrary, if

$$w(\overline{g}_k) \leq \sum_{i=1}^{k-1} w(\overline{g}_i),\ k = 3, 4, \ldots, M \tag{6}$$

with $M \geq 3$, then there exists G_M such that the space–time code generated by $G_M(D)$ with coefficient matrix G_M does not have full diversity order in the asynchronous cooperative communication.

Proof: It is obvious that the row permutation operation does not affect the full diversity requirement of the asynchronous cooperative communication. For any shift of a row, its weight does not change.

We first prove the sufficiency part. Assume that a shifted version of G_M is

$$G_M^a = \begin{bmatrix} \overline{g}_1^a \\ \overline{g}_2^a \\ \vdots \\ \overline{g}_M^a \end{bmatrix}$$

then $w(\overline{g}_i^a) = w(\overline{g}_i)$ for $i = 1, 2, \ldots, M$. Assume that $\overline{g}^a$ is a binary linear combination of the rows, i.e., $\overline{g}^a = a_1\overline{g}_1^a \oplus a_2\overline{g}_2^a \oplus \cdots \oplus a_M\overline{g}_M^a$, where $a_i \in \mathbb{F}$ for $i = 1, 2, \ldots, M$. We show that the binary row vector $\overline{g}^a$ is not an all zero vector unless $a_i = 0$ for $i = 1, 2, \ldots, M$.

Assume that M' is the largest value in $1, 2, \ldots, M$, such that $a_{M'} \neq 0$ and $a_k = 0$ for $M' + 1 \leq k \leq M$. Then, there should be a 1 in $\overline{g}_{M'}$, where in its column position the kth row $\overline{g}_k$, for $1 \leq k \leq M' - 1$, has 0 as the component. Otherwise, the condition $w(\overline{g}_{M'}^a) > \sum_{i=1}^{M'-1} w(\overline{g}_i^a)$ can not be satisfied. This shows $\overline{g}^a \neq 0$ unless $a_i = 0$ for $i = 1, 2, \ldots, M$, i.e., the sufficiency part of the theorem.

We use counterexamples to show the necessity

$$G_M = \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ & \vdots & & & \vdots & & & & & & & \\ 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \cdots & 1 & 1 & \cdots & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

where the weight of the kth row vector $\overline{g}_k$ satisfies the following identity:

$$w(\overline{g}_k) = \sum_{i=1}^{k-1} w(\overline{g}_i),\ \ k = 3, 4, \ldots, M.$$

The previous matrix does not have the full rank since the sum of all rows is the all zero vector, which shows the necessity part of the theorem. ∎

From Theorem 3, we can give a construction G_M easily. For example, we can construct G_2 as

$$G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{7}$$

G_3 as

$$G_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{8}$$

and G_4 can be constructed as

$$G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}. \tag{9}$$

For the construction in Theorem 3, the number of columns in G_M grows very quickly with the number of rows. For example, if $M = 4$ then the number of columns in G_4 should be at least 8. While if we only need to satisfy the full rank of G_4 itself, the least number of columns is only 4. Although the necessity part in Theorem 3 implies that if the weight condition (5) does not hold, there exists a counter example that does not satisfy the full rank property of G_M^a for some shifts. However, the weight requirement (5) in Theorem 3 is still a sufficient condition, i.e., we can construct a G_M which satisfies the full diversity requirement in asynchronous cooperative communication with much lower number of columns. For example, in Theorem 2, for $M = 4$ the number of columns in G_4 is 5, but we should check more conditions than that in Theorem 3. The disadvantage of having large number of columns is that the decoding complexity would be high. Since large number of rows means large maximum memory order. For the Viterbi decoder, the complexity grows exponentially with the maximum memory order. For the construction from Theorem 3, the minimum requirement of the weights is

$$w(\overline{g}_k) = \sum_{i=1}^{k-1} w(\overline{g}_i) + 1$$

for $k = 2, \ldots, M$, from which we can easily calculate that the minimum required number of columns of G_M is 2^{M-1}.

