

Simple Analysis of XCP Equilibrium Performance

Peng Wang

Department of Electrical & Computer Engineering
University of Delaware
Newark, DE, USA
Email: pwangee@udel.edu

David L. Mills

Department of Electrical & Computer Engineering
University of Delaware
Newark, DE, USA
Email: mills@ece.udel.edu

Abstract—The recently developed eXplicit Control Protocol (XCP) is a congestion control system well suited for high Bandwidth Delay Product (BDP) networks. Low proves that the XCP equilibrium solves a constrained max-min fairness problem, and derives the explicit formulas for the link utilization and the common flow rate bottlenecked at the link by applying the derived window-based dynamic model. Furthermore, a lower bound and an upper bound of link utilization are given in Low’s paper.

However, the derived window-based dynamic model is so complicated that it can not be used easily. In this paper, a simple rate-based model is proposed for analyzing XCP equilibrium performance. Most results that appeared in Low’s paper can be easily reproduced by applying our simple dynamic model. Furthermore, we prove that the shuffling parameter γ is no more than the parameter α to achieve high link utilization.

I. INTRODUCTION

The current Internet provides a connectionless, best-effort, and end-to-end packet service by using the IP protocol. The majority of the Internet traffic including HTTP, FTP, TELNET, and email traffic is carried by TCP protocol. The Van Jacobson’s congestion control algorithms [1] for TCP guarantee the stability of today’s Internet. However, Jacobson’s algorithm also exhibits several shortcomings, such as its poor link utilization at high BDP, its unfairness at long round-trip times, and confusion by lossy links. Due to the interest in the wired network, we focus on the first two shortcomings.

Several ways are developed to improve the performance in these environments [2], [3], [4], [5], [6], [7]. Of these, the recently developed Explicit Control Protocol (XCP) [4] is a congestion control system well suited for high Bandwidth Delay Product (BDP) networks. XCP executing in the routers explicitly allocates the feedback to each packet, and the minimum feedback for each packet along the path is sent back to the sender. The per-flow state information is carried in the congestion header and the router does not maintain per-flow state to avoid the scalability problem. XCP achieves high link utilization, fairness among flows with different RTTs, and reduces the packet losses. XCP is proved to be stable in the single link case when all flows have the same RTT. Simulations show that XCP is stable in more complicated

topologies, although the stability with a complicated topology is hard to be analytically proven.

Low [8] studies the XCP’s equilibrium performance and gives some interesting results. He builds a dynamic window-based model of XCP, proves that the XCP equilibrium solves a constrained max-min fairness problem, and derives the explicit formulas for the link utilization and the common flow rate bottlenecked at the link. Furthermore, a lower bound and an upper bound of link utilization are given in Low’s paper. However, although a beautiful mathematic description of XCP is given, the derived window-based dynamic model is so complicated that it can not be used easily. Another shortcoming is that the shuffling parameter γ and the parameter α can not be chosen arbitrarily which is not proven in Low’s paper.

In this paper, we also focus on analysis of XCP’s equilibrium performance. A simple rate-based model is proposed for analyzing XCP’s equilibrium performance. Most results that appeared in Low’s paper can be easily reproduced by applying our simple dynamic model. Furthermore, we prove that the shuffling parameter γ is no more than the parameter α to achieve high link utilization, which is an important constraint for XCP.

The remainder of the paper is structured as follows. In the next section, XCP and Low’s window-based model are introduced briefly. Then, the simple rate-based model is derived in detail. In Section III, we prove the important constraint that $\gamma \leq \alpha$ and analyze the equilibrium performance of XCP briefly. Finally, we conclude in Section IV.

II. MODEL

A. XCP Congestion Control

XCP is a window-based congestion control algorithm. The per-flow state information is carried in the congestion header and the router does not maintain per-flow state to avoid the scalability problem. The congestion header includes three fields: H_cwnd (sender’s current cwnd), H_rtt (sender’s rtt estimate) and H_feedback (initialized to sender’s demands).

