
Digital Communications: the Past, Current, and Future

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Outline

- Where is University of Delaware
 - Digital communications has changed our daily life
 - Digital Communications: Basics
 - Cellular Communications: Standards
 - Orthogonal Frequency Division Multiplexing (OFDM) Systems
 - Vector OFDM Systems
 - Conclusions
-



美国特拉华州 (State of Delaware)

特拉华大学 (University of Delaware)

**电子与计算机工程系 (Department of
Electrical and Computer Engineering)**

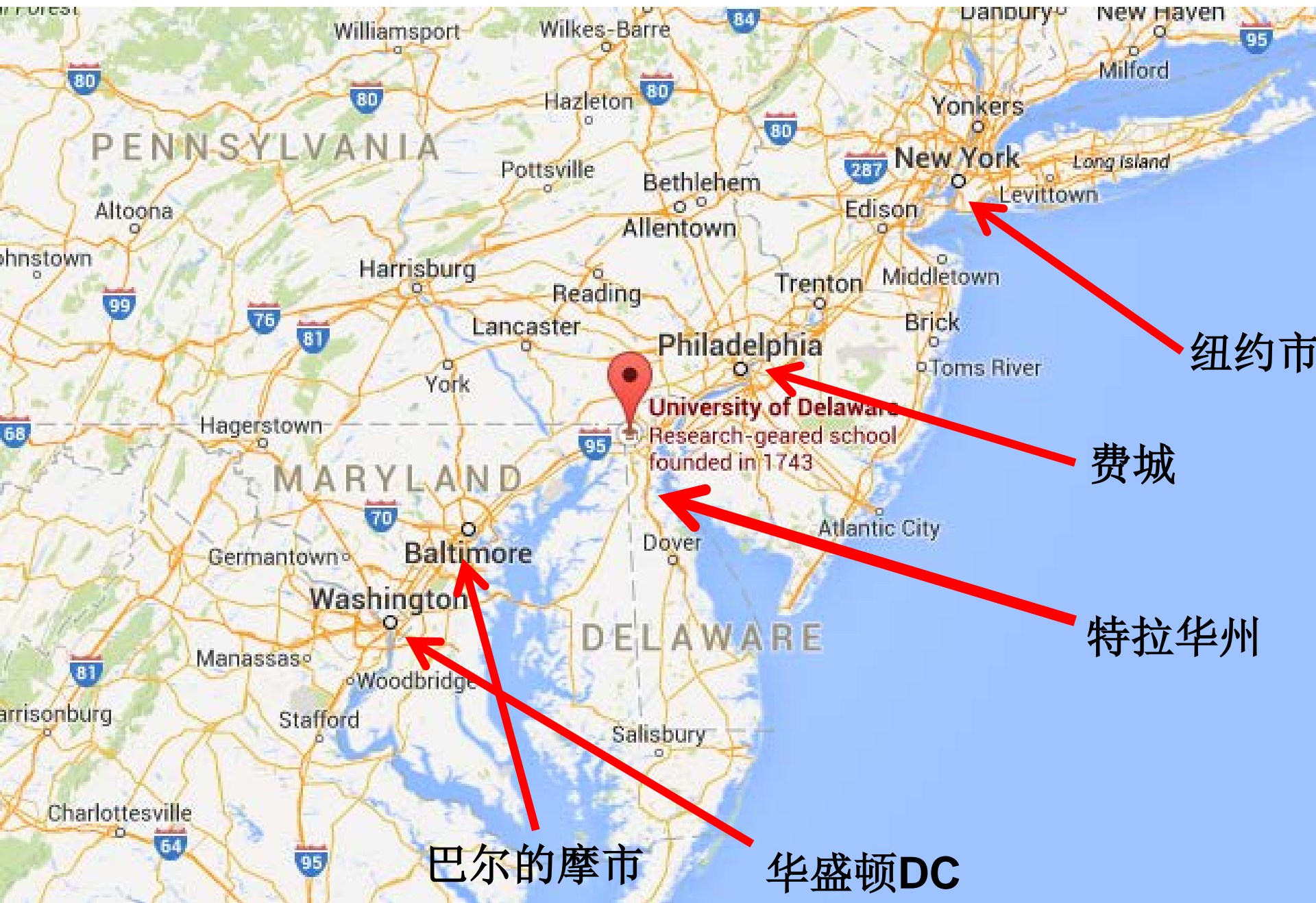


特拉华州

第二小州

雅名:

The First State



纽约市

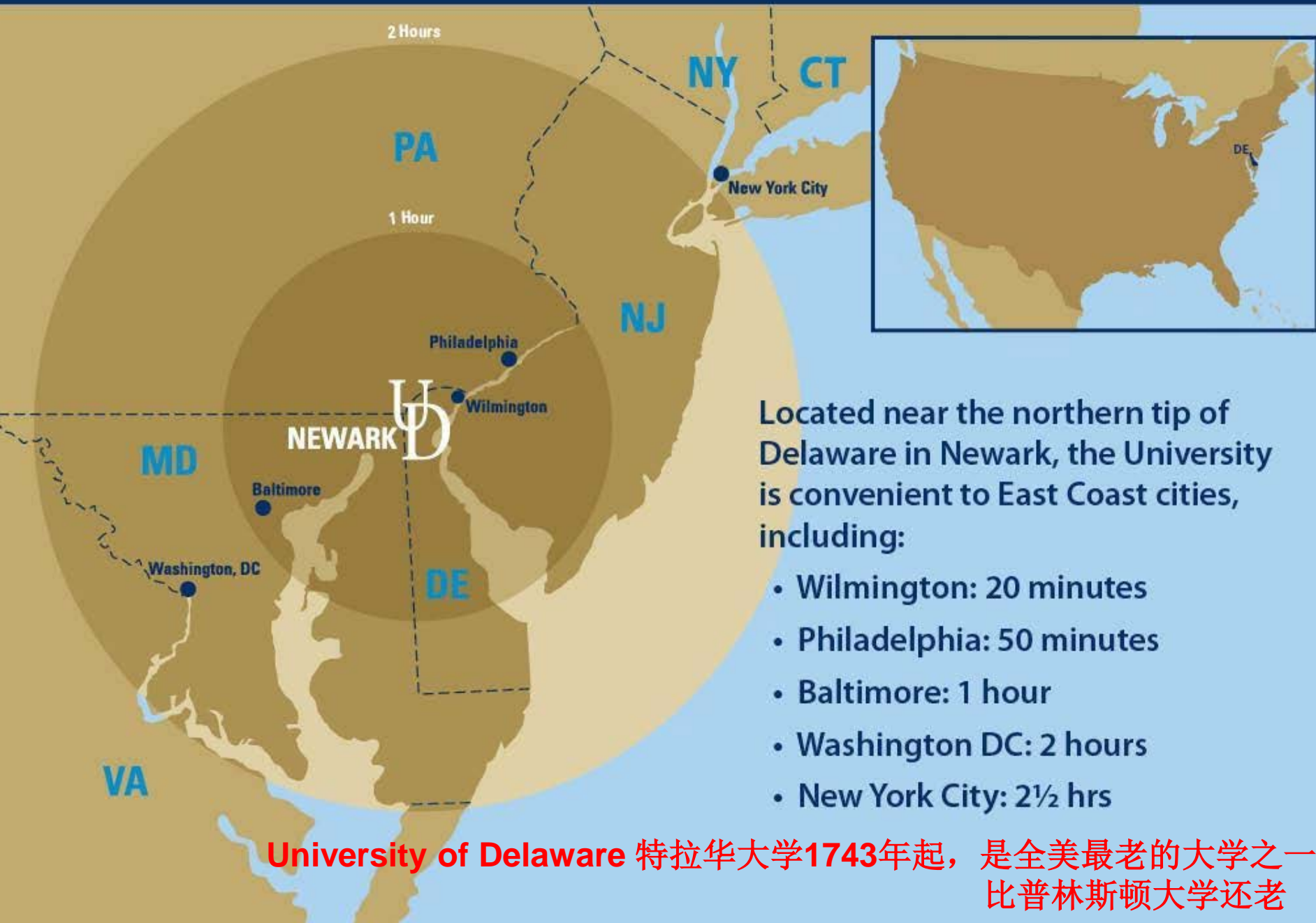
费城

特拉华州

巴尔的摩市

华盛顿DC

Our Location at the Center of the East Coast of the United States

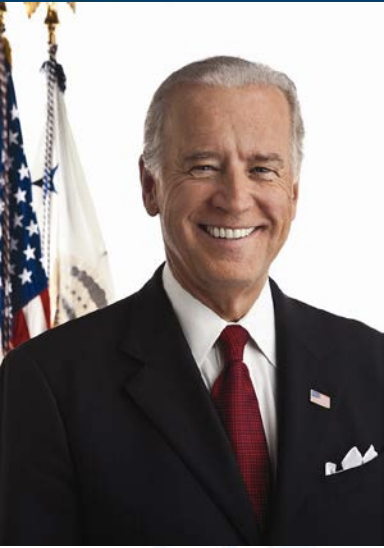


秋天的校园





Famous Alumni



- Joe Biden, the President of USA.
- Chris Christie, Governor of New Jersey (Former)



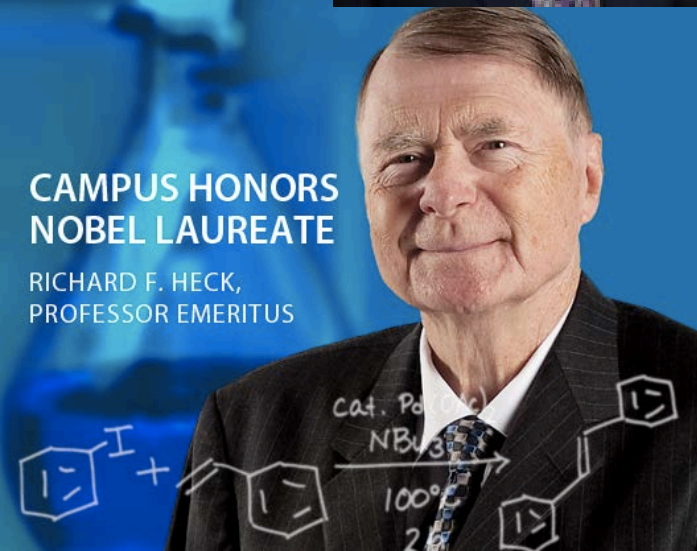
- Joe Flacco, NFL Super Bowl MVP (most valuable player).