The difficulty of constructing G_M with less number of columns lies in the fact that even when G_M satisfies the full diversity order requirement of the asynchronous cooperative communication, adding a column to G_M, the resulting G'_M may lose the full diversity order property in the asynchronous cooperative communication. To construct G_M by using the construction G_{M-1}, add a column to existing construction is necessary. For example, we can verify by using Theorem 2 that

$$G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

has full diversity in the asynchronous cooperative diversity, but

$$G'_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & \mathbf{1} & 1 \\ 1 & 0 & 0 & 1 & 1 & \mathbf{0} & 1 \\ 1 & 1 & 1 & 1 & 0 & \mathbf{0} & 1 \\ 1 & 1 & 0 & 1 & 1 & \mathbf{0} & 1 \end{bmatrix}$$

which is generated by adding a column $[1\ 0\ 0\ 0]^T$ to G_4, loses the full diversity order property in the asynchronous cooperative communication when it is shifted to be

$$G'_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & \mathbf{1} & 1 & \mathbf{0} \\ 1 & 0 & 0 & 1 & 1 & \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & 1 & 1 & 1 & 1 & 0 & \mathbf{0} & 1 \\ \mathbf{0} & 1 & 1 & 0 & 1 & 1 & \mathbf{0} & 1 \end{bmatrix}.$$

Constructing G_M with a smaller number of columns is an interesting problem. The shifted full rank construction with minimum number of column for arbitrary M has been recently found in [23].

Having the BPSK modulated space–time codes that have full diversity order, using Lu–Kumar's unified construction [16], we can generalize them to other PAM, QAM, or PSK modulated space–time codes, as shown in the following theorem.

Theorem 4: Let K, U be integers with $K > 0$, $U > 0$. Let

$$\{\mathcal{C}_{i,j} | 0 \le i \le U-1,\ 0 \le j \le K-1\}$$

be a collection of UK sets of $M \times (L_u + \nu)$ binary matrices generated by $G_M(D)$ using (4) with UK independent binary vectors $\overline{u}^{(i,j)}$ of dimension L_u. Let θ be a primitive 2^K-th root of unity. Let $\eta \in \mathbb{Z}[\theta]$, $\eta \ne 0$, such that η belongs to the ideal $2\mathbb{Z}[\theta]$ generated by 2 in $\mathbb{Z}[\theta]$. Let

$$f : \mathcal{C}_{0,0} \times \mathcal{C}_{0,1} \times \cdots \times \mathcal{C}_{U-1,K-1} \to \mathcal{X} \in \mathbb{C}^{M \times (L_u+\nu)}$$

be the map defined by

$$(C_{0,0}, C_{0,1}, \ldots, C_{U-1,K-1}) \to \kappa \sum_{i=0}^{U-1} \eta^i \theta^{\sum_{j=0}^{K-1} 2^j C_{i,j}}$$

where κ is a nonzero complex number, $C_{i,j}$ is a matrix in the binary matrix set $\mathcal{C}_{i,j}$, and the multiplication and exponential of $C_{i,j}$ to θ are carried out entry by entry. Then, if $G_M(D)$ satisfies the condition of full diversity order in asynchronous cooperative communication the same in Theorem 1, then the space–time code $\mathcal{X}$ generated by the above map f also has full diversity order in the asynchronous cooperative communication.

This theorem can be proved similarly to that in [16]. The most important thing to notice is that in this construction the operations in the map f are carried out entry by entry, which is similar to the BPSK modulation, and the timing errors in relays can be mapped to the shifts in binary matrix sets $\mathcal{C}_{i,j}$ for $i = 0, 1, \ldots, U-1$, $j = 0, 1, \ldots, K-1$, which is then reflected in $G_M(D)$. In this case, the dumb symbol a relay sends is $\star = \kappa \sum_{i=0}^{U-1} \eta^i$.

Similar to the BPSK modulation case, if the modulation is 2^{UK} PAM, QAM or PSK, and the maximum allowed timing error in one packet/frame is L_e, when the binary information bit sequence length is UKL_u, then the transmission rate is $UKL_u/(L_u + \nu + L_e)$. When the length L_u of the information bit sequence approaches infinity, the rate approaches UK b/s/Hz.