XCP uses Efficiency Controller (EC) and Fairness Controller to achieve efficiency and fairness respectively. EC determines the aggregate feedback in one control interval and does not care about the per-flow feedback. The aggregate feedback ϕ is computed each control interval:

$$\phi = \alpha \cdot d \cdot S - \beta \cdot Q$$

¹This research is sponsored by the NSWCCD grant N00178-04-1-9001. Views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies or endorsements of NSWCCD or the US government.

where α and β are set to 0.4 and 0.226 respectively based on the stability analysis, d is the average RTT which is the control interval, S is the spare bandwidth which is the difference between the flow load and the link bandwidth, and Q is the persistent queue size.

The fairness controller (FC) adopts Additive-Increase Multiplicative-Decrease (AIMD) policy to achieve fairness. Additive-Increase means that the throughput of each flow increases same when $\phi > 0$, and Multiplicative-Decrease means that the flow's throughput decreases proportionally to its throughput when $\phi < 0$.

Moreover, XCP uses bandwidth shuffling to prevent convergence stalling when efficiency is close to optimal. It means allocation and deallocation bandwidth are executed at the same time. The shuffled traffic is:

$$h = \max(0, \gamma \cdot y - |\phi|)$$

where y is the input traffic in an average RTT and γ is a constant set to 0.1. This equation assures that at least 10% of traffic is redistributed according to AIMD at every average RTT.

The feedback assigned to packet i is a combination of a positive feedback p_i and a negative feedback n_i .

$$H_feedback_i = p_i - n_i$$

The per-packet positive feedback is proportional to the square of the flow's RTT, and inversely proportional to its congestion window divided by its packet size. The per-packet negative feedback is proportional to the packet size multiplied by its flow's RTT.

$$p_i = \xi_p \frac{rtt_i^2 \cdot s_i}{cwnd_i} \quad (1)$$

$$n_i = \xi_n \cdot rtt_i \cdot s_i \quad (2)$$

where

$$\xi_p = \frac{h + \max(\phi, 0)}{d \cdot \sum_L \frac{rtt_i \cdot s_i}{cwnd_i}}$$

$$\xi_n = \frac{h + \max(-\phi, 0)}{d \cdot \sum_L s_i}$$

ξ_p and ξ_n are computed each control interval.

B. Low's Window-based Dynamic Model

Low derives a window-based dynamic model for XCP with multiple links and multiple flows. Consider a network with L links shared by N flows. Let R be the $L \times N$ routing matrix: $R_{li} = 1$ if flow i uses link l and 0 otherwise. Let $L(i)$ be the set of links in the path of flow i : $L(i) := \{l | R_{li} = 1\}$ and $I(l)$ be the set of flows that use link l : $I(l) := \{i | R_{li} = 1\}$. For flow i , four variables are defined:

$w_i(t)$: window size at time t , in packets.

τ_i : round-trip propagation (and fixed processing) delay.

$T_i(t)$: round-trip time (RTT) at time t .

$x_i(t) = w_i(t)/T_i(t)$: flow rate at time t .

For link l , four variables are defined:

n_l : the number of flows at link l .

C_l : link capacity, in packets/sec.

$b_l(t)$: backlog at time t , in packets.

$y_l(t) = \sum_i R_{li} x_i(t)$: aggregate input rate at link l at time t .