- Xin Wang, builder of RenRen Net (人人网) 美团, 系友
- Wayne Westerman, inventor of multi-touch interface. 系友



在美国只有5所大学产生过上面两种校友



Famous Faculty

- **Dave Farber, Internet pioneer.**
Pioneer's Circle of Internet Hall of Fame
网络先驱者名人墙
- **Dave Mills, Internet pioneer and inventor of the Network Time Protocol(NTP)**
- **Richard Heck, 2010 Nobel Prize in Chemistry.**



Innovating in leading tech sectors

FingerWorks, a company started by Electrical and Computer Engineering Professor John Elias and UD alumnus Wayne Westerman, developed the key technology in the iPhone's multi-touch interface.

系友，智能手机的出现改变了人们的日常生活

“The iPhone would not have been possible without the engineering solutions of Professors John Elias and Wayne Westerman of the University of Delaware who developed multi-touch sensing capabilities” --- Steve Jobs' biography



**2005年Apple公司买了FingerWorks公司后
2007年才有iPhone**

而真正的智能手机又是从iPhone开始的

Our daily life has been changed by electronics

- I came to USA in 1988 and I do not feel anything else (clothing, food, housing, transportation) but our electronics, such as computers, phones, internet, are changed/improved over the past 34 years
 - Among all the electronics, due to the chip developments, computers and digital communications have changed the fastest.
-

Digital vs. Analog Communications

家具搬运

工厂

住家

过去：
搬运现存的家具
两边都简单
但：慢！



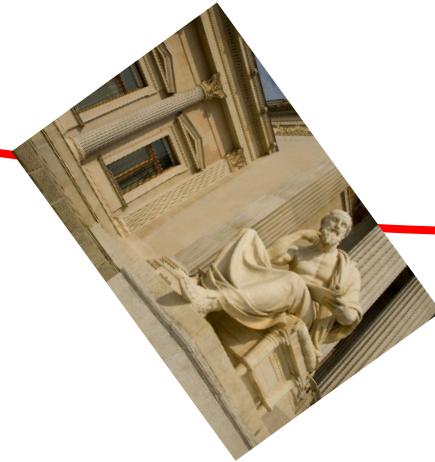
现代：
把家具拆成最小块
再把小块装箱
只搬运规定好的箱子
两边都不简单，要拆要装
但：快！



Digital vs. Analog Communications

通信：信息的搬运

过去：
模拟通信
发送现存图像
两端方便
但送：慢



现代：
数字通信
把图像拆成最小单元
的0和1序列
打包发送
两端不方便，要求高
但送：快



01101001
11001011
10101010



香农数字通信理论

将来：
发送更快，延时更短

频谱是固定的
地球大小是固定的

地球上的人口在增加：现已超过80亿

数字通信：快，但发收两端麻烦

■ 正需要两大类人才

□ 一类是芯片等硬件人才

- 芯片越来越小，越来越快：可以小和快到在你的手机上实时地实现FFT等等处理

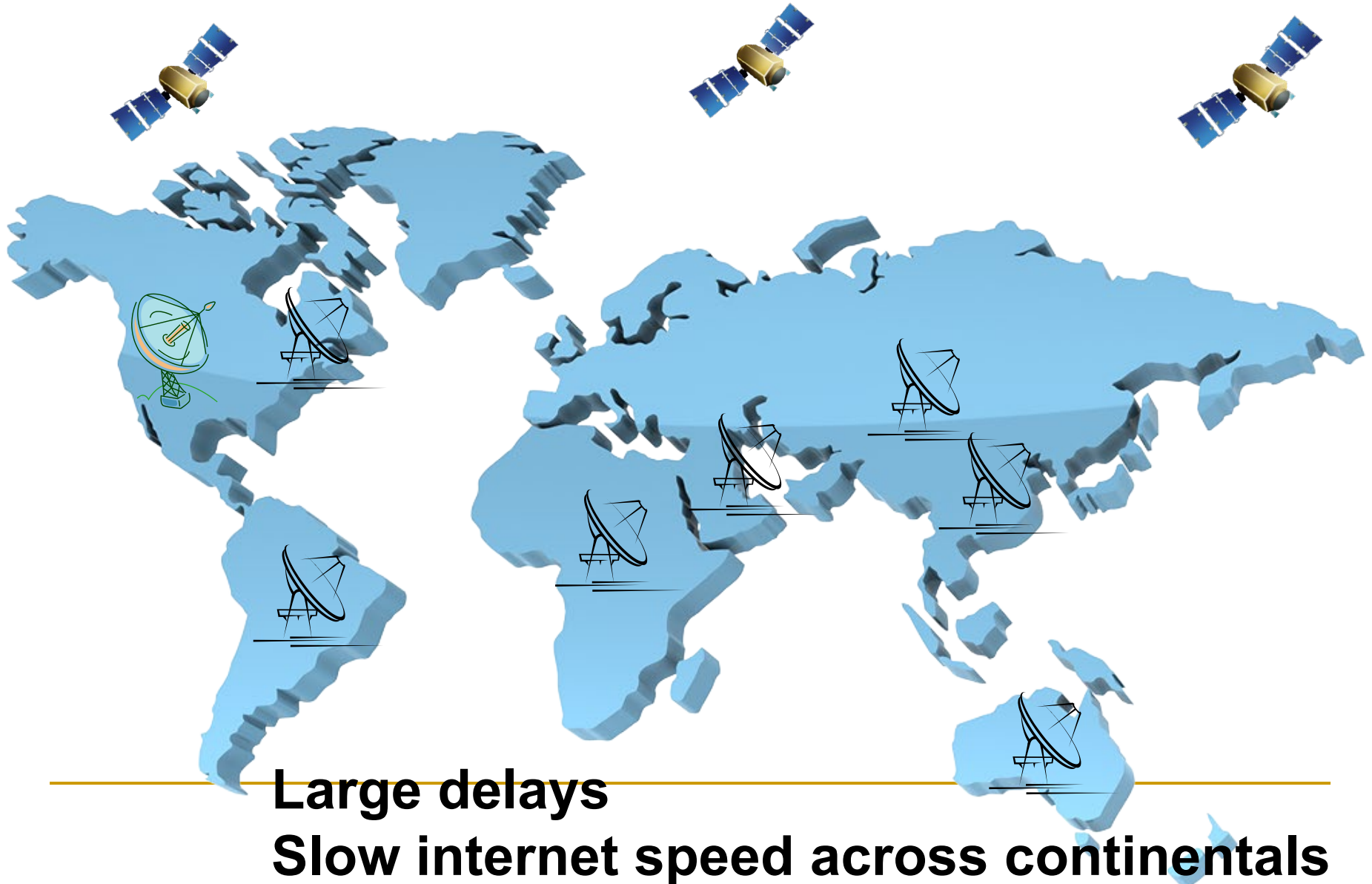
□ 一类是算法等软件人才

- 各种调制，编码，压缩等先进算法的发现



今天的重点

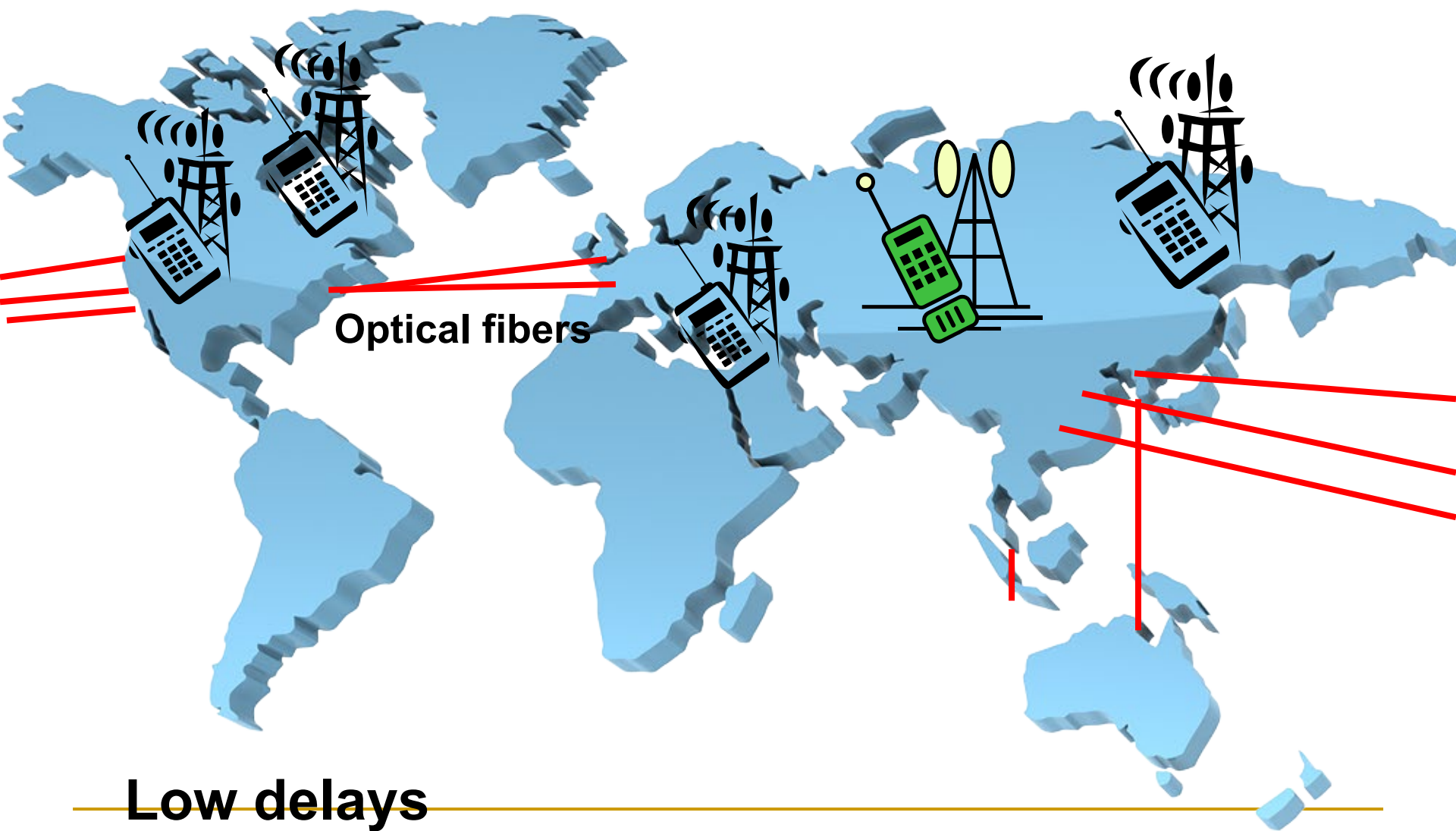
Connectivity: Before 1990's: the world is connected through stationary satellites