We now give two examples. Obviously, if we choose $U = 1$, $K = 1$, $\eta = 2$, $\theta = -1$ and $\kappa = 1$, then the construction is the same as the BPSK construction shown before. If each entry of the space–time code matrix $X_r \in \mathcal{X}$ is chosen to be a 16-QAM symbol, then, we can set $U = 2$, $K = 2$, $\eta = 2$, $\theta = \mathbf{j}$ and $\kappa = 1 + \mathbf{j}$, where $\mathbf{j} = \sqrt{-1}$. The space–time code $\mathcal{X}$ is the set $\mathcal{X} =$

$$\left\{(1+\mathbf{j})\left[\mathbf{j}^{C_{0,0}+2C_{0,1}} + 2\mathbf{j}^{C_{1,0}+2C_{1,1}}\right] : C_{i,j} \in \mathcal{C}_{i,j}; i, j \in \{0,1\}\right\}.$$

In the construction, $C_{0,0}, C_{0,1}, C_{1,0}, C_{1,1}$ are the space–time code matrices generated by four independent information binary vectors $\overline{u}^{0,0}, \overline{u}^{0,1}, \overline{u}^{1,0}, \overline{u}^{1,1}$, each of which has dimension L_u, from the generator matrix $G_M(D)$ using (4).

IV. Diversity Product Analysis

When a space–time code has full diversity, its diversity product is another important factor to determine its performance. Assume X_1 and X_2 are two different matrices in a full diversity space–time code $\mathcal{X}$, $\Delta X \triangleq X_1 - X_2$ and assume $\lambda_1, \lambda_2, \ldots, \lambda_M$ are the eigenvalues of $\Delta X (\Delta X)^H$, then the diversity product of the space–time code $\mathcal{X}$ is defined as

$$\zeta(\mathcal{X}) = \min_{X_1 \ne X_2, X_1 \in \mathcal{X}, X_2 \in \mathcal{X}} \left(\prod_{i=1}^{M} \lambda_i\right)^{1/M}.$$

Based on the arithmetic mean and geometric mean inequality, the diversity product is upper bounded by

$$\begin{aligned}\zeta(\mathcal{X}) &\le \min_{X_1 \ne X_2, X_1 \in \mathcal{X}, X_2 \in \mathcal{X}} \frac{\sum_{i=1}^{M} \lambda_i}{M} \\ &= \min_{X_1 \ne X_2, X_1 \in \mathcal{X}, X_2 \in \mathcal{X}} \frac{\|\Delta X\|^2}{M} \\ &\triangleq \zeta^{ub}(\mathcal{X})\end{aligned}$$

where $\|\Delta X\|^2$ is the squared Frobenius norm of matrix ΔX. According to the asynchronous nature, we have the following theorem which states the relationship of the upper bound of the

TABLE I
RATE $1/M$ CONVOLUTIONAL CODES WITH OPTIMAL FREE HAMMING DISTANCE AND FULL DIVERSITY IN ASYNCHRONOUS COOPERATIVE COMMUNICATION

M	ν	$g_1(D)$	$g_2(D)$	$g_3(D)$	$g_4(D)$	$g_5(D)$	d_{free}
2	2	5	7				5
	3	64	74				6
	4	46	72				7
	5	65	57				8
3	3	54	64	74			10
	4	52	66	76			12
	5	47	53	75			13
	6	554	624	764			15
4	5	53	67	71	75		18
	7	472	572	626	736		22
	8	463	535	733	745		24
	9	4474	5724	7154	7254		27
	10	4656	4726	5562	6372		29
	11	4767	5723	6265	7455		32
5	7	536	466	646	562	736	28

diversity products between the space–time code $\mathcal{X}$ and its asynchronous version $\mathcal{X}^a$.