The window-based dynamic model of XCP is described by the following equations:

$$\dot{w}_i(t) = \frac{w_i(t)}{d^2} \min_{l \in L(i)} F_{li}(t)$$

$$\dot{b}_l(t) = \begin{cases} y_l(t) - C_l & \text{if } b_l(t) > 0 \\ \max(y_l(t) - C_l, 0) & \text{if } b_l(t) = 0 \end{cases}$$

where

$$F_{li}(t) = \frac{h_l(t) + \phi_l^+(t)}{n_l x_i(t)} - \frac{h_l(t) + \phi_l^-(t)}{y_l(t)}$$

$$\phi_l(t) = \alpha d (C_l - y_l(t)) - \beta b_l(t)$$

$$h_l(t) = \max(\gamma d y_l(t) - |\phi_l(t)|, 0)$$

$$x_i(t) = \frac{w_i(t)}{T_i(t)}$$

$$y_l(t) = \sum_i R_{li} x_i(t)$$

$$T_i(t) = \tau_i + \sum_l R_{li} \frac{b_l(t)}{C_l}$$

The complicated mathematic description prevents us from analyzing the equilibrium performance of XCP intuitively. In other words, the window-based model is not easily to be used.

C. Simple Rate-based Dynamic Model

In this section, we derive a simple rate-based dynamic model to study the equilibrium performance of XCP. We also do not consider the feedback delay because we only care about the equilibrium performance.

From the previous section of XCP, the aggregate positive change in one control interval is $h_l + \max(\phi_l, 0)$, and the aggregate negative change in one control interval is $h_l + \max(-\phi_l, 0)$ for any link l . It is interesting to see the values of aggregate positive and negative feedbacks of link l based on the relationship of ϕ_l and γy_l .

$$P_l = h_l + \max(\phi_l, 0) = \max(0, \gamma y_l - |\phi_l|) + \max(\phi_l, 0)$$

$$= \begin{cases} 0 & \phi_l \leq -\gamma y_l \\ \gamma y_l - |\phi_l| & -\gamma y_l < \phi_l \leq 0 \\ \gamma y_l & 0 < \phi_l \leq \gamma y_l \\ \phi_l & \gamma y_l < \phi_l \end{cases}$$

$$N_l = h_l + \max(-\phi_l, 0) = \max(0, \gamma y_l - |\phi_l|) + \max(-\phi_l, 0)$$

$$= \begin{cases} |\phi_l| & \phi_l \leq -\gamma y_l \\ \gamma y_l & -\gamma y_l < \phi_l \leq 0 \\ \gamma y_l - \phi_l & 0 < \phi_l \leq \gamma y_l \\ 0 & \gamma y_l < \phi_l \end{cases}$$

Thus, the aggregate feedback for link l is

$$P_l - N_l = \begin{cases} -|\phi_l| & \phi_l \leq -\gamma y_l \\ -|\phi_l| & -\gamma y_l < \phi_l \leq 0 \\ \phi_l & 0 < \phi_l \leq \gamma y_l \\ \phi_l & \gamma y_l < \phi_l \end{cases}$$

The simultaneous allocation and deallocation of bandwidth do not change the aggregate feedback ϕ_l at link l . Therefore, it shows that the efficient controller (EC) is decoupled from the fairness controller (FC).

Assume the packets from the same flow have the same $cwnd_i$, s_i , and rtt_i during a sufficiently small time interval dt . The number of flow i 's packets during the time interval dt is

$$num_i = \frac{cwnd_i \cdot dt}{rtt_i \cdot s_i}$$

where $cwnd_i$ is the flow's congestion window size, s_i is the packet size, rtt_i is the flow's round trip time. Thus, the flow i 's aggregate positive congestion window feedback for the time interval dt at link l is derived from (1):

$$p_{ia,l} = p_{i,l} \cdot num_i = dt \cdot rtt_i \cdot \xi_{pl}$$

where $p_{i,l}$ is p_i at link l . Furthermore, the flow i 's aggregate positive throughput feedback during the interval dt at link l is:

$$p_{ithrough,l} = \frac{p_{ia,l}}{rtt_i} = dt \cdot \xi_{pl}$$

Similarly, the flow i 's aggregate negative congestion window and throughput feedbacks can be obtained respectively for the interval dt at link l .

$$n_{ia,l} = n_{i,l} \cdot num_i = dt \cdot cwnd_i \cdot \xi_{nl}$$

$$n_{ithrough,l} = \frac{n_{ia,l}}{rtt_i} = dt \cdot \frac{cwnd_i}{rtt_i} \cdot \xi_{nl}$$