Large delays

Slow internet speed across continentals

Connectivity: After 1990's: the world is connected through optical fibers and cellphone towers



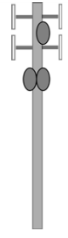
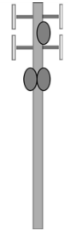
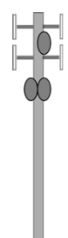
Optical fibers

Low delays

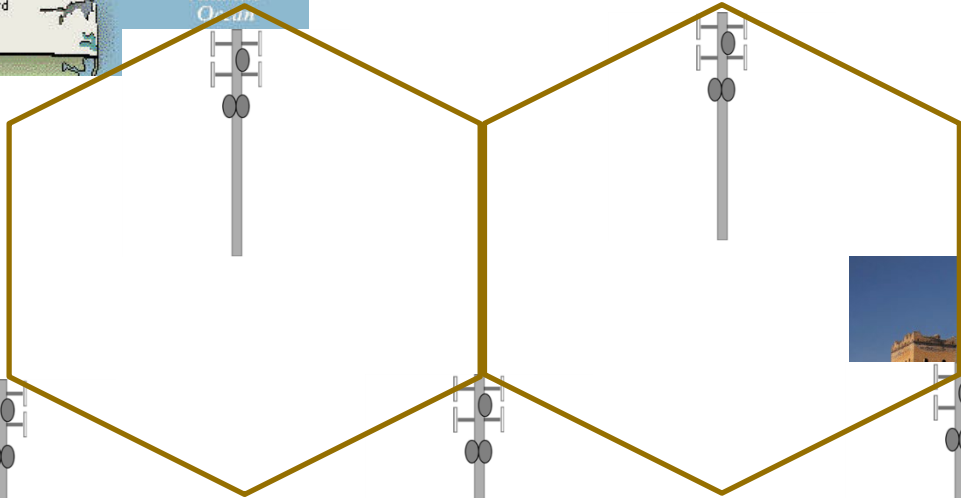
Fast internet speed across continentals

Cellular Systems

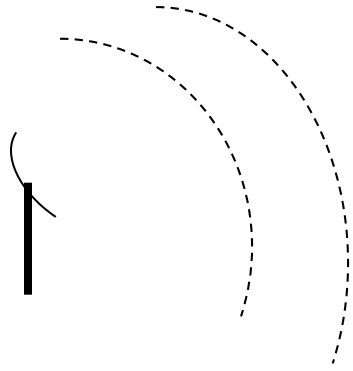
Thank to the development of digital communications, you can communicate with your friends anywhere anytime now (probably not anything yet)



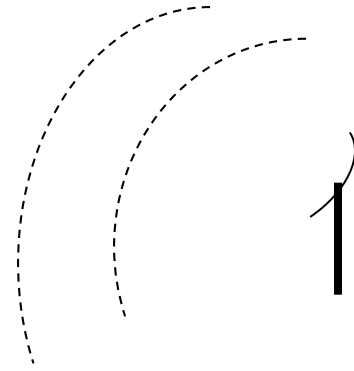
Cellphone towers



RF Communications

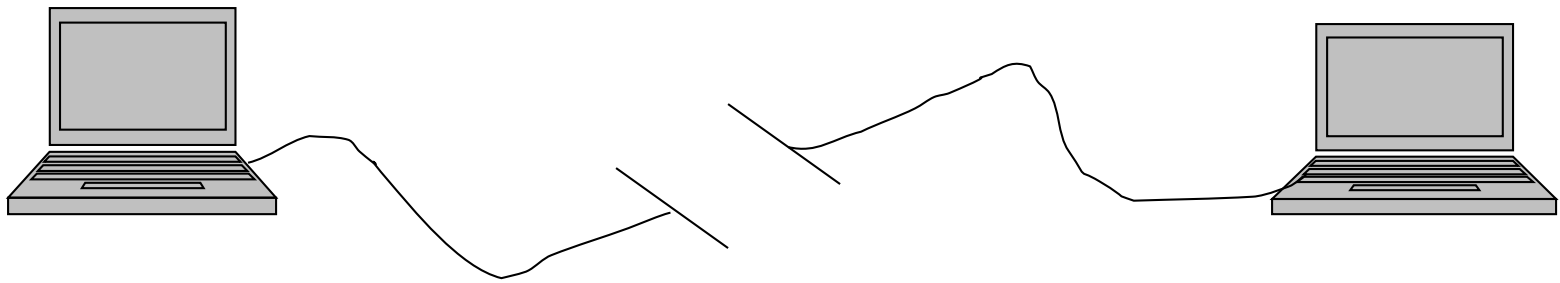


wireless



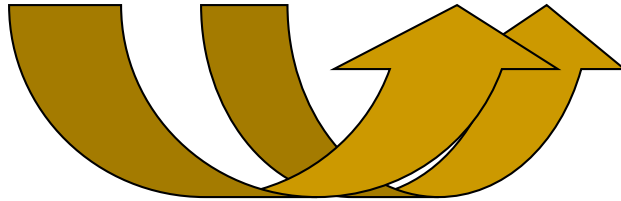
transmission through EM wave propagation

wired



Signal:

$A(t) \cos(p(t))$ carry information

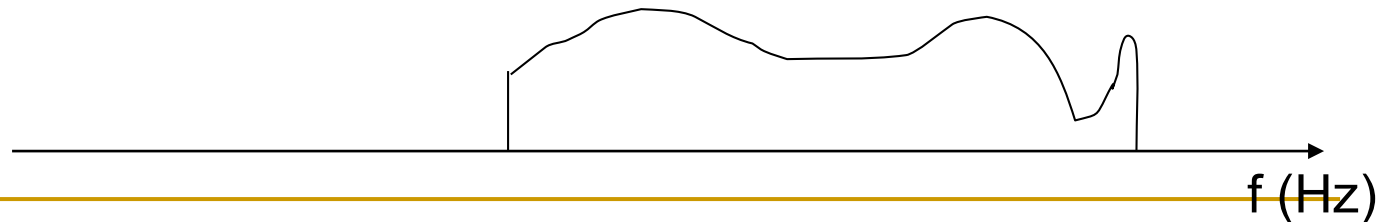


Channel:

a media that wave carrying information propagates through

→ Approximately a linear system

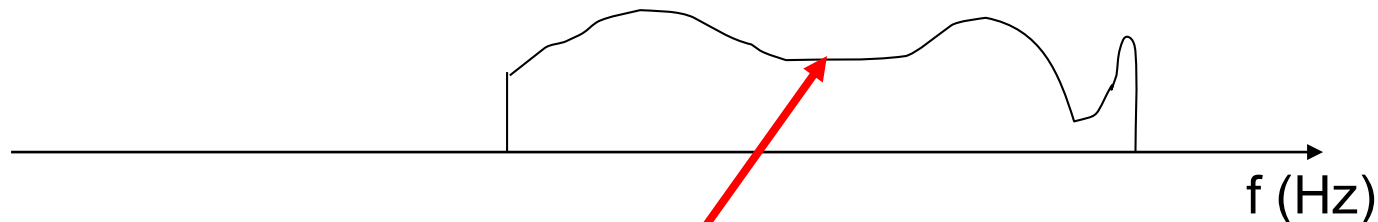
system frequency response



signal
 $\cos(p(t))$

channel

System/channel frequency response



(approximately) flat

none flat

additive white Gaussian (AWGN)

$$y(n) = a x(n) + w(n)$$

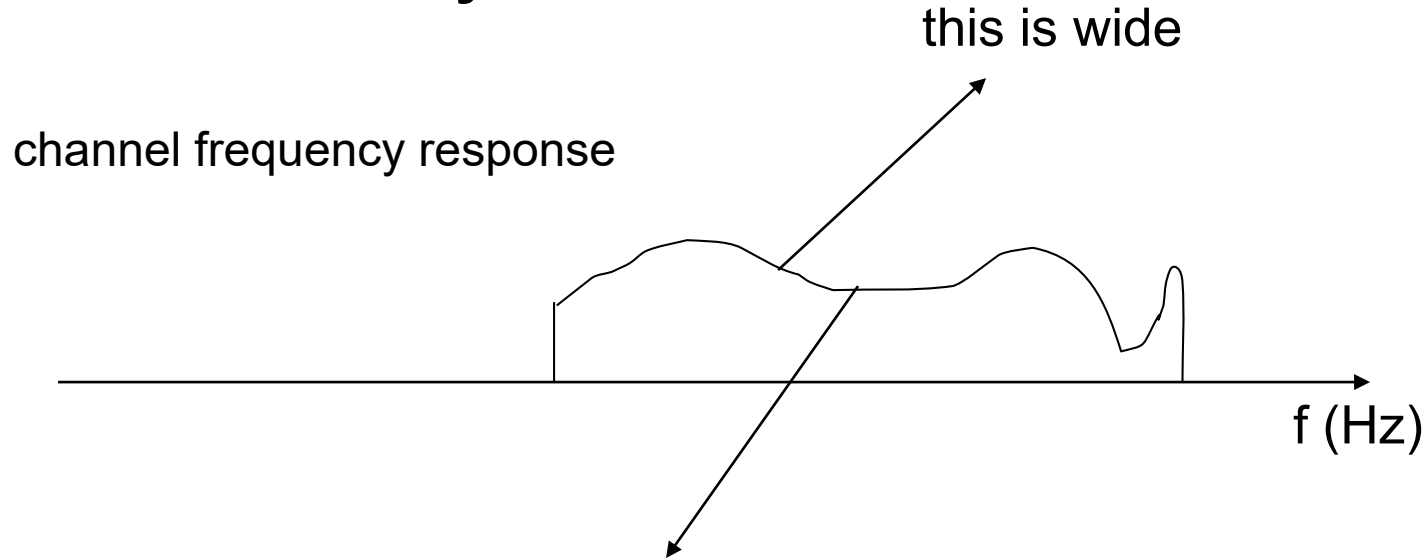
where a is a constant

intersymbol interference (ISI)