Theorem 5: Assume $\zeta^{ub}(\mathcal{X})$ is the diversity product upper bound of the space–time code $\mathcal{X}$ and $\zeta^{ub}(\mathcal{X}^a)$ is the diversity product upper bound of $\mathcal{X}^a$, which is the asynchronous version of the space–time code $\mathcal{X}$, as previously defined, then we have

$$\zeta^{ub}(\mathcal{X}) = \zeta^{ub}(\mathcal{X}^a).$$

Proof: Since the padded dumb symbols $\star$ for the asynchronous version $\mathcal{X}^a$ of $\mathcal{X}$, are the same for all the space–time code matrices, the squared Frobenius norm of the difference matrix $\|\Delta X^a\|^2 \triangleq \|X_1^a - X_2^a\|^2$ in $\mathcal{X}^a$ should be same as the corresponding $\|\Delta X\|^2 \triangleq \|X_1 - X_2\|^2$ in $\mathcal{X}$. So, their diversity product upper bounds are the same, i.e., $\zeta^{ub}(\mathcal{X}) = \zeta^{ub}(\mathcal{X}^a)$. ■

For the space–time codes constructed in Section III, $\|\Delta X\|^2$ is a function of the Hamming distance between elements of the binary matrices C_1 and C_2 in the binary matrix set $\mathcal{C}$ which is used to construct the space–time code $\mathcal{X}$. This Hamming distance is equal to the Hamming distance of the rate $1/M$ convolutional code generated by the corresponding $G_M(D)$. The larger this Hamming distance is, the larger the upper bound of the diversity product is. Therefore, it is suggested that we choose the rate $1/M$ convolutional code $G_M(D)$ that has the maximum free Hamming distance. Table I gives $G_M(D)$ with the optimal free Hamming distances and full diversity order in asynchronous cooperative communication for M =2, 3, 4, and 5. For $M \geq 6$, we find that the optimal minimum free Hamming distance convolutional codes do not satisfy the full diversity order property in asynchronous cooperative communication.

Although the diversity product upper bound of the space–time code $\mathcal{X}$ is the same as its asynchronous version $\mathcal{X}^a$, if we represent $\mathcal{X}$ and $\mathcal{X}^a$ in trellis, as those represented in Fig. 3(a) and (b), their minimum error event lengths are not the same but they have the following relationship.

Theorem 6: Assume that L_{error} is the minimum length of the error event path of the space–time code $\mathcal{X}$ generated by $G_M(D) = [g_1(D),\ g_2(D),\ \ldots,\ g_M(D)]$, and L_{error}^a is the minimum length of the error event path of $\mathcal{X}^a$, the asynchronous version of $\mathcal{X}$ generated by $G_M^a(D) = [D^{k_1}g_1(D),\ D^{k_2}g_2(D),\ \ldots,\ D^{k_M}g_M(D)]$. Assume that $L_e' \triangleq \max\{k_1, k_2, \ldots, k_M\} - \min\{k_1, k_2, \ldots, k_M\}$. Then, we have

$$L_{\text{error}}^a = L_{error} + L_e'.$$

Proof: Based on the construction of the space–time code $\mathcal{X}$, the branches in the error event path of the trellis generated by $G_M(D) = [g_1(D),\ g_2(D),\ \ldots,\ g_M(D)]$ correspond to the columns from the first nonzero column to the last nonzero column in the corresponding difference matrix of $\mathcal{X}$. And the length of the shortest error event path is equal to the smallest number of such columns in the difference matrices of $\mathcal{X}$. For the asynchronous version $\mathcal{X}^a$, the number of columns from the first nonzero column to the last nonzero column for all the difference matrices of $\mathcal{X}$ is L_e' larger than the corresponding difference matrices in $\mathcal{X}$, so the smallest number of columns is also L_e' larger. Thus, the length of the shortest error event path of the trellis generated by $G_M(D)$ is L_e' larger than that of the trellis generated by $G_M^a(D)$, i.e., $L_{\text{error}}^a = L_{error} + L_e'$. ■

As a remark, it is usually difficult to compare the exact relationship between the diversity product of the space–time code $\mathcal{X}$ and that of its asynchronous version $\mathcal{X}^a$, since when $\Delta X = X_1 - X_2$ achieves the minimum diversity product in $\mathcal{X}$, its asynchronous version $\Delta X^a = X_1^a - X_2^a$ may not achieve the minimum diversity product in $\mathcal{X}^a$.