The change of the flow i 's congestion window size dw_i is equal to the minimum aggregate congestion window feedback during the interval dt along the path.

$$dw_i = \min_{l \in L(i)} \{dt \cdot rtt_i \cdot \xi_{pl} - dt \cdot cwnd_i \cdot \xi_{nl}\}$$

Assume the flow i 's throughput x_i is equal to $\frac{cwnd_i}{rtt_i}$. The change of the flow's throughput dx_i is equal to the aggregate throughput feedback during the interval dt . Thus,

$$dx_i = \frac{dw_i}{rtt_i} = \min_{l \in L(i)} \{dt \cdot (\xi_{pl} - x_i \cdot \xi_{nl})\}$$

Therefore, the equation can be written as:

$$\frac{dx_i(t)}{dt} = \min_{l \in L(i)} \{\xi_{pl} - x_i(t) \cdot \xi_{nl}\} \quad (3)$$

This shows that XCP implements the AIMD algorithm for each flow at each link. In the equilibrium, all flows bottlenecked in the same link have the same flow rate, and the flows bottlenecked in other links cause under-utilization problem for this link.

Since XCP is a window-based congestion control protocol, some variables are redefined to match the rate-based model. We redefine the parameter $\phi_{new} = \phi/d$ as the aggregate throughput feedback.

$$\phi_{new} = \alpha \cdot S - \beta \cdot \frac{Q}{d}$$

And we redefine the parameter $y_{new} = y/d$ as the aggregate flow rate. In the following sections, we still use the notation ϕ to represent the aggregate throughput feedback and y to represent the aggregate flow rate for convenience. Then, P and N represent the aggregate positive and negative throughput feedbacks respectively. From (3) we know,

$$P_l = n_l \cdot \xi_p$$

$$N_l = \sum_{i \in I(l)} x_i(t) \cdot \xi_n = y_l \cdot \xi_n$$

where y_l is the aggregate flow rate and n_l is the number of flows at link l .

In summary, the simple rate-based dynamic model is described by the following equations.

$$\frac{dx_i(t)}{dt} = \min_{l \in L(i)} \{\xi_{pl} - x_i(t) \cdot \xi_{nl}\} \quad (4)$$

where

$$\xi_{pl} = \begin{cases} 0 & \phi_l \leq -\gamma y_l \\ \frac{\gamma y_l - |\phi_l|}{n_l} & -\gamma y_l < \phi_l \leq 0 \\ \frac{\gamma y_l}{n_l} & 0 < \phi_l \leq \gamma y_l \\ \frac{\phi_l}{n_l} & \gamma y_l < \phi_l \end{cases} \quad (5)$$

$$\xi_{nl} = \begin{cases} \frac{|\phi_l|}{y_l} & \phi_l \leq -\gamma y_l \\ r & -\gamma y_l < \phi_l \leq 0 \\ \frac{\gamma y_l - \phi_l}{y_l} & 0 < \phi_l \leq \gamma y_l \\ 0 & \gamma y_l < \phi_l \end{cases} \quad (6)$$

Note: The flow i 's rate is determined by the bottleneck link. This means that we only need to consider the bottleneck link for flow i to determine the flow i 's rate.

D. Simple analysis of link utilization

In this section, we analyze the XCP's equilibrium performance briefly. Let us consider a link l . The flows traversing through link l are classified into three sets. One set denoted by φ contains the flows which are bottlenecked at the downstream links. Another set denoted by χ contains the flows which are bottlenecked at the upstream links. The last set denoted by ψ contains the flows which are bottlenecked at link l itself.

Furthermore, all links can be classified into five typical classes based on the flows traversing through.

Class 1: $\psi = \emptyset$; χ do not care; φ do not care
 All flows are bottlenecked elsewhere. Since the router can not allocate any spare bandwidth to flows, it is not meaningful to discuss the link utilization in this case. The link utilization can be arbitrarily small as the link bandwidth is arbitrarily large when all flows are controlled by other routers.