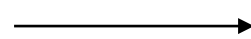
$$y(n) = a x(n) + b x(n-1) + w(n)$$

where a and b are constants

In broadband systems



It is not possible to be flat



ISI occurs

More details will follow

Wireless:

Multipath

Wired:

None flat ISI channel
None ideal

Wired (modem): Channel is fixed and has high SNR

Telnet

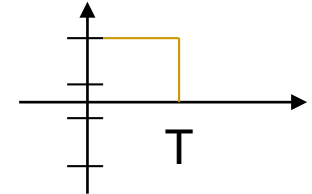
< 9.6 kbs/s

equalization (Lucky 60s)

Squeeze more bits to a symbol

9.6 kbs/s

TCM +equalization (DFE)



time to send one symbol

14.4 kbs/s

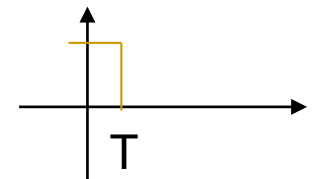
TCM + equalization

28.8 kbs/s

TCM/shaping+equalization

56 kbs/s

Mod/Code Demod/decode



Use more bandwidth

Asymmetric Digital Subscriber Line (**ADSL**)

Ethernet 6 Mbs/s

orthogonal frequency division multiplexing (OFDM)

or called discrete multi-tone (DMT)

Data Rate	Wire Size	Distance
1.5 or 2 Mbps	0.5 mm	5.5 Km
1.5 or 2 Mbps	0.4 mm	4.6 Km
6.1 Mbps	0.5 mm	3.7 Km
6.1 Mbps	0.4 mm	2.7 Km

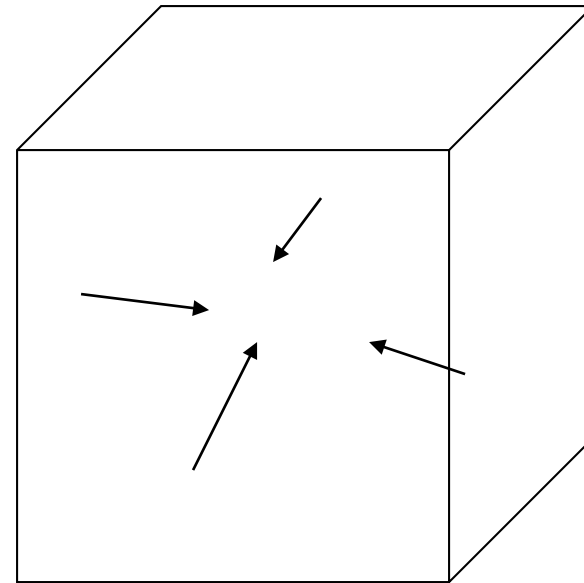
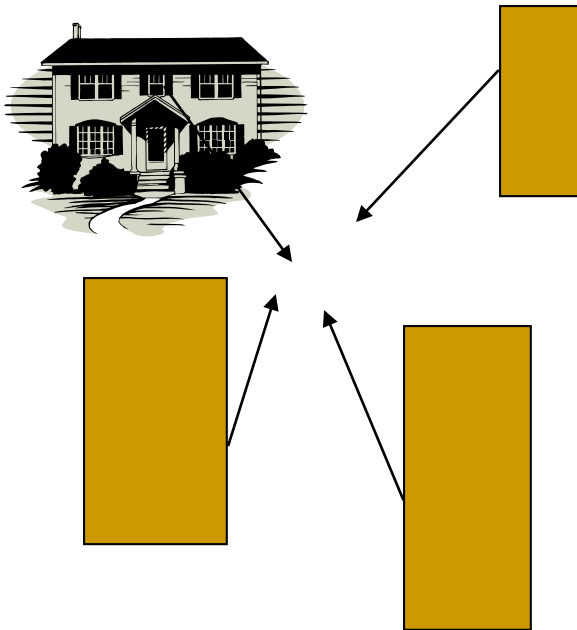
Computer modem was the most important business in communications in the 1990's

Wireless Systems: Channel varies/fades and not high SNR

outdoor

indoor

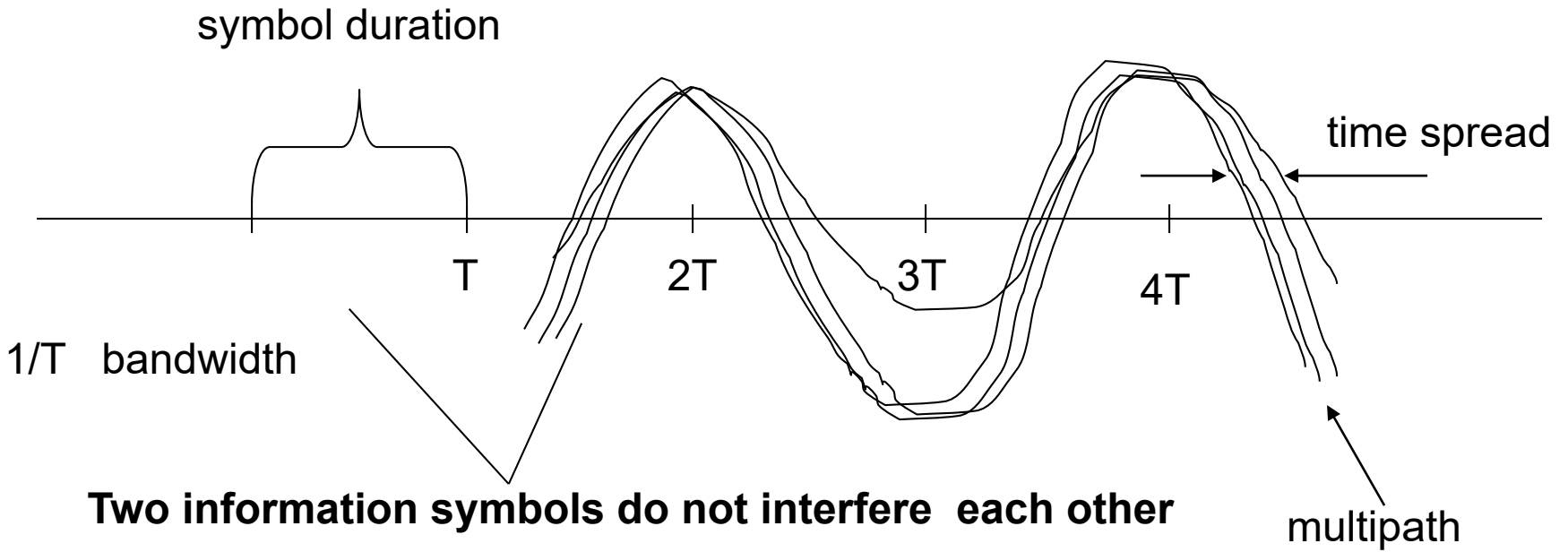
multipaths



multiple reflections

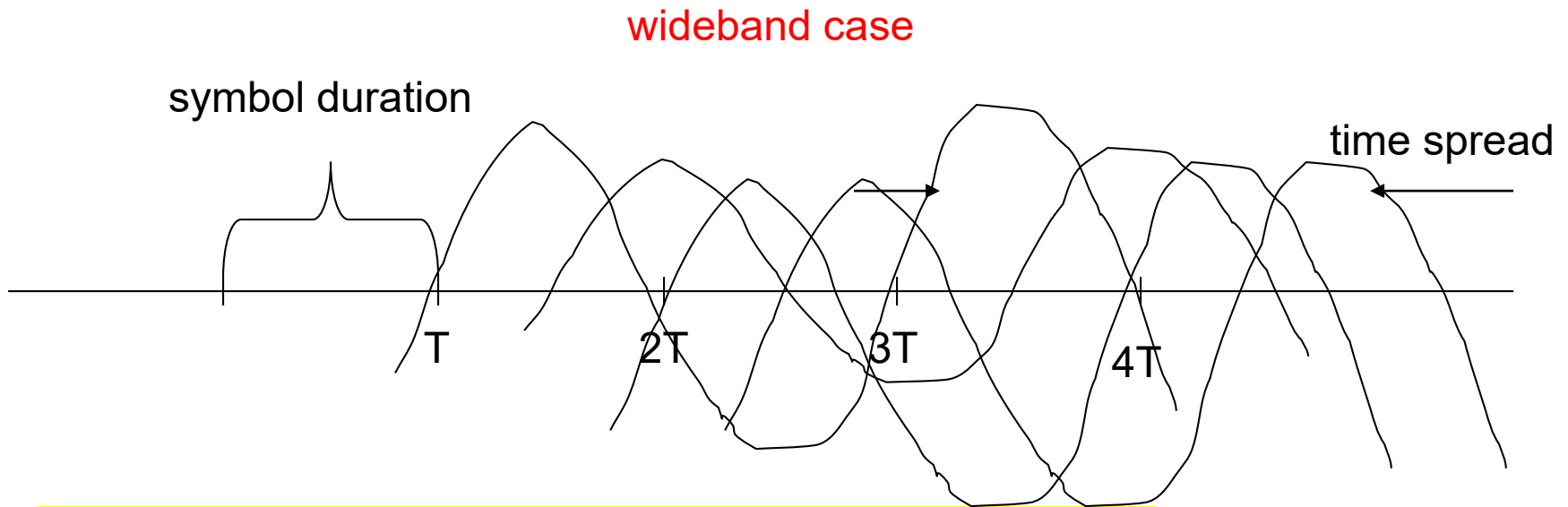
Multipath

narrowband case



No intersymbol interferences

When the bandwidth is too wide or T is too small, the time spread may be across over multiple symbols. In this case, intersymbol interference (ISI) occurs.



$x(t)$: transmitted signal; $r(t)$: received signal

$$r(t) = \sum_{n=0}^L h(n)x(t - nT) + W(t)$$

ISI

AWGN

Modulation Methods for Multi-users

FDMA: frequency division multiple access

1G

TDMA: time division multiple access

2G

CDMA: code division multiple access

2G and 3G

OFDMA: orthogonal frequency division
multiplexing access

4G and 5G

Aug. 11, 1942.