V. SIMPLIFIED DECODING AND SIMULATION RESULTS

To decode the newly constructed space–time trellis code, the optimal decoding method is Viterbi algorithm and its complexity grows exponentially with the maximum memory order of the generating polynomial $G_M(D)$, which equals to the number of columns in the coefficient matrix G_M. Thus, the decoding complexity grows exponentially with $L_e + \nu$. For the construction in Theorem 3, to ensure the full diversity order, the minimum number of columns of G_M equals 2^{M-1}. When the number of involved relays is large, the decoding complexity of Viterbi algorithm is high. However, when there is only one antenna in the destination receiver, the performance gain achieved by more than four transmit antennas is marginal comparing with that achieved by four transmit antennas [13], [3]. Hence, four enrolled relays could be enough when only one destination antenna is available. On the other hand, L_e is an undetermined parameter in the system. Although it has an upper limit because of the packet/frame synchronization, it may be not small. Therefore, it may be necessary to investigate the performance of a simplified decoding. The recent shift-full-rank matrices constructed in [23] have minimum number of columns, thus the decoding complexity is low.

A natural simplified decoding is sequential decoding, whose complexity is independent of the maximum memory order of the generating polynomial $G_M(D)$. Usually, there are three kinds of sequential decoding algorithms [20], [21]: one is *breadth-first* algorithms which include M-algorithm, T-algorithm, etc.; one is *depth-first* algorithms which include Fano algorithm, etc.; and the other is *metric-first* algorithms, which include stack algorithm etc. For *depth-first* algorithms, *metric-first* algorithms and T-algorithm, their complexities are

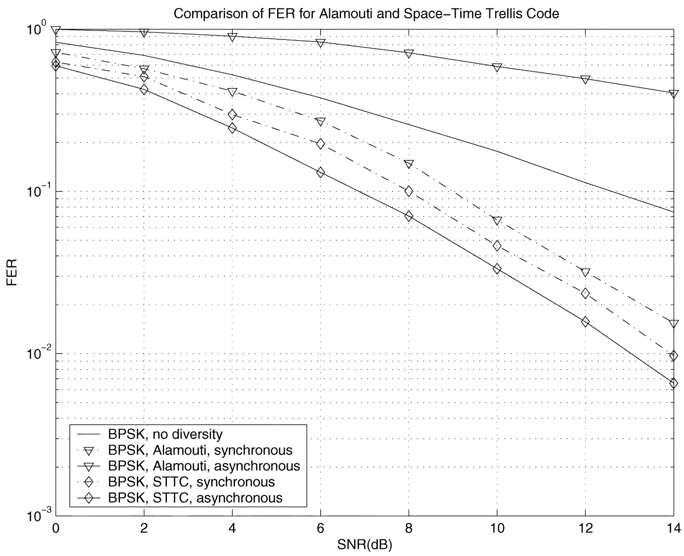

Fig. 4. Comparison of FER for Alamouti's code and space–time trellis code generated by $G_2(D) = [1 + D^2,\ \ 1 + D + D^2]$.

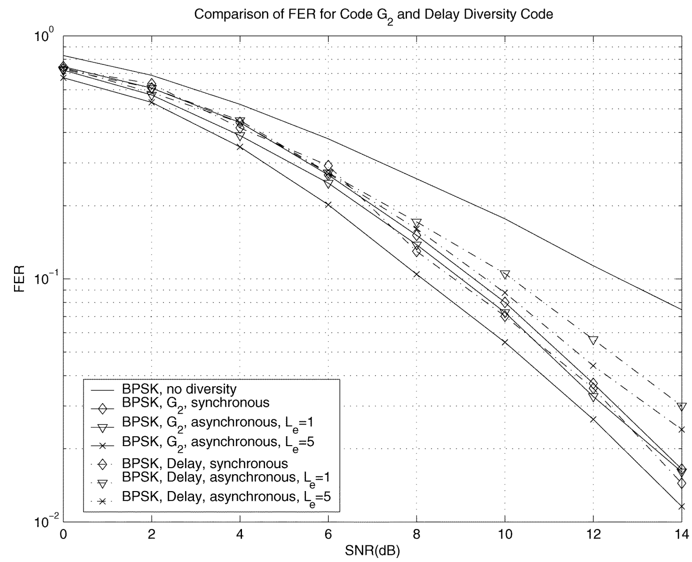

Fig. 5. Comparison of FER for code G_2 (solid lines) and the delay diversity code (dotted–dashed lines).

random variables depending on the quality of the channel. Also, there is stack overflow problem in Fano algorithm and stack algorithm, while M-algorithm does not have the problem of either random complexity or stack overflow. Furthermore, in M-algorithm, the complexity and the performance can be easily traded off by controlling the number of branches existed in the searching. Therefore, in our simulations, we adopt M-algorithm and compare it with the optimal Viterbi algorithm.