Class 2: $\psi \neq \emptyset$; $\chi = \emptyset$; $\varphi = \emptyset$

In the equilibrium, all flows in set ψ have the flow rate:

$$x_i = \frac{\xi_{pl}}{\xi_{nl}}$$

From the equation (7),

$$y_l = N_l \cdot x_i = \frac{\gamma y_l^2}{\gamma y_l - \alpha \cdot (C_l - y_l)}$$

Thus, the aggregate flow rate y_l is equal to the link capacity C_l , and the link is fully utilized. Furthermore, the per-flow rate is $x_i = C_l/N_l$ and the max-min fairness is achieved.

Class 3: $\psi \neq \emptyset$; $\chi = \emptyset$; $\varphi \neq \emptyset$

All flows are bottlenecked in the downstream links and the current link. In the equilibrium the feedback for flows in set ψ is

$$\xi_{pl} - x_i \cdot \xi_{nl} = 0 \quad i \in \psi$$

It is obvious that the flows in set ψ have a higher flow rate than the flows in set φ . Otherwise, the flows will be bottlenecked in the current link. Thus, the feedback for flows in set φ is

$$\xi_{pl} - x_j \cdot \xi_{nl} > 0 \quad j \in \varphi$$

The feedback for flows $j \in \varphi$ is eliminated at the downstream links, and the under-utilization problem of XCP appears. The detailed discussion is given in the following section.

Class 4: $\psi \neq \emptyset$; $\chi \neq \emptyset$; $\varphi = \emptyset$

All flows are bottlenecked in the upstream links and the current link. In the equilibrium, the feedback for flows in set χ is 0, but the current link still allocates feedbacks to flows in set χ . In fact, XCP treats the flows in set χ the same as the flows in set ψ due to the AIMD. It causes the same under-utilization problem as that of Class 3.

Since the flows in set χ can be detected at the current link in the equilibrium, Katabi [4] tries to alleviate the negative effects caused by the flows in set χ with the pseudo code in Fig. 1.

```

feedback = pos_fbk - neg_fbk
if (H_feedback ≥ feedback) then
  .....
else
  if (H_feedback ≥ 0)
    residue_pos_fbk := H_feedback / H_rtt
    residue_neg_fbk := (feedback - H_feedback) / H_rtt
  else
    residue_pos_fbk += H_feedback / H_rtt
    if (feedback ≥ 0) then residue_neg_fbk := feedback / H_rtt
  
```

Fig. 1. Compensation pseudo code of XCP

The compensation policy affects the equilibrium, but it only increases the link utilization by several percentage points. The deterministic factor is still the derivative equation which is the description of XCP. The detailed analysis of the above compensation policy is a topic for further research.

Class 5: $\psi \neq \emptyset$; $\chi \neq \emptyset$; $\varphi \neq \emptyset$

In the equilibrium, the link utilization should be not worse than that of Class 3 and not better than that of Class 4 if the compensation policy is adopted. Otherwise, it is same as Class 3.

In summary, we call the network analysis Downstream case if the compensation policy is not adopted. Otherwise, we call it Upstream case. In Low's analysis, the compensation policy is not considered. The flows bottlenecked at the upstream links and downstream links are treated equally according to AIMD. Therefore, Low's analysis is same as our analysis of Downstream case. In the following section, we focus on studying Downstream case and give some interesting analysis.

III. LINK UTILIZATION OF DOWNSTREAM CASE

If all flows are bottlenecked at other links and link l can not allocate any spare bandwidth to any flows, it is not meaningful to analyze the link utilization in this case since the link utilization can be arbitrarily small as the link capacity approaches to infinite. Thus, we have an important assumption for all of our analysis.

Assumption: Assume the set ψ of link l is not empty in all analysis.