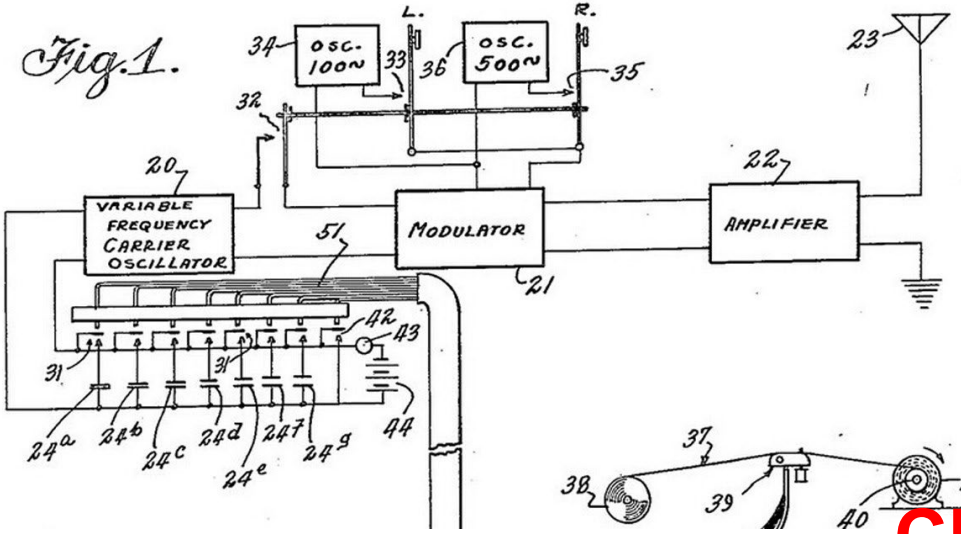
H. K. MARKEY ET AL

SECRET COMMUNICATION SYSTEM

Filed June 10, 1941

2 Sheets-Sheet 1

Patent



CDMA is based on the famous star actress Hedy Lamarr's invention of frequency hopping spread spectrum communications

(88 frequencies, using piano role unpredictably change the signal, 88 black and white keys on a keyboard)

to protect the allies wireless signals for radio-controlled torpedoes to avoid jamming, thus improve the hitting rate



Number of Multipaths vs. Modulation Methods in Wireless Applications

Bandwidth 带宽

2G (IS-95)	1.23 MHz	Almost optimal for single path (or equivalent)	
3G (WCDMA CDMA2000)	< 11 MHz	6--8 multipaths (or equivalent) almost the break point to use CDMA	
IEEE 802.11b (LAN)	similar to 3G		
IEEE 802.11a (LAN) WiFi	20 MHz	16 multipaths (or equivalent) OFDM	past
IEEE 802.11n (LAN) WiFi	20 & 40MHz	40MHz doubles everything in 20MHz OFDM	
4G LTE	20 MHz	16 multipaths (or equivalent) OFDM and SC-FDE Downlink Uplink	
5G	100 MHz		current
6G	??	???	future

Digital Wireless Standards vs. Bandwidth (#of Multipaths)

- A standard is determined by a bandwidth (so far)
- 2G: 1.23MHz, almost the highest for non-ISI (or highest for TDMA in cellular systems)
 - Both TDMA and CDMA (DS spread spectrum) work well
- 3G: ~10 MHz, a few multipaths, highest for CDMA
 - Due to the ISI and wireless varying channels, time domain equalization may not work well, TDMA is not used, but CDMA (DS spread spectrum) is used in all standards since it is good to resist a few chip level time delays (RAKE receiver)
- 4G: 20 MHz, more multipaths
 - **Even CDMA RAKE receiver may not work well**
 - **OFDM is adopted (down link)**
 - Due to wireless channel varying, the number of subcarriers, $N=64$, is used, 25% data overhead for the cyclic prefix (CP) to deal with the multipaths
- 5G: 100 MHz, OFDM

Some Comments on These Standards

- The modulation schemes for all these standards are determined by the way to deal with ISI.
 - In my opinion, multi-access or multi-cells is **NOT** the problem to determine which basic modulation is used.
 - Adding more antennas or not is the hardware choice and may not determine a basic modulation (?)
 - A basic modulation has to be simple.
- Dealing with ISI is the key !

The Most Noticeable Things in Digital Communications in the Past

Communications is always one of the most important tasks among any animals.

- Channel Coding (Always the most impacted, 海底捞针)
 - Shannon's Channel Coding Theorem and Capacity (如来佛)
 - Reed-Solomon Codes (BCH Codes) (孙悟空)
 - Viterbi Decoding
 - Trellis Coded Modulation (TCM)
 - Turbo Codes (LDPC Codes, Iterative Decoding)

The Most Noticeable Things in Communications in the Past

- Source Coding (Compression, 精益求精)
 - Shannon's Source Coding (both lossless and lossy) Theorem
 - Lossless Coding
 - Huffman Coding
 - Lempel-Ziv-Welch Algorithm
 - Lossy Coding
 - DCT, DWT
 - Coded Excited Linear Prediction (CELP) in speech coding
-

The Most Noticeable Things in Communications in the Past

- Systems: Modulations (排兵布阵)
 - CDMA
 - OFDM
 - MIMO: This is natural and not surprising
-

The Most Noticeable Things in Communications in the Past

- Systems: Receiver and Signal Processing (画蛇点睛)
 - Matched Filtering
 - Decision Feedback Equalizer (DFE)
-

The Most Noticeable Things in Communications in the Past

- Techniques/Skills (鲁班在世)
 - Synchronization (Phase Locked Loop)

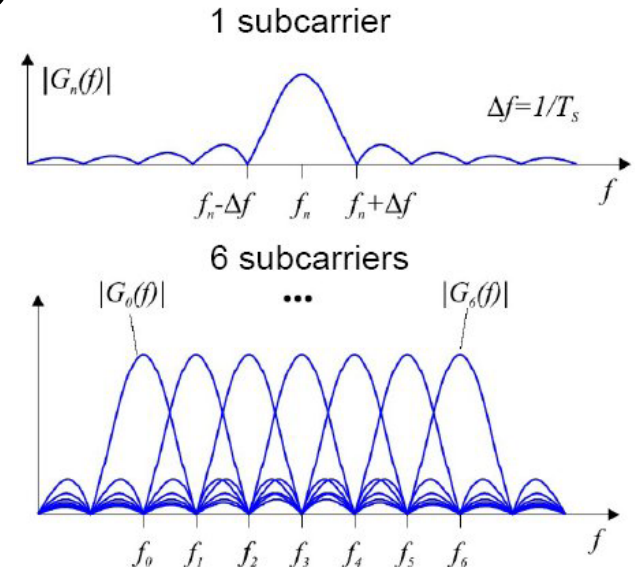


6G: Bandwidth $\gg 20$ MHz (?)

- Can OFDM Still Work?
 - Much more multipaths exist
 - much large CP length to deal with multipaths
 - much large number N of subcarriers/IFFT_size
 - may lead to break down OFDM??
 - High PAPR (?)
 - Time varying channels (?)
- Is multiband OFDM bandwidth efficient?
 - Five 20 MHz bandwidth OFDM systems to form 100 MHz band
- What bandwidth will be the breakpoint for OFDM in **cellular systems**? How large will a bandwidth go?
 - Can we make it work with a **fixed N** while it still can deal with the increased # of multipaths?
- **We next think about single antenna VOFDM [Xia, TCOM, August, 2001, also ICC 2000]**

OFDM and VOFDM

- OFDM: orthogonal frequency division multiplexing

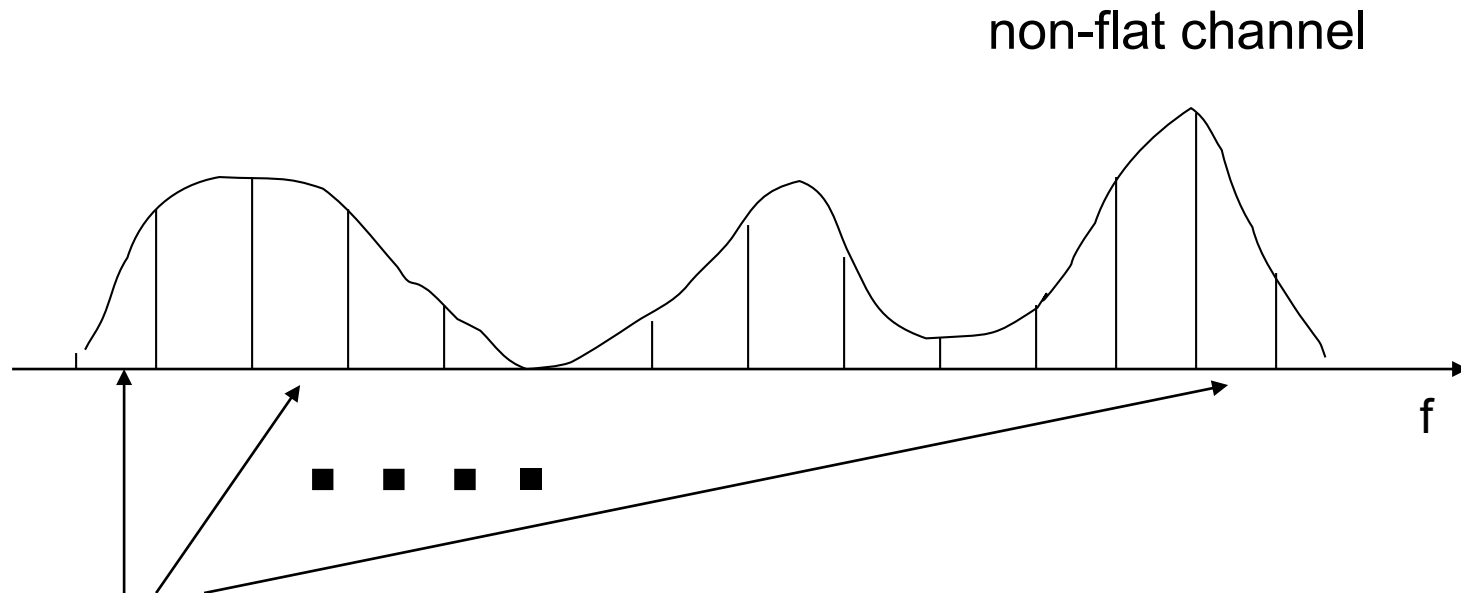


- VOFDM: vector OFDM

- It is nature for multiple antenna systems, when every antenna employs OFDM
 - Cisco's VOFDM for MIMO systems (MIMO-OFDM)
- It is not trivial for single transmit antenna systems