In all our simulations, we assume that packet/frame length is 200 information bits, and the channel is quasi-static Rayleigh flat fading, i.e., the channel keeps constant in one packet/frame and changes independently in the next packet/frame. We further assume that in Phase I transmission, there is no errors in relays during a packet/frame, i.e., what relays detected are the same as what the source terminal has sent. We also assume that there is only one antenna in the destination terminal.

Fig. 4 compares the frame error rate (FER) performances for Alamouti's code and the space–time trellis code generated by $G_2(D) = [1 + D^2,\ 1 + D + D^2]$ from Theorem 2 and Table I when two relays are involved. Both the case with symbol synchronization and the case without symbol synchronization are shown. When simulating the case without symbol synchronization, we assume that the maximum timing error is $L_e = 3$ and that the instant timing error is uniformly distributed in the set $\{0, 1, \ldots, L_e\}$. We also assume that the destination receiver knows these timing errors. For the space–time trellis code used in Fig. 4, we use Viterbi algorithm for the decoding. From Fig. 4 we can see that when there is no symbol synchronization, the performance of Alamouti's code degrades significantly and it is even worse than the case when there is no diversity at all. This is because that the transmission energy is distributed among relays but the destination receiver can not effectively collect this energy when the symbols are not synchronized.

Fig. 5 compares the performance of G_2 in (7) and the performance of the delay diversity codes [12], [17] in synchronous case, i.e., $L_e = 0$ and in asynchronous cases for $L_e = 1$ and $L_e = 5$. In asynchronous cases, the relative timing errors

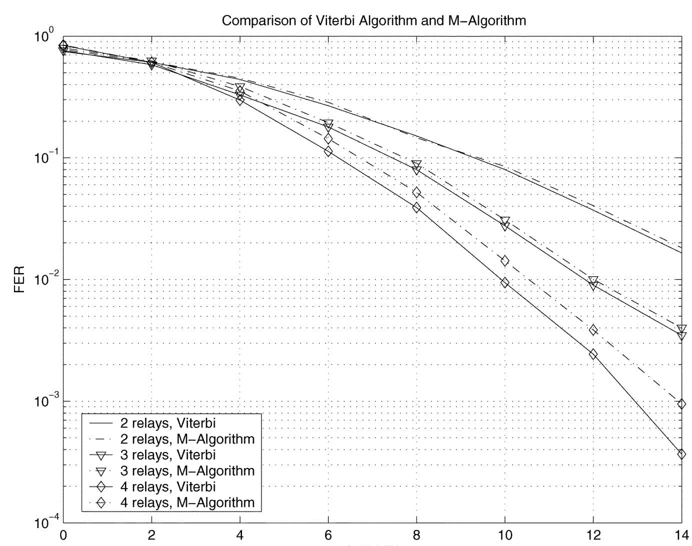

Fig. 6. Comparison of FER for Viterbi algorithm and M-algorithm without timing error, i.e., $L_e = 0$, for the proposed space–time trellis codes.

are uniformly generated from the set $\{0, 1, \ldots, L_e\}$. In Fig. 5, Viterbi algorithm is used. As we have analyzed, for G_2 in (7), full diversity order can be achieved for arbitrary relative timing errors, while it is not the case for the delay diversity codes. This can be clearly seen in Fig. 5. In synchronous case G_2 performs slightly better while in asynchronous cases G_2 performs significantly better than the delay diversity code. Note that G_2 and the delay diversity code have the same decoding complexity, since both of them have the same number of states. An interesting property of our newly proposed codes is that their performance improves when the relative timing error range increases, which does not hold for the delay diversity code or Alamouti's code as discussed previously.