A. Simple and Important Results

In this section, we derive some simple and important results of XCP.

Lemma 1: In the equilibrium, $0 \leq \phi_l \leq \gamma y_l$.

Proof: In the equilibrium, assume $\gamma y_l < \phi_l$, the aggregate positive feedback $P_l = \phi_l$ and the aggregate negative feedback $N_l = 0$. Thus, the negative feedback for all flows is 0. However, the flows in set ψ satisfy

$$\frac{dx_i}{dt} = \xi_{pl} > 0$$

It means that these flows are not in the equilibrium. By contradiction, we know that $\phi_l \leq \gamma y_l$.

If $\phi_l < 0$, the feedback filled in the packet header is negative no matter whether the flow is bottlenecked in the upstream link or this link. Moreover, downstream links can replace this negative feedback only if they have more negative feedback. Thus, the final feedback is always negative and the flow rate has to be decreased. It means that the flows are not in the equilibrium. We conclude that $0 \leq \phi_l \leq \gamma y_l$. ■

Lemma 2: In the equilibrium, the queue Q_l is empty and the aggregate throughput feedback is $\phi_l = \alpha \cdot (C_l - y_l)$.

Proof: In the equilibrium, the aggregate flow rate is no more than the link capacity. Otherwise, the flows can not achieve the equilibrium.

If the aggregate flow rate y_l is less than the link capacity C_l , it is obviously that the queue Q_l is empty. If the aggregate

flow rate y_l is equal to the link capacity C_l , we assume the queue Q_l is not empty in the equilibrium. Then

$$\phi_l = \alpha \cdot (C_l - y_l) - \beta \cdot \frac{Q_l}{d} < 0$$

Thus, the flows are not in the equilibrium from Lemma 1. We conclude that Q_l is empty. Furthermore, the aggregate throughput feedback is $\phi_l = \alpha \cdot (C_l - y_l)$. ■

Theorem 1: In the equilibrium, if all links have $\gamma \rightarrow 0$, the network approaches the max-min fairness.

Proof: We only need to consider the bottlenecks in the network. From lemma 1 and 2, we know that set ψ of link l is not empty and link l has the aggregate feedback $\phi_l = \alpha(C_l - y_l) \rightarrow 0$ as $\gamma \rightarrow 0$. Then, link l is fully utilized and all flows bottlenecked at link l have the same flow rate $x_l = \xi_{pl}/\xi_{nl}$ which is also the maximum allowable flow rate at link l . Moreover, x_l can not be increased without decreasing the rates of flows which are not bottlenecked at link l because x_l is the maximum flow rate and link l is fully utilized. Similarly, every bottleneck link works like link l . Thus, the network approaches the max-min fairness. ■

Note: If we substitute $r = 0$ to Low's derived common flow rate formula, the max-min fairness can also be proven.

B. Reproduce Low's Results

Low derives a dynamic window-based model for XCP and analyzes the link utilization and the fairness. It gives many encouraging results. To compare with Low's results, we use the same notations in our following analysis.

N_l : the number of flows at link l

N_{l0} : the number of flows bottlenecked at other routers

y_{l0} : the aggregate flow rate bottlenecked at other routers

$\rho_l := N_{l0}/N_l$

$\sigma_l := y_{l0}/C_l$

Since the compensation policy is not considered here, the flows bottlenecked at the upstream links and downstream links are treated equally by XCP. We can use the above notations without differentiating the upstream and downstream bottlenecked flows in the theoretical analysis.

Let us see Low's work first. The formula given by Low to calculate the equilibrium flow rate bottlenecked at link l is based on the following equation.

$$\frac{y_l - y_{l0}}{N_l - N_{l0}} = r^l = \frac{\gamma \cdot y_l^2}{N_l \cdot (\gamma y_l - \alpha(C_l - y_l))}$$

This is a key equation in Low's paper. Most of his results are obtained from this equation directly or indirectly, such as the aggregate flow rate y_l , the equilibrium flow rate x_r at link l , and the upper and lower bounds of link utilization.