■ **Today's focus**

Why OFDM ? ----- Rough Idea



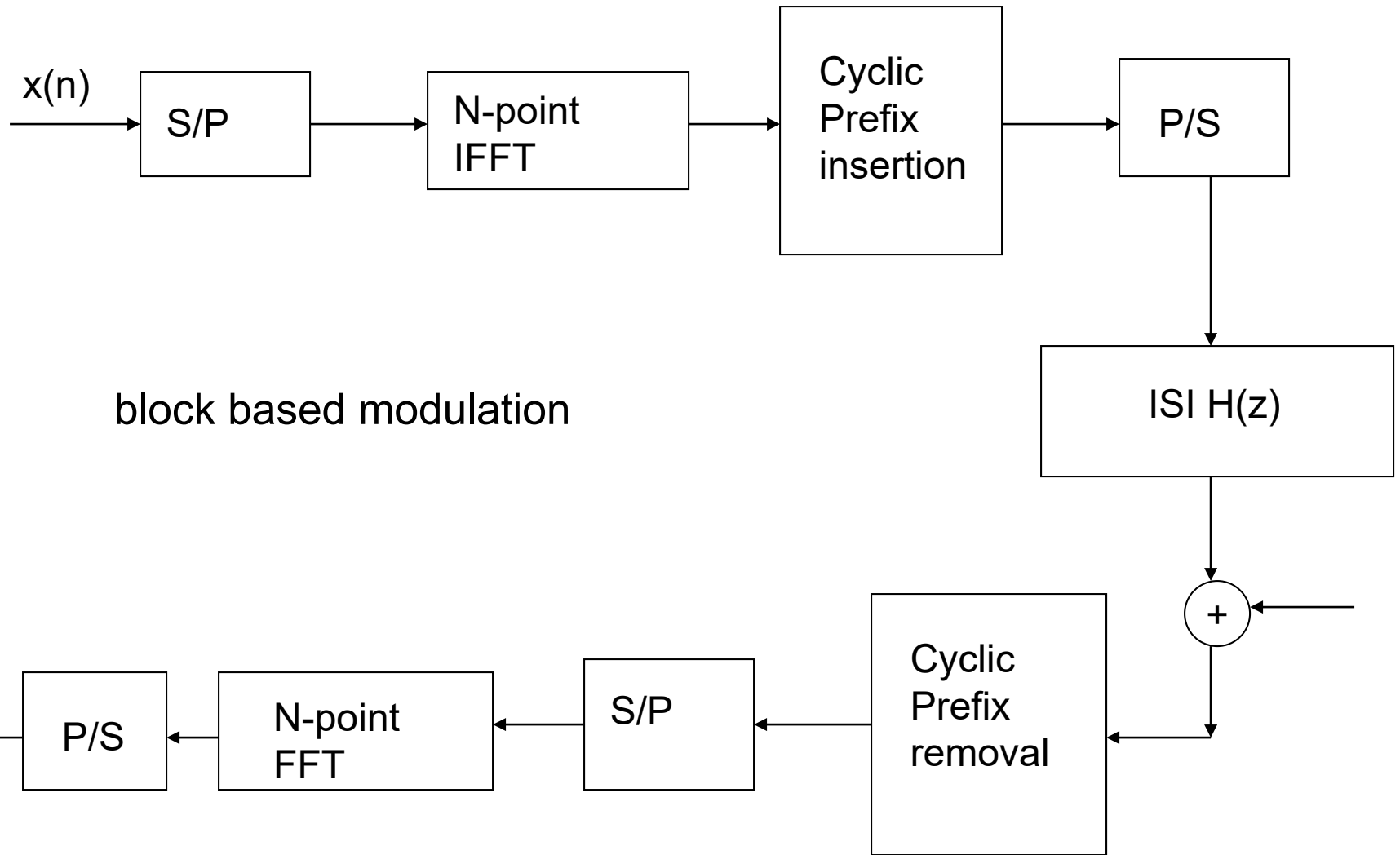
Narrow subchannels using multiple subcarriers

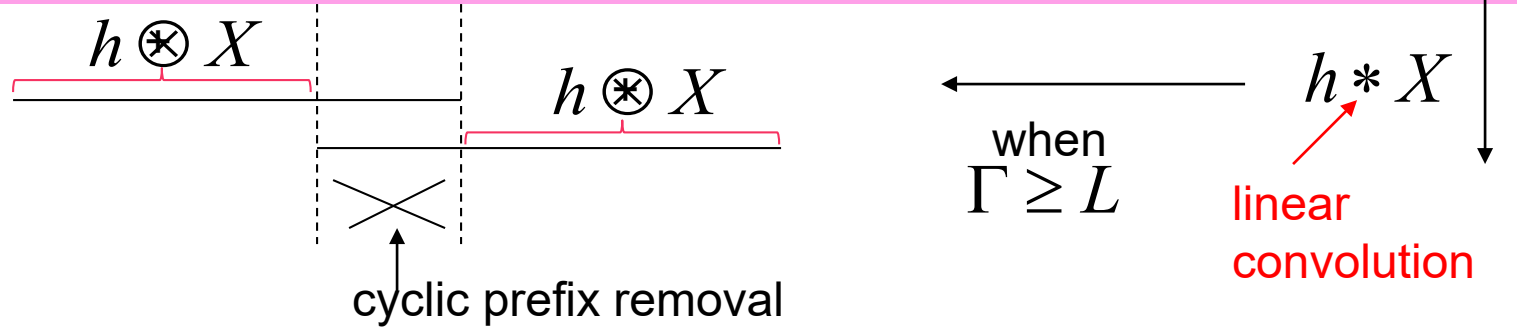
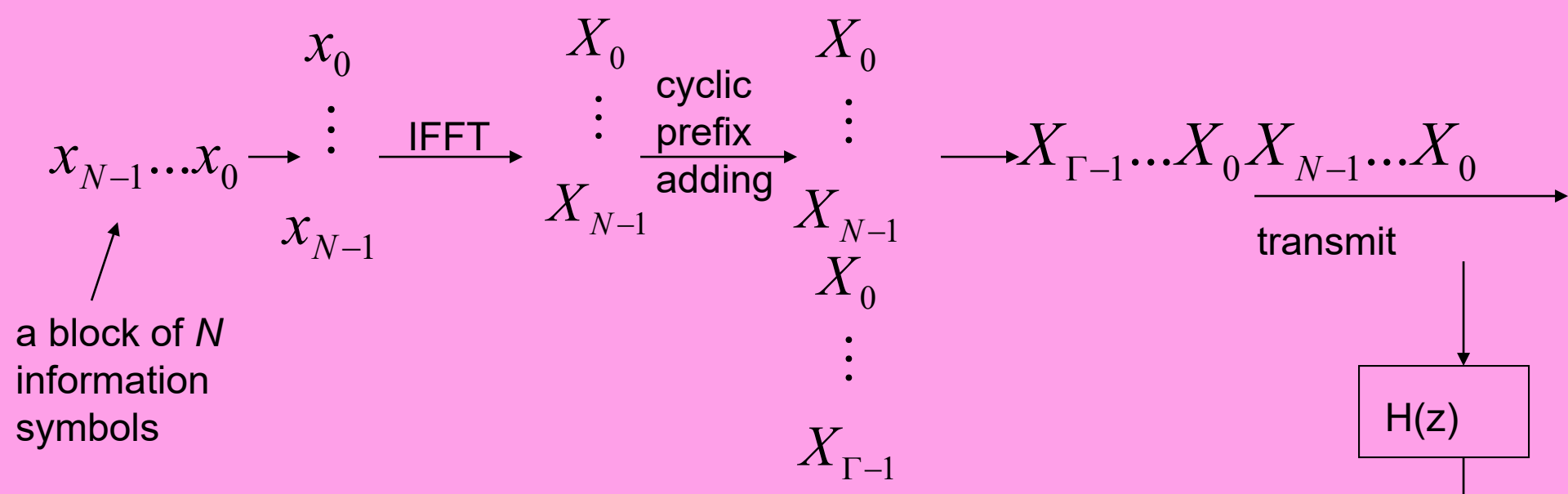
- These subchannels may have overlapped spectrums. So, OFDM is bandwidth efficient
 - The analog signals in these subchannels are not orthogonal each other.
 - Their discrete/sampled signals are orthogonal each other.

Each subchannel is narrow and therefore more flat

- It does not have ISI.

OFDM



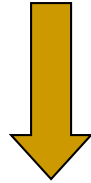


$Y_k = H_k \cdot x_k, k = 0, 1, \dots, N-1$ $\xleftarrow{\text{FFT}}$ $h \circledast X$

$H_k = H(e^{j2\pi \frac{k}{N}}) = \sum_{n=0}^L h(n) e^{-j \frac{2\pi nk}{N}}$ (cyclic convolution)

ISI channel

$$y(k) = \sum_{n=0}^L h(n)x(k-n) + w(k)$$



adding cyclic prefix as an additional data rate **overhead**

N ISI-free subchannels

$$Y_k = H_k \cdot x_k + W_k, \quad k = 0, 1, \dots, N-1$$

Each subchannel corresponds to a DFT component H_k of the ISI channel.
If the frequency component H_k is small (bad), then this subchannel is bad.