Fig. 6 compares the performance of M-algorithm and Viterbi algorithm. In M-algorithm we choose the number of total branches surviving in the search as 4. The codes are constructed

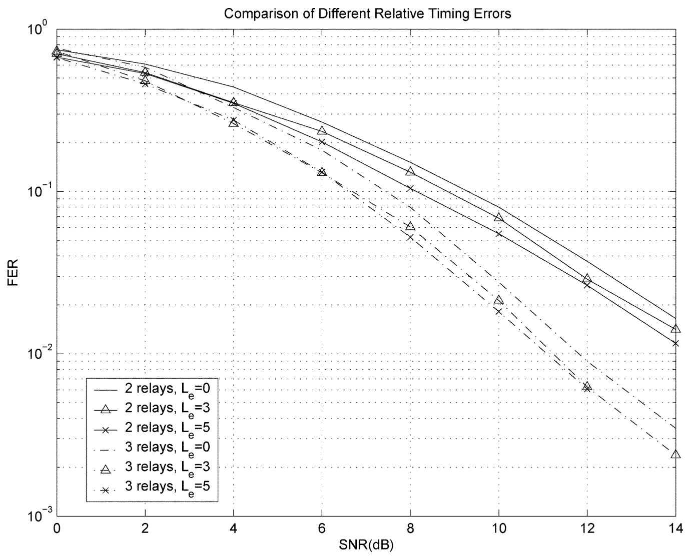

Fig. 7. Comparison of the effects of different ranges of timing errors to the performance of Viterbi algorithm for the proposed space–time trellis codes.

from Theorem 3. For M =2, 3, and 4, the coefficient matrices are chosen as in (7)–(9), respectively. In this figure, we assume no relative timing errors, i.e., $L_e = 0$ or synchronized, between involved relays. From this figure we can see for $M = 2$, when the complexity of M-algorithm is comparable with Viterbi algorithm, their performances are also comparable. For $M = 3$, the number of states in M-algorithm is still remained as 4, but the number of states in Viterbi algorithm has increased to 16. From the simulation we can see that in this case, there is only 0.2-dB performance loss of M-algorithm with respect to Viterbi algorithm. When $M = 4$, because of the larger difference of the numbers of states in M-algorithm and Viterbi algorithm, the performance gap between M-algorithm and Viterbi algorithm becomes larger.

Fig. 7 shows the simulation results of the comparison of the effects of different ranges of relative timing errors to the performances when these relative timing errors are known in the receiver and when the optimal Viterbi algorithm is used. In the simulation, the space–time trellis codes we choose are generated from Theorem 3, and the coefficient matrices are (7) and (8), respectively, for $M = 2$ and $M = 3$. The maximal relative timing errors are $L_e = 0$, i.e., no timing error, $L_e = 3$ and $L_e = 5$. When we implement the simulation, the timing errors for each relay are generated uniformly from the set $\{0, 1, \ldots, L_e\}$. We can see from Fig. 7 that, as the timing error ranges L_e increase, the performance improves. Although we have seen in Section IV that as L_e increases, the diversity product upper bound of the constructed space–time trellis codes does not increase, for the construction in Theorem 3 we can see from the simulation that, as L_e increases, the diversity product is suspected to increase too.

VI. Conclusion

In this paper, to consider the asynchronous nature of the cooperative diversity, we construct a family of space–time trellis codes that have full diversity order without the symbol synchronization requirement. This family of space–time trellis codes can be used for BPSK, QAM and PSK modulation schemes. Some diversity product properties of this family of space–time trellis codes are studied, and M-algorithm is used to decode this family of space–time trellis codes to reduce the decoding complexity. The simulation results are presented to illustrate the promising performances of the newly constructed space–time trellis codes for the asynchronous cooperative communication. Also, from the simulations we find that, as the relative timing error ranges increase, when the timing errors are known at the receiver and the optimal Viterbi algorithm is used, the performance of the newly constructed space–time trellis codes improves.

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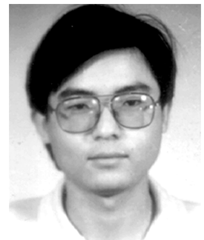

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