In our model, the same formula can be easily derived for Downstream case. The equilibrium flow rate bottlenecked at link l is

$$x_r = \frac{\xi_{pl}}{\xi_{nl}}$$

where

$$\xi_{pl} = \frac{\gamma \cdot y_l}{N_l}$$

$$\xi_{nl} = \frac{\gamma \cdot y_l - \alpha(C_l - y_l)}{y_l}$$

Thus,

$$\frac{y_l - y_{l0}}{N_l - N_{l0}} = x_r = \frac{\gamma \cdot y_l^2}{N_l \cdot (\gamma y_l - \alpha(C_l - y_l))} \quad (7)$$

which is same as Low's formula. Therefore, most of Low's results can be reproduced by applying our model.

C. γ and α

1) *The relationship of γ and α* : Low does not give a constraint on the relationship of γ and α , but his simulations show that the link utilization is very low if γ/α approaches to a large value. In fact, the shuffling parameter γ and the parameter α can not be chosen arbitrarily. We show that the shuffling parameter γ is no more than the parameter α to achieve high link utilization in this section.

Let us see the relationship between $x_r N_l$ and the link capacity C_l first where x_r represents the common flow rate bottlenecked at link l . If all flows belong to set ψ , we can get $x_r N_l = C_l$ which is easy to check. On the other hand, if some flows belong to set χ and set φ , the common flow rate x_r is obviously greater than the flow rate $x_j, j \in \chi, \varphi$. If $x_r N_l \leq C_l$, the under-utilization problem is obviously worse. It is necessary to guarantee $x_r N_l \geq C_l$.

Theorem 2: To guarantee $x_r N_l \geq C_l$ where x_r represents the common flow rate bottlenecked at the current link l , the shuffling parameter γ and the parameter α must satisfy $\gamma \leq \alpha \frac{C_l}{y_l}$. Furthermore, we choose $\gamma \leq \alpha$ to guarantee that $\gamma \leq \alpha \frac{C_l}{y_l}$.

Proof: In the equilibrium, the flows in set ψ have a common flow rate x_r , and the feedback of these flows is 0. Thus,

$$\xi_{nl} = \frac{\gamma \cdot y_l - \alpha(C_l - y_l)}{y_l}$$

$$\xi_{pl} = x_r \cdot \xi_{nl} = x_r \cdot \frac{\gamma \cdot y_l - \alpha(C_l - y_l)}{y_l}$$

The bandwidth wasted by the flows in set φ or set χ is equal to the allocated bandwidth ϕ_l in the equilibrium. Then,

$$\begin{aligned} \alpha(C_l - y_l) &= \sum_{j \in \varphi, \chi} (\xi_{pl} - x_j \cdot \xi_{nl}) \\ &= \frac{\gamma \cdot y_l - \alpha(C_l - y_l)}{y_l} \cdot \sum_{j \in \varphi, \chi} (x_r - x_j) \\ &= \frac{\alpha y_l + \gamma y_l - \alpha C_l}{y_l} \cdot (x_r N_{l0} - y_{l0}) \end{aligned}$$

Finally,

$$x_r N_{l0} - y_{l0} = \frac{\alpha}{\alpha + \frac{\gamma y_l - \alpha C_l}{y_l}} \cdot (C_l - y_l)$$

If $\gamma \leq \alpha \frac{C_l}{y_l}$, then

$$x_r N_{l0} - y_{l0} + y_l \geq C_l$$

Therefore,

$$x_r N_l \geq C_l$$

Furthermore, since we know that $y_l \leq C_l$, we choose $\gamma \leq \alpha$ to guarantee that $\gamma \leq \alpha \frac{C_l}{y_l}$. ■

2) *Choosing values of γ and α* : Low gives the lower and upper bounds of link l 's utilization:

$$\frac{\alpha}{\gamma \rho_l + \alpha} \leq \frac{y_l}{C_l} \leq 1 - \frac{\gamma \sigma_l (\rho_l - \sigma_l)}{\gamma \rho_l + \alpha}$$