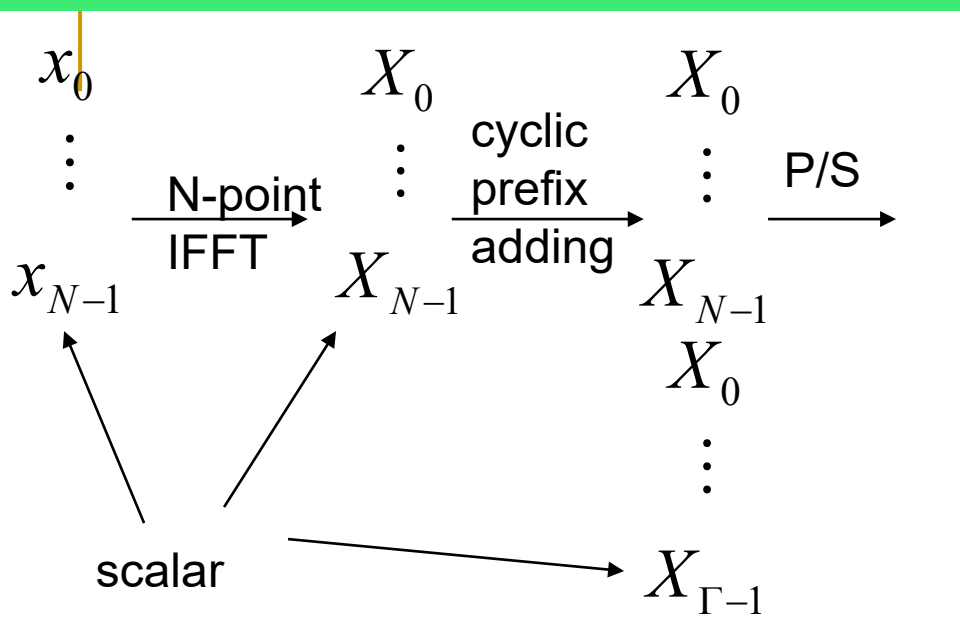
For 20 MHz Channel,
OFDM

$$L \leq 16$$

$$N=64$$

$$\Gamma=L=16, \quad 25\% \text{ data rate overhead}$$

Basic Idea for Vector OFDM Single Transmit Antenna System

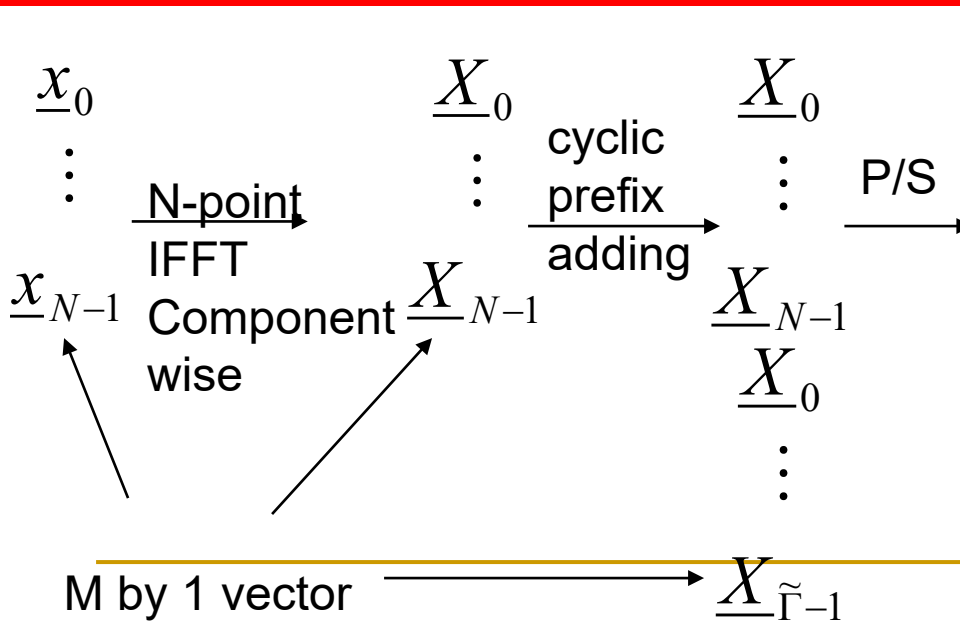


OFDM, when data rate overhead $\Gamma \geq L$

Receiver:

$$Y_k = H_k x_k + W_k$$

N scalar channels/equations



VOFDM, when data rate overhead $\tilde{\Gamma} \geq \tilde{L} \approx \frac{L}{M}$

Receiver:

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{W}_k$$

N M by 1 vector channels/equations

Comments on VOFDM and OTFS

- A pulse shaping filter $p(t)$ is skipped here but can be always added to VOFDM signals in real transmission similar to OFDM.
- The CP length does not have to be exactly a multiple of the vector size M : $\tilde{\Gamma}M$.
 - The CP part can be truncated to any length that is not less than the channel length L to avoid the inter-block-interference.
- The transmission of VOFDM is exactly the same as that of OTFS.

OTFS: orthogonal time frequency space, recently attracted a lot attentions

VOFDM: Vectorized Channel

- The ISI channel $H(z)$ is converted into N vector channels

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{W}_k, \quad k=0, 1, \dots, N-1 \quad (1)$$

where \underline{H}_k is the M by M blocked version of the original frequency responses of the ISI $H(z)$:

$$\underline{H}_k = \underline{H}(e^{j2\pi k/N}), \quad \underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{M-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{M-1}(z) & H_{M-2}(z) & \cdots & H_0(z) \end{bmatrix}$$

$$H_m(z) = \sum_{l=0}^{\tilde{L}'} h(Ml + m)z^{-l}, \quad 0 \leq m \leq M-1.$$

m th polyphase component of $H(z)$

$$\tilde{L}' = \left\lfloor \frac{L}{M} \right\rfloor$$

Why VOFDM Is Good for Channels with Doppler Spread

- The vectorized channel matrix $\underline{H}(z)$ is pseudo-circulant and can be diagonalized by M -point DFT matrix \mathbf{W}_M with a diagonal phase shift matrix $\Lambda(z) = \text{diag}(1, z^{-1}, \dots, z^{-M+1})$ as follows:

$$\underline{H}(z^M) = \left(\mathbf{W}_M^* \Lambda(z) \right)^{-1} \text{diag}(H(z), H(zW_M), \dots, H(zW_M^{M-1})) \mathbf{W}_M^* \Lambda(z)$$

where $W_M = e^{-j\frac{2\pi}{M}}$. Thus, matrices $\underline{H}_k = \underline{H}(W_N^{-k})$ can be diagonalized by $\mathbf{W}_M^* \Lambda(W_N^{-k})$, $k=0, 1, \dots, N-1$.

- The receiver equation (1) becomes

$$\underline{\tilde{Y}}_k = \text{diag}(H(W_{MN}^{-k}), H(W_{MN}^{-k}W_M), \dots, H(W_{MN}^{-k}W_M^{M-1})) \mathbf{W}_M^* \text{diag}(1, W_{MN}^k, \dots, W_{MN}^{k(M-1)}) \underline{x}_k + \underline{\tilde{\eta}}_k$$

This frequency domain part is similar to the channel in time domain for single antenna systems
Or
diagonal space-time coded MIMO system

This part is similar to the precoding to achieve signal space diversity
Or
diagonal space-time block coding
To achieve spatial diversity

When channel varies with Doppler spread, it can collect multipath diversity and/or signal space diversity. This will be seen later even with the MMSE receiver.

VOFDM vs OTFS

- The VOFDM receiver equation to demodulate

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{\eta}_k$$

- It coincides with that of OTFS when the channel is stationary/quasi-static at both Tx and Rx.

Y. Ge, Q. Deng, P. C. Ching, and Z. Ding, "OTFS Signaling for Uplink NOMA of Heterogeneous Mobility Users," *IEEE Trans. on Commu.*, vol. 69, no. 5, pp. 3147-3161, May 2021.

R. Patchava, Y. Hong, and E. Viterbo, "OTFS Performance on Static Multipath Channels", *IEEE Wireless Commu. Lett.*, vol. 8, no. 3, pp. 745 – 748, 2019.

- **In Fact, the transmission of OTFS is the same as that of VOFDM, no matter the channel is stationary or not.**
 - **The transmitted signals of OTFS and VOFDM are the same in either discrete-time sequence or continuous-time waveform.**
 - X.-G. Xia, "The transmitted signals of OTFS and VOFDM are the same," *IEEE Trans. Wireless Commun.*, DOI 10.1109/TWC.2022.3190437, 2022, Feb. 2023.
- Some other names proposed later in the literature:
OSDM, Quadrature OFDMA (or A-OFDM)

Vectorized Channel Example

If $H(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.6z^{-3} + 0.5z^{-4} - 0.4z^{-5}$, vector size $M=2$,

then, its polyphase components are

$$H_0(z) = 1 - 0.8z^{-1} + 0.5z^{-2}, \quad H_1(z) = 0.9 + 0.6z^{-1} - 0.4z^{-2}$$

and the vector channel coefficient matrices are

$$\underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_1(z) \\ H_1(z) & H_0(z) \end{bmatrix}$$

$$L = 5$$

$$\tilde{L} = \left\lceil \frac{L}{M} \right\rceil = \left\lceil \frac{5}{2} \right\rceil = 3$$

$$= \begin{bmatrix} 1 & 0 \\ 0.9 & 1 \end{bmatrix} + \begin{bmatrix} -0.8 & 0.9 \\ 0.6 & -0.8 \end{bmatrix} z^{-1} + \begin{bmatrix} 0.5 & 0.6 \\ -0.4 & 0.5 \end{bmatrix} z^{-2} + \begin{bmatrix} 0 & -0.4 \\ 0 & 0 \end{bmatrix} z^{-3}$$

VOFDM/OTFS, OFDM, SC-FDE

- When $M=1$, VOFDM=OFDM
 - When $M=N$ and the FFT size is 1, VOFDM=SC-FDE:
 - at the transmitter, no IFFT is implemented (so the PAPR is not changed) but just CP of the information symbols is inserted; low PAPR
 - at the receiver, both FFT and IFFT, and frequency domain equalizer are implemented
 - VOFDM is a bridge between OFDM and SC-FDE
 - Its ML receiver complexity is also in the middle
-