From the control theorem, we know that the parameters γ and α are controllable, while the parameters ρ_l and σ_l are observable instead of controllable. Due to the diversity of ρ_l and σ_l , the relaxed bounds are derived only based on the controllable variables γ and α . Since the set ψ is not empty, ρ_l belongs to the interval $[0, 1)$. The relaxed bounds are given as

$$\frac{\alpha}{\alpha + \gamma} < \frac{y_l}{C_l} \leq 1$$

Let us define u as the desired link utilization, and u belongs to the interval $(0.5, 1]$ because $\gamma \leq \alpha$. Therefore, the chosen lower bound must be no less than u to guarantee the desired link utilization u . Thus, we get a relationship of γ and α .

$$\frac{\gamma}{\alpha} \leq \frac{(1 - u)}{u}$$

Fig. 2 shows the feasible values of γ/α for the given desired link utilization u .

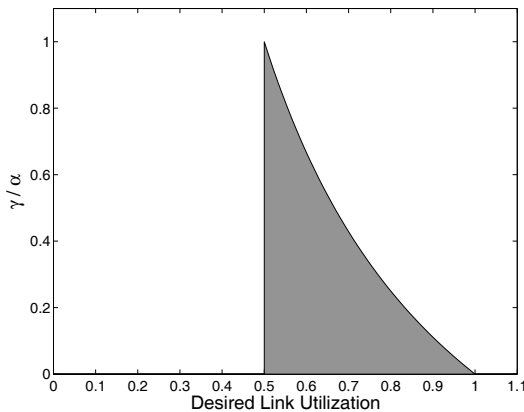


Fig. 2. Feasible values of $\frac{\alpha}{\gamma}$ for Desired Link Utilization

IV. CONCLUSION

In this paper, we focus on analysis of XCP's equilibrium performance. A simple rate-based model is proposed for analyzing the XCP's equilibrium performance. Most results that appeared in Low's paper can be easily reproduced by applying our simple dynamic model. Furthermore, we prove that the shuffling parameter γ is no more than the parameter α to achieve high link utilization, which is an important constraint for XCP.

Further research includes analyzing the effect of the compensation policy. The compensation policy increases the link utilization by several percentage points, and we try to find the rules behind it. Another topic is that the link utilization may have more stringent bounds provided with the new constraint $\gamma \leq \alpha$.

REFERENCES

- [1] V. Jacobson, "Congestion avoidance and control," in *Proc. ACM SIGCOMM '88*, Stanford, CA, USA, 1988, pp. 314–329.
- [2] A. Falk, T. Faber, J. Bannister, A. Chien, R. Grossman, and J. Leigh, "Transport protocols for high performance," *Communications of ACM*, vol. 46, no. 11, pp. 43–49, 2003.
- [3] F. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, vol. 49, no. 3, pp. 237–252, 1998.
- [4] D. Katabi, M. Handley, and C. Rohrs, "Congestion control for high bandwidth-delay product networks," in *Proc. ACM SIGCOMM'02*, Pittsburgh, PA, USA, 2002, pp. 89–102.
- [5] T. Kelly, "Scalable tcp: Improving performance in highspeed wide area networks," *ACM SIGCOMM Computer Communication Review*, vol. 33, no. 2, pp. 83–91, 2003.
- [6] C. Jin, D. Wei, and S. Low, "Fast tcp: Motivation, architecture, algorithms, and performance," in *Proc. IEEE Infocom*, Hongkong, 2004.
- [7] S. Floyd, "HighSpeed TCP for large congestion windows," RFC 3649, 2003.
- [8] S. H. Low, L. L. H. Andrew, and B. P. Wyrowski, "Understanding XCP: Equilibrium and fairness," in *Proc. IEEE Infocom*, Miami, FL, USA, 2005.