Time domain single carrier
vs. equalization

Maximum # symbols in ISI



VOFDM

No, or 2, or 3, ...,
or Maximum #
(**you choose**)
symbols in ISI



Frequency domain
OFDM

No ISI

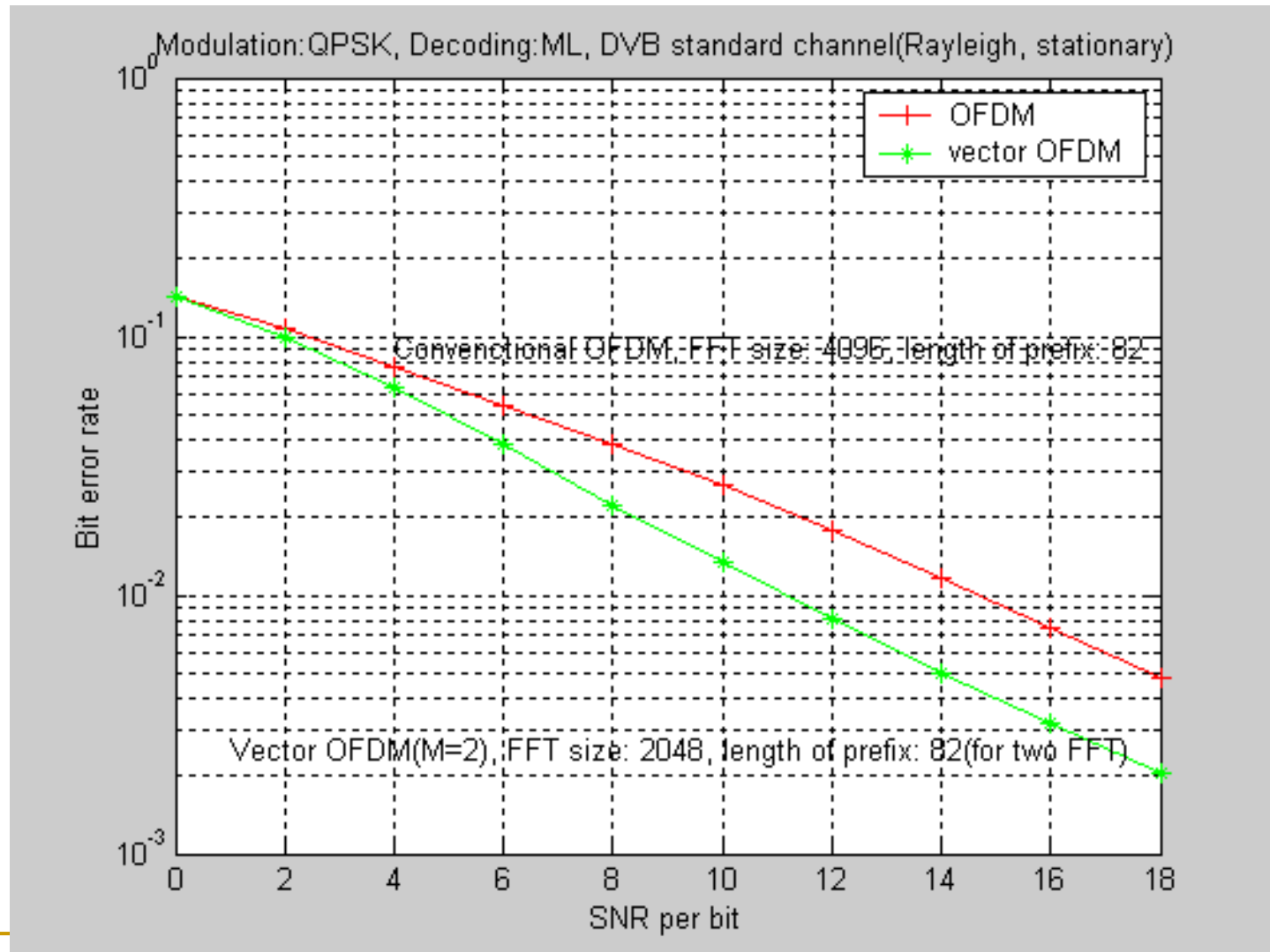


VOFDM/OTFS: Advantages

- Cyclic prefix data rate overhead reduction when the FFT/IFFT size is fixed
 - For OFDM, it is $\frac{L}{N}$
 - For VOFDM, it is $\frac{L}{MN}$
- For fixed cyclic data rate overhead, the FFT/IFFT size can be reduced by M times
 - The IFFT size reduction reduces the peak-to-average power ratio (PAPR), which is important in cellular communications.
- VOFDM achieves multipath diversity even with the MMSE linear receiver, good to both frequency selective fading and time selective fading/Doppler

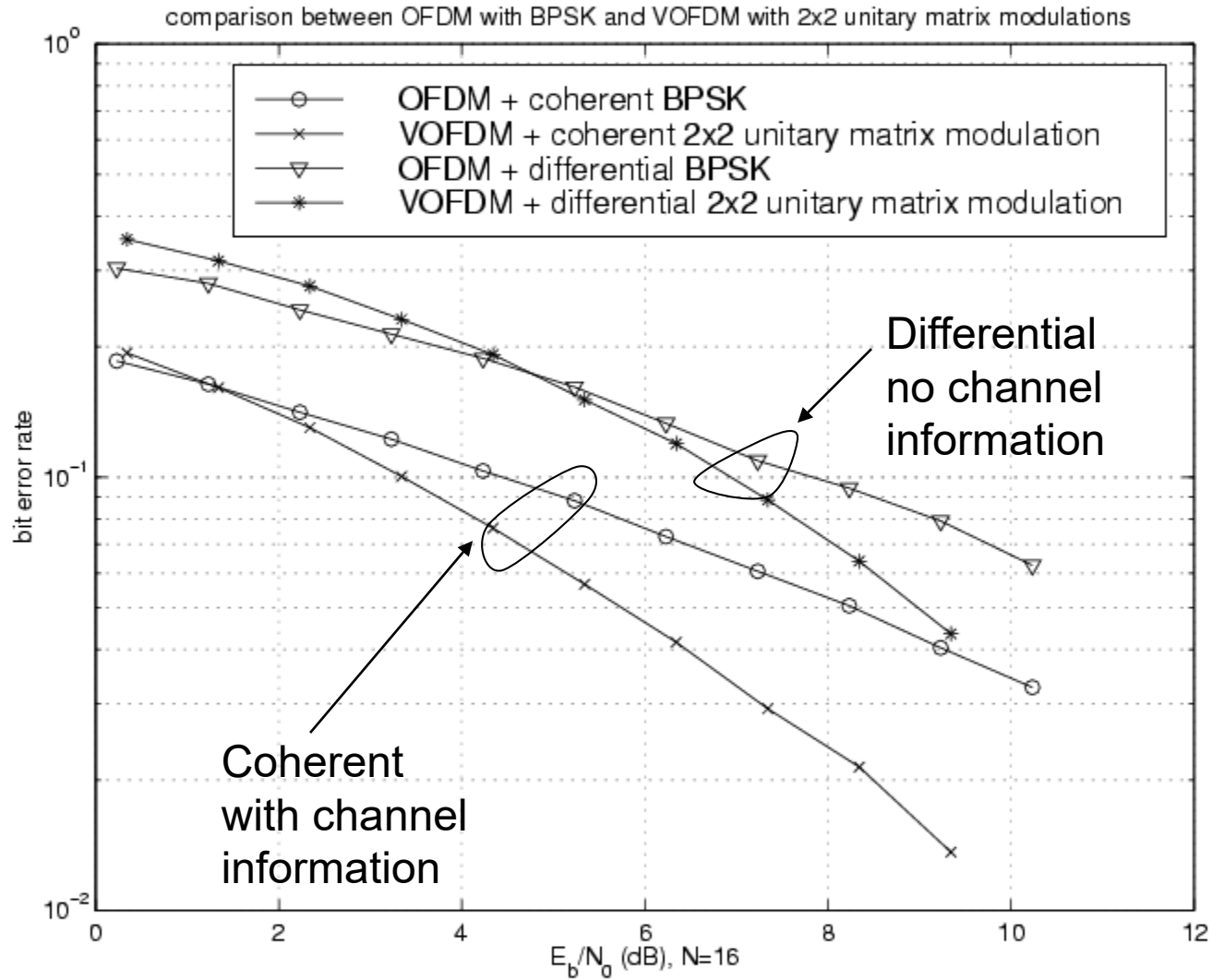
Simulations

DVB



CP data rate overhead is the same for the two curves, matrix modulation is not used.

ML receivers



Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD can be used to collect both spatial and multipath diversities in a MIMO-OFDM systems

$$\begin{array}{ccc} \underbrace{h_{11}, h_{12}, \dots, h_{1L}}_{\rightarrow} & & \\ \vdots & \xrightarrow{\text{After CDD}} & \underbrace{h_{11}, \dots, h_{1L}, \dots, h_{n_t1}, \dots, h_{n_tL}}_{\rightarrow} \\ \underbrace{h_{n_t1}, h_{n_t2}, \dots, h_{n_tL}}_{\rightarrow} & \text{It is equivalent to} & \\ & \text{if } N \geq n_t L & \end{array}$$

When the bandwidth is larger, the number L of multipaths will be larger too. Then, CDD in this case may not be able to collect full spatial and multipath diversities anymore.

Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD VOFDM can collect both spatial and multipath diversities despite of a large bandwidth

$$\begin{array}{ccc}
 \begin{array}{c}
 H_{11}, H_{12}, \dots, H_{1\frac{L}{M}} \\
 \xrightarrow{\hspace{10em}} \\
 \vdots \\
 H_{n_t1}, H_{n_t2}, \dots, H_{n_t\frac{L}{M}} \\
 \xrightarrow{\hspace{10em}}
 \end{array}
 &
 \begin{array}{c}
 \text{After CDD} \\
 \xrightarrow{\hspace{10em}} \\
 \text{It is equivalent to} \\
 \text{if } N \geq n_t \frac{L}{M}
 \end{array}
 &
 \begin{array}{c}
 H_{11}, \dots, H_{1\frac{L}{M}}, \dots, H_{n_t1}, \dots, H_{n_t\frac{L}{M}} \\
 \xrightarrow{\hspace{10em}}
 \end{array}
 \end{array}$$

The number of multipaths is equivalently reduced by M times for VOFDM with a vector size M

Recall Physical Layer Communications Developments in Recent Decades for Both Wireless and Wired Systems

- It has been always on dealing with inter-symbol interference (ISI)

Time domain single carrier
vs. equalization

Maximum # symbols in ISI



VOFDM

No, or 2, or 3, ...,
or Maximum #
(**you choose**)
symbols in ISI



Frequency domain
OFDM

No ISI



Is this VOFDM something to think
about after OFDM?

Or what's next???

Conclusion

- VOFDM provides a tradeoff between the receiver complexity, performance, PAPR, CP overhead for an ISI channel.
 - VOFDM is in the middle between single carrier and OFDM systems in terms of dealing with ISI.
 - OTFS coincides with VOFDM
 - **The transmission of OTFS is the same as that of VOFDM no matter the channel is stationary or not.**
 - CDD VOFDM for multi-antennas can collect both spatial and multipath diversities, where CDD OFDM is not be able to do so in a large bandwidth system.
-

Conclusion

- **More bandwidth will be used in the next 6G cellular systems**
- **In my opinion, a basic modulation format for a standard is still determined by how to deal with the ISI**
- **Low latency high speed real-time communications, with applications in autonomous cars, auto factories, VR etc.**

Conclusion: Modulations

- Wireless Communications Can Be Categorized as
 - Narrowband: both TDMA and CDMA work well
 - 2G
 - Low wideband: CDMA
 - 3G
 - Wideband: OFDM
 - 4G, 5G
 - High wideband: VOFDM (It is scalable to bandwidth)
 - 6G?
-

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Thank you